

# A method for the efficient computation of an exact state space abstraction for Petri nets with stopwatches

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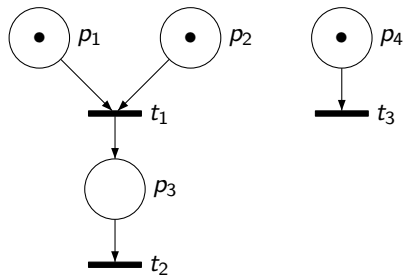
CISS – Aalborg University, DENMARK

27 August 2005

# Introduction

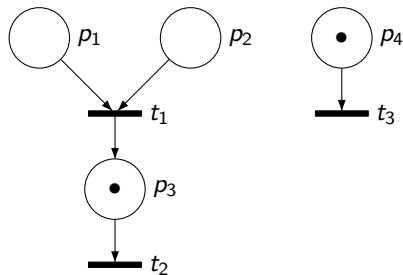
- ▶ Formal verification of real-time systems in the context of a **preemptive** scheduling policy
- ▶ Extensions of Time Petri Nets to model *stopped/resumed* actions
  - ▶ Scheduling Time Petri Nets [Roux, JESA'02]
  - ▶ Preemptive Time Petri Nets [Bucci, TSE'04]
  - ▶ **Inhibitor Hyperarcs Time Petri Nets** [Roux, ICATPN'04]
- ▶ “Everything” is undecidable (even for **bounded** nets)
- ▶ Computing abstractions requires **general convex polyhedra**
- ▶ Contributions
  - ▶ a necessary and sufficient condition for the need of polyhedra
  - ▶ an algorithm taking advantage of it
  - ▶ an implementation

# The Model: IHTPNs



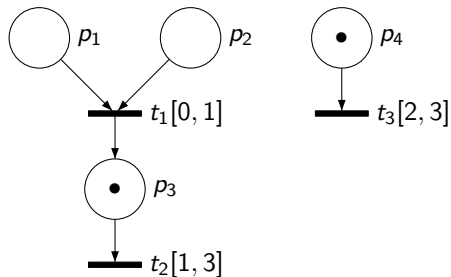
A Petri Net

# The Model: IHTPNs



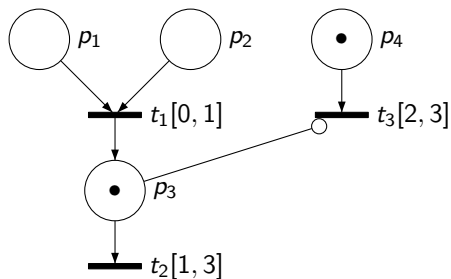
A Petri Net

# The Model: IHTPNs



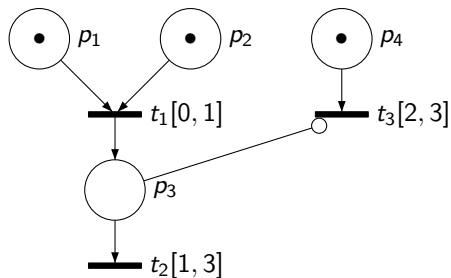
A **Time** Petri Net

# The Model: IHTPNs



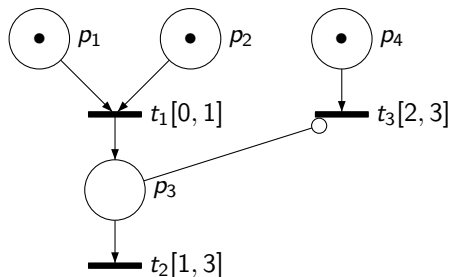
An **Inhibitor Hyperarc** Time Petri Net

## IHTPNs: Semantics



$$\begin{array}{l} \nu(t_1) = 0 \\ \nu(t_3) = 0 \end{array} \xrightarrow{0.7}$$

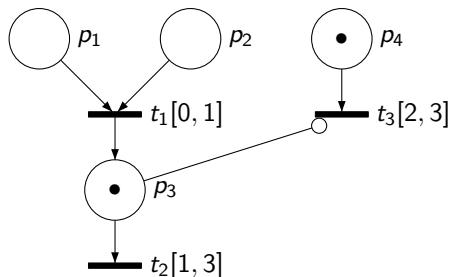
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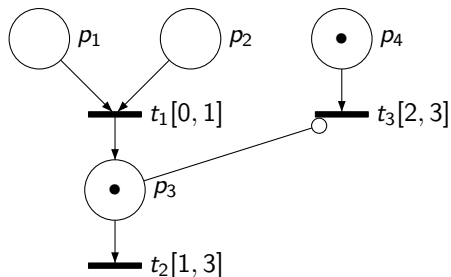


## IHTPNs: Semantics



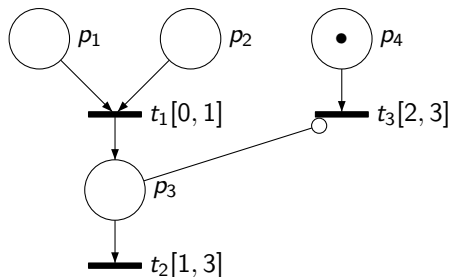
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 \end{array}
 \xrightarrow{1.4}$$

## IHTPNs: Semantics



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 \end{array}
 \xrightarrow{t_2}$$

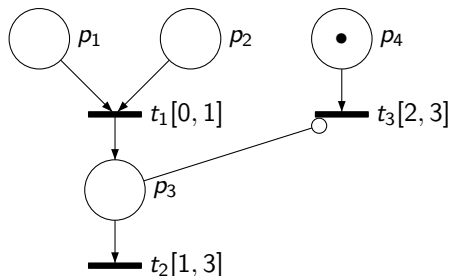
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$$\nu(t_3) = 0.7 \quad \xrightarrow{0.5}$$

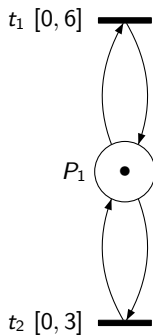
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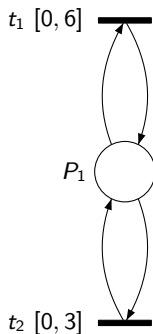
$$\nu(t_3) = 0.7 \quad \xrightarrow{0.5} \quad \nu(t_3) = 1.2 \quad \xrightarrow{t_3}$$

# About newly enabled transitions



We fire  $t_1$

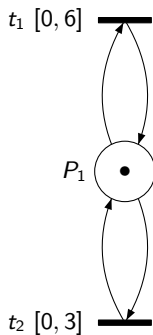
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$t_1$  and  $t_2$  are not enabled by  $M - \bullet t_1$

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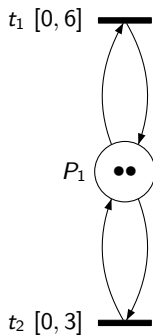


We fire  $t_1$

$t_1$  and  $t_2$  are not enabled by  $M - \bullet t_1$

$t_1$  and  $t_2$  are **newly** enabled

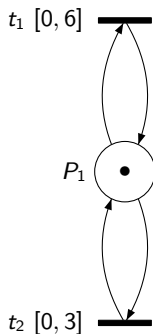
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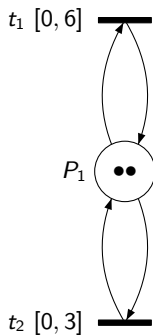
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$t_1$  and  $t_2$  are enabled by  $M - \bullet t_1$  but  $t_1$  is the fired transition  
 $t_2$  **remains** enabled,  $t_1$  is **newly** enabled

# Abstractions

- ▶ **Infinite** state-space  $\Rightarrow$  Abstractions
- ▶ TPNs: Zone-based simulation graph [Gardey, TPLP'05]
- ▶ TPNs: **State class graph** [Berthomieux, TSE'91]
- ▶ TPNs w/ stopwatches (IHTPNs, . . .): **State class graph** [Lime, FET'03, Roux, ICATPN'04, Bucci, TSE'04]

# Basic Algorithm

**begin**

$Passed = \emptyset$

$Waiting = \{C_0\}$

**while**  $Waiting \neq \emptyset$

$C = \text{pop}(Waiting)$

$Passed = Passed \cup C$

**for**  $t$  firable from  $C$

$C' = \text{AbstractSuccessor}(C, t)$

**if**  $C' \notin Passed$

$Waiting = Waiting \cup C'$


**end if**

**end for**

**end while**

**end**

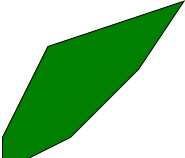
## State Class

$$C = \left\{ \left( \begin{array}{c} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{array} \right), \text{  \right\}$$

TPNs: Zone (encoded by a Difference Bound Matrix (DBM)  $[d_{ij}]_{i,j \in [0..n]}$ ):

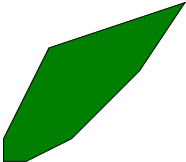
$$\begin{cases} -d_{0i} \leq \theta_i - \mathbf{0} \leq d_{i0}, \\ \theta_i - \theta_j \leq d_{ij} \end{cases}$$

## State Class

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IHTPNs: General polyhedron:  $A\bar{\Theta} \leq B$

# Over-approximation

$$C = \left\{ \left( \begin{array}{c} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{array} \right), \text{  \right\}$$

Over-approximation using the **smallest englobing** zone

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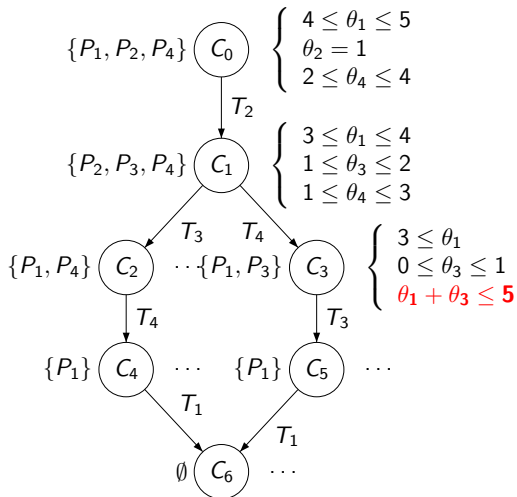
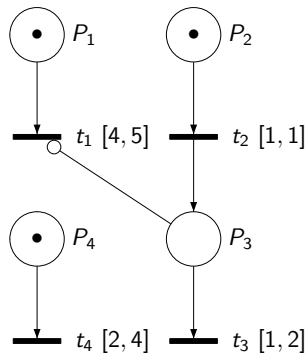
# Computing the state class graphs

Let  $C = (M, D)$  and  $D = (A.\Theta \leq B)$ . We fire  $t_f$ .

- ▶  $M' = M - \bullet t_f + t_f \bullet$
- ▶  $D'$  is computed by:
  - ▶ for all **non-inhibited** enabled transitions  $t_i$ , constrain by  $\theta_f \leq \theta_i$
  - ▶ for all **non-inhibited** enabled transitions  $t_i$ ,  $\theta'_i = \theta_i - \theta_f$
  - ▶ eliminate variables for **disabled** transitions (e.g. using Fourier-Motzkin method)
  - ▶ add new variables for **newly** enabled transitions  $t_i$ :

$$\alpha(t_i) \leq \theta_i \leq \beta(t_i)$$

## Example



# A Comparison

For **bounded** IHTPNs ( $n$ : number of clocks *i.e.* of enabled transitions):

	Memory	Time	properties
Exact comp.	$O(\exp(n))$	$O(\exp(n))$	LTL/CTL
Over-appr.	$O(n^2)$	$O(n^3)$	LTL/CTL ( <b>safety</b> )

	#markings	#polyhedra
Exact comp.	<b>bounded</b>	<b>unbounded</b>
Over-appr.	<b>unbounded</b>	<b>bounded</b>

## The Way of the Middle (1/2)

For IHTPNs:

- ▶ The polyhedron in the **initial** state class is **always a zone**.
- ▶ The successor of a non-zone polyhedron might be a zone
- ▶ The successor of a zone might not be a zone

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For IHTPNs:

- ▶ The polyhedron in the **initial** state class is **always a zone**.
- ▶ The successor of a non-zone polyhedron might be a zone (**easy to check**:  $O(n^2)$ )
- ▶ The successor of a zone might not be a zone (**not so easy to check**)

**We want to start to compute with zones and fall back to general polyhedra when needed (and return to zones asap).**

## Abstractions: The Way of the Middle (2/2)

$D = [d_{ij}]_{i,j \in [0..n]}$  and  $(M', D') = \text{AbstractSuccessor}((M, D), t_f)$ .

$D'$  is **not a zone** iff there are at least three enabled transitions  $t_i, t_j, t_k$  in  $D$  such that:

1.  $t_i, t_j, t_k$  are not disabled when firing  $t_f$
2.  $t_i$  and  $t_k$  are not inhibited
3.  $t_j$  is inhibited
4.  $d_{j0} + d_{ki} > d_{k0} + d_{ji}$  or  $d_{0j} - d_{ik} < d_{0k} - d_{ij}$

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4.  $d_{j0} + d_{ki} > d_{k0} + d_{ji}$  or  $d_{0j} - d_{ik} < d_{0k} - d_{ij}$

**Complexity:**  $O(n^3)$



# Experimental results

Scheduling-TPN	Overapproximation			Exact computation					
				Polyhedral algo			Mixed algo		
	Time	Nodes	Transitions	Time	Nodes	Transitions	Time	Nodes	Transitions
Example 1	6.63 s	2260	5700	33.51 s	2260	5700	6.69 s	2260	5700
Example 2	18.88 s	11167	25856	80.9 s	11167	25856	19.08 s	11167	25856
Example 3	6.45 s	2225	5371	23.51 s	2225	5371	6.61 s	2225	5371
Example 4	85.52 s	15178	49135	NA	NA	NA	85.59 s	15178	49135
Example 5	99.66 s	16323	54688	NA	NA	NA	99.73 s	16323	54688
Example 6	NA	NA	NA	0.19 s	19	24	0.19 s	19	24
Example 7	NA	NA	NA	26.92 s	4528	8699	13.24 s	4528	8699
Example 8	NA	NA	NA	NA	NA	NA	115.11 s	16650	32865
Example 9	1.72	478	1137	NA	NA	NA	NA	NA	NA

# A Tool: Romeo

## Features:

- ▶ **Graphical** editing of (Stopwatch) Time Petri Nets
- ▶ Symbolic **simulation**
- ▶ (A fragment of) **TCTL** Model-checking
- ▶ **State-space** computations
  - ▶ timed automata-like
  - ▶ State class-based (for stopwatches: Over-approximation with DBM, exact with polyhedra, exact with mixed method)
- ▶ **Translations** to Timed/Stopwatch Automata
  - ▶ State-space based
  - ▶ Structural

# Conclusion and Further Work





## Summary:

- ▶ An **exact** but **efficient** computation of the state class abstraction for TPN with **stopwatches**
- ▶ **Polynomial** checks for switching from DBMs to polyhedra (and the other way around)




## Further work:

- ▶ Adaptation to timed automata-like forward computation
- ▶ Model checking based on this method
- ▶ Investigation of discrete time

## References

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-  Bucci, G., A. Fedeli, L. Sassoli and E. Vicario, *Time state space analysis of real-time preemptive systems*, *IEEE transactions on software engineering* **30** (2004), pp. 97–111.
-  Berthomieu, B. and M. Diaz, *Modeling and verification of time dependent systems using time Petri nets*, *IEEE transactions on software engineering* **17** (1991), pp. 259–273.

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-  Gardey, G., Roux, O., Roux, O., *State space computation and analysis of time Petri nets*. Theory and Practice of Logic Programming (TPLP). Special Issue on Specification Analysis and Verification of Reactive Systems (2005) to appear.
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