Computation of Generalized Aspect of Parallel Manipulators

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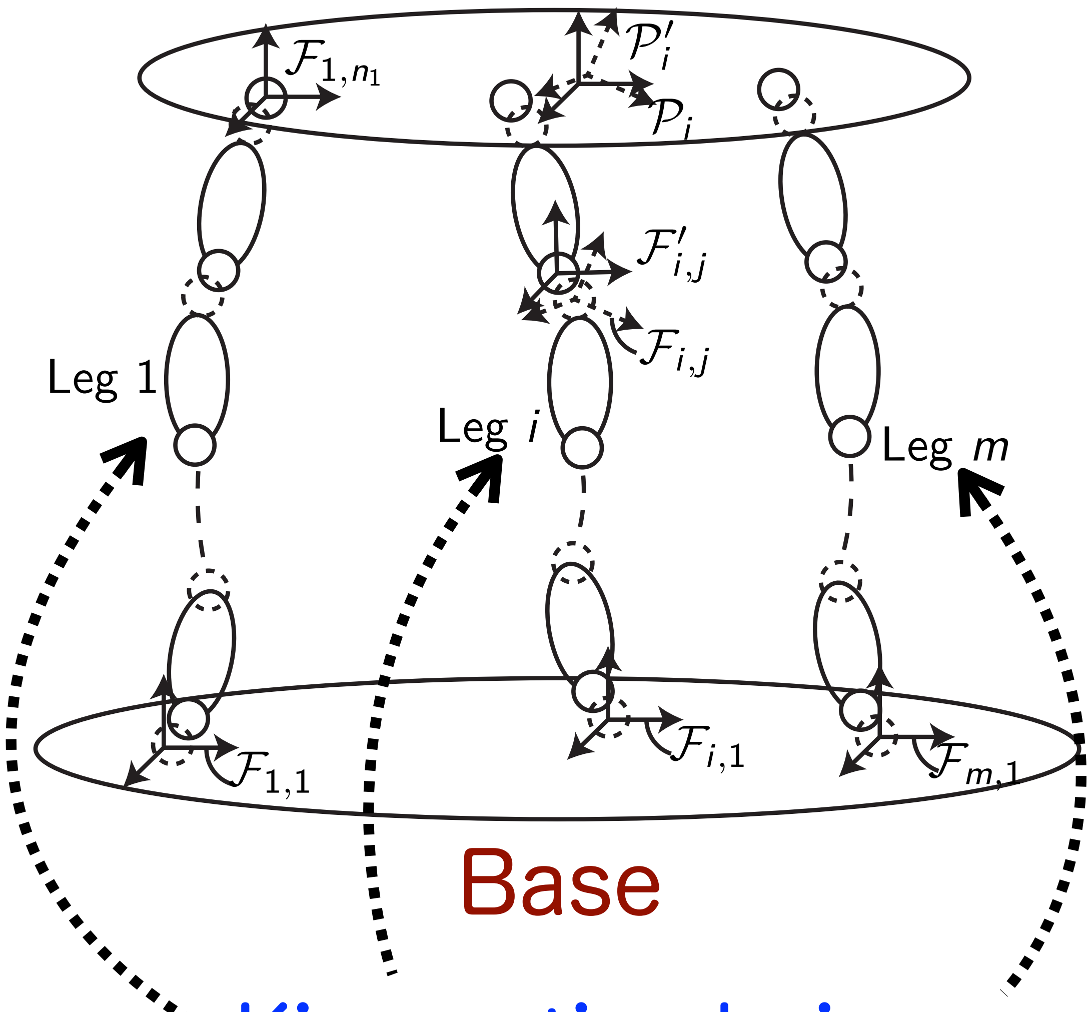
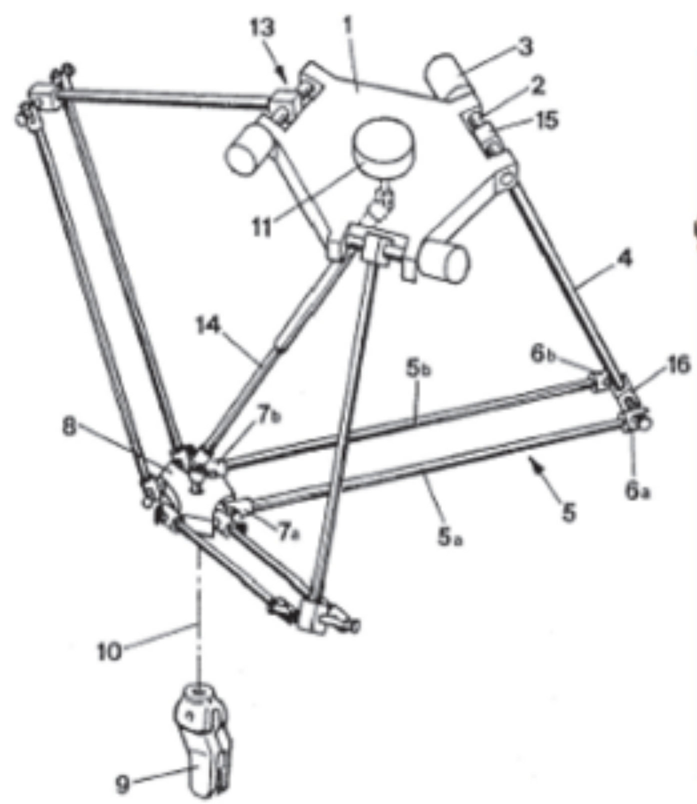
Parallel Mechanism (Manipulator)

- **Closed loop mechanism** in which the end-effector is connected to the base by at least two independent **kinematic chains**

End-effector

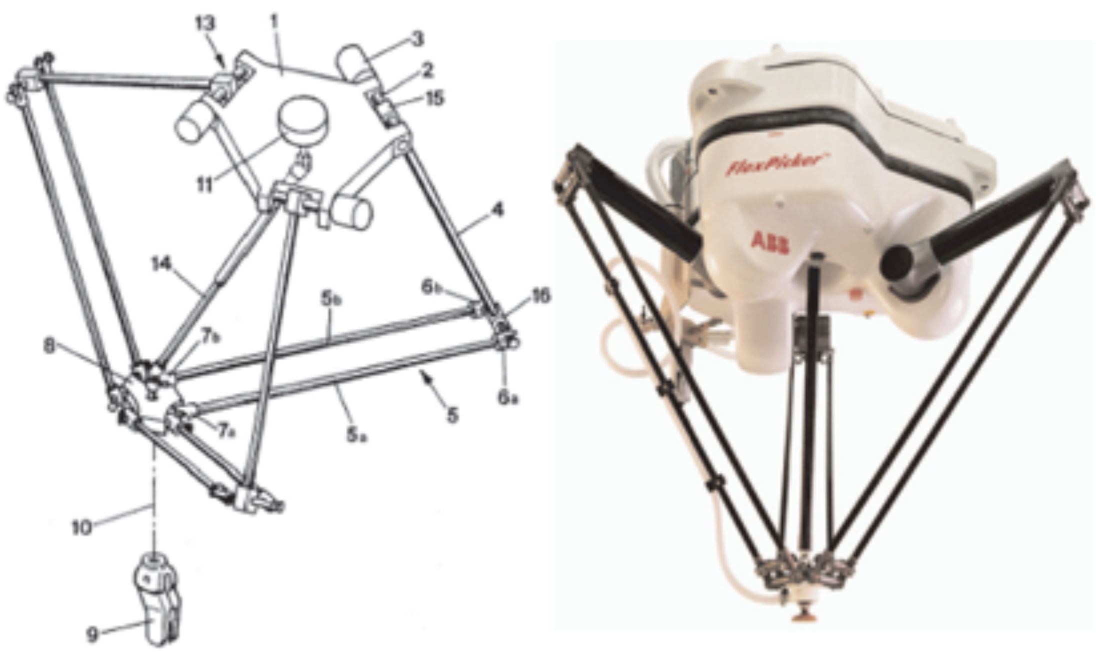


Kinematic chains: Coupling of links via kinematic pairs



Delta robot [80]

Parallel vs. Serial Manipulators



Delta robot

Puma robot

	Parallel manip.	Serial manip.
Kinematic chain(s)	Closed	Opened
Workspace	Limited	Large
Accuracy	Good	Low
Payload	High	Low
Stiffness	High	Low

Aspect Computation

- **Parallel manipulators** may have multiple inverse and direct kinematic solutions
  - A given end-effector pose
    → several control inputs
  - A given control input
    → several end-effector poses

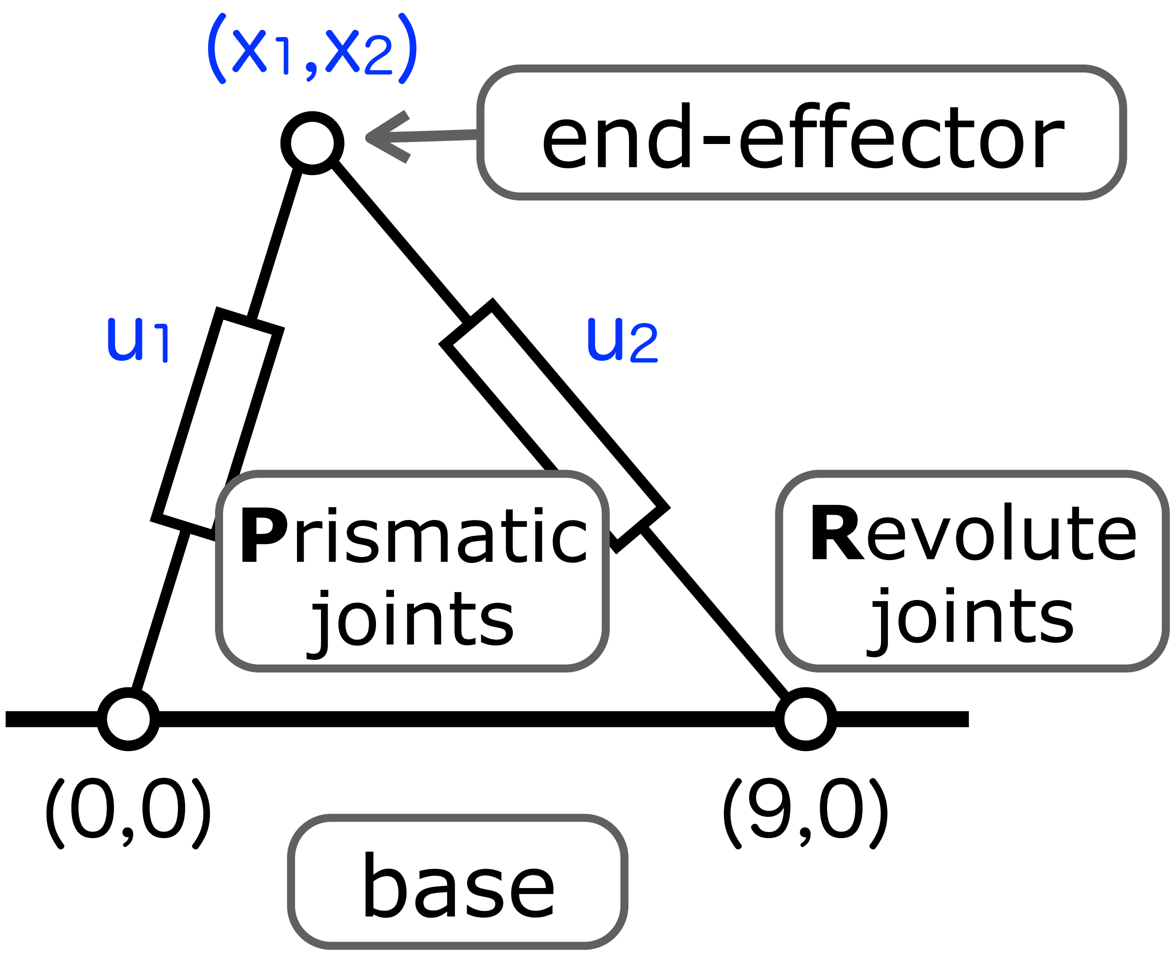
- **Domain with multiple solutions contains singular solutions**

- **Aspect** [Chablat, 2007]:
  Maximal singularity-free region within the domain

➡ Our aim: **Rigorous computation of aspects**

Example: 2-RPR Manipulator

- Inputs:
  - Variables
    - Control Variables: u_1, u_2
    - Pose Variables: x_1, x_2
  - Initial domain:
    $u_1 \in [2, 6]$, $u_2 \in [3, 9]$,
    $x_1, x_2 \in [-20, 20]$



  - Model:

$$f(u,x) = \begin{pmatrix} u_1^2 - (x_1^2 + x_2^2) \\ u_2^2 - ((x_1 - 9)^2 + x_2^2) \end{pmatrix} = 0$$

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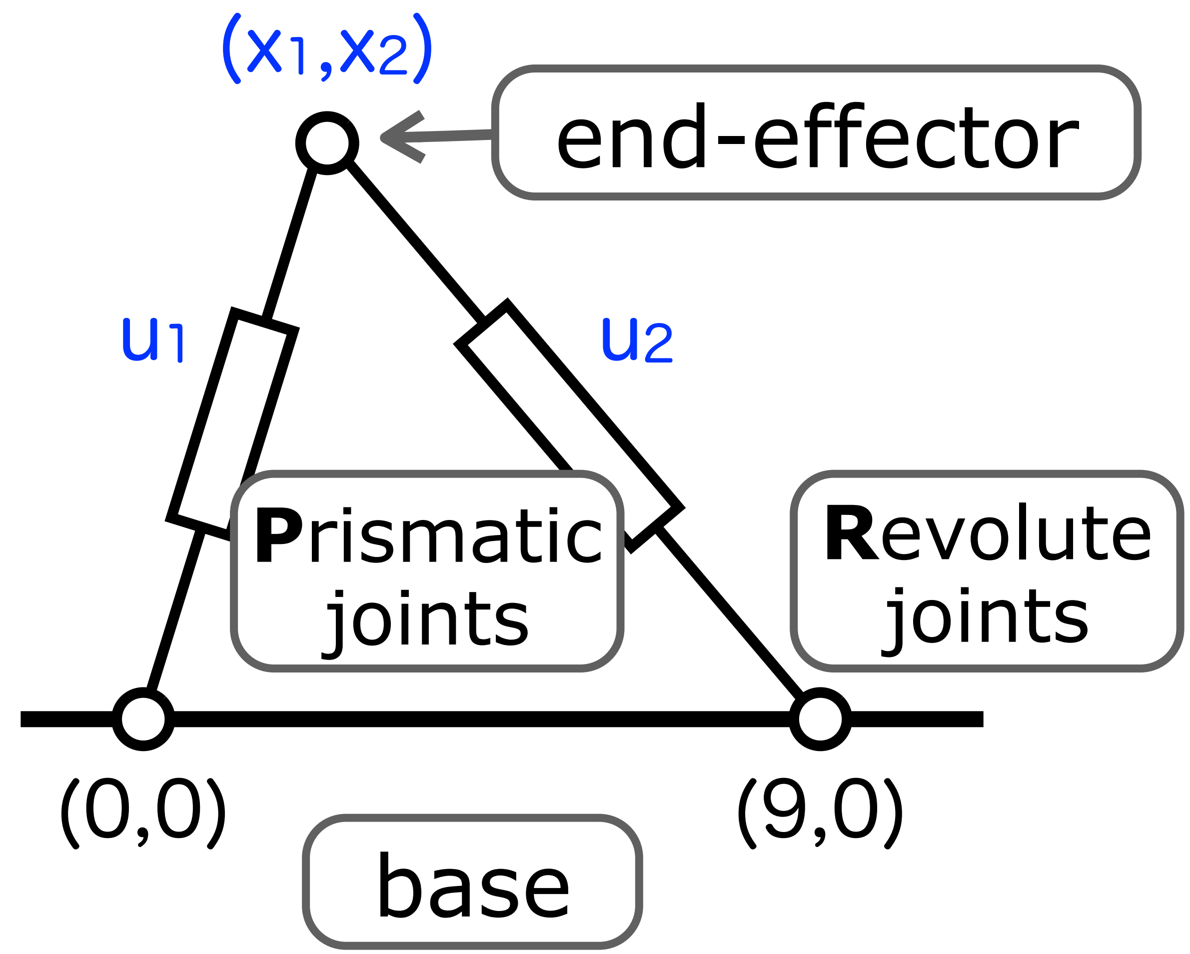

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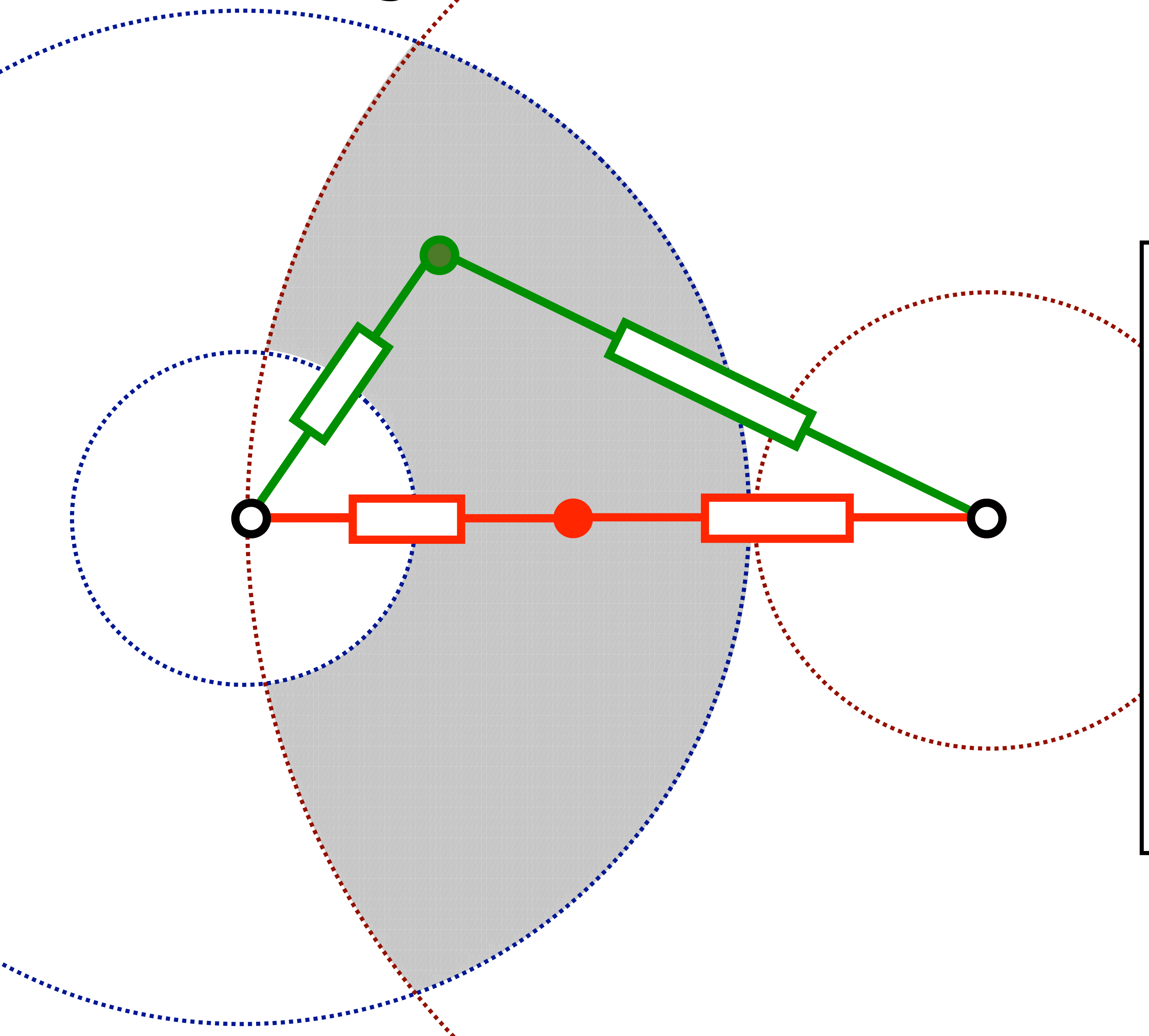
Example: 2-RPR Manipulator

- **Safe configuration**
- **Singular configuration**
  - Change of control variables ⇒ robot breakdown



Algebraic characterization of singularity:

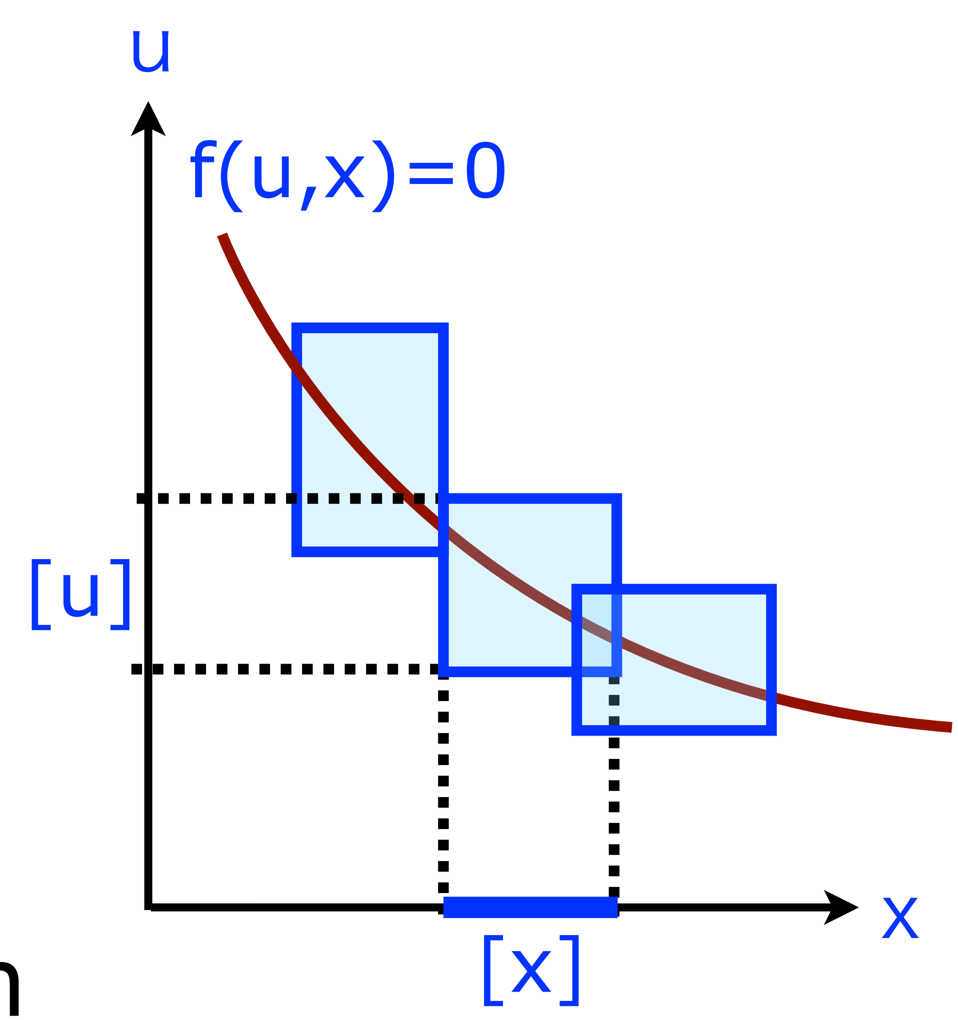
$\det D_x f(u,x) = 0$

or

$\det D_u f(u,x) = 0$

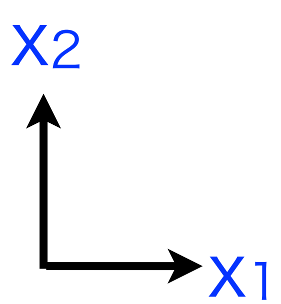
Singularity Free Connected Components (SFCCs)

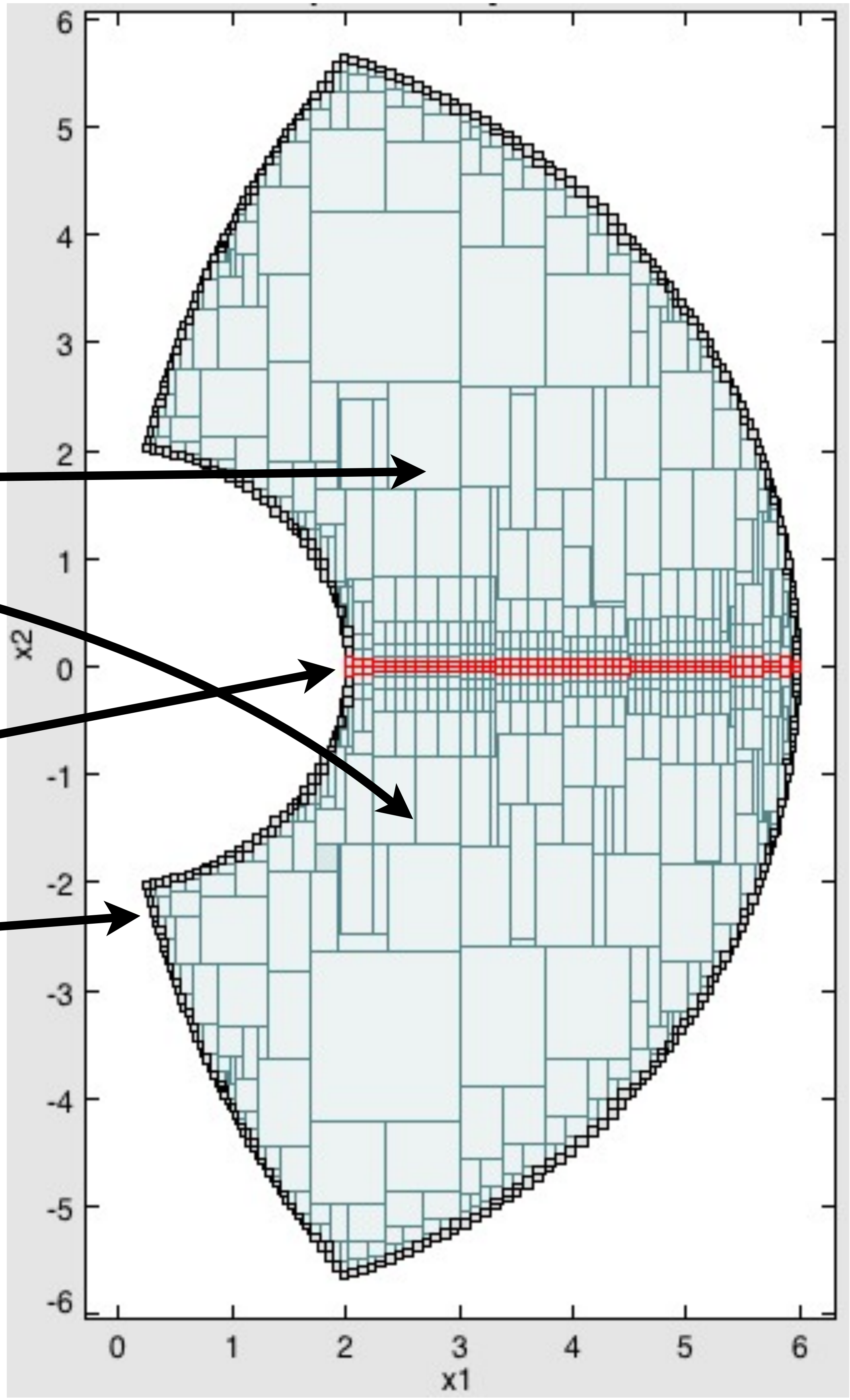
- Consider a manipulator modeled with 2n variables $(u, x) \in \mathbf{R}^{2n}$
- An **SFCC** is a set of boxes $[u]\times[x] \in \mathbf{IR}^{2n}$ that are
  - connected
  - not containing any singular configuration
  - proved to contain configurations
- **SFCC is an inner approximation of aspect**
  - Robot can move safely within the x projection of a given SFCC



Example: 2-RPR Manipulator

- Output:
  - **2 SFCCs**
    (projected on the workspace)
  - Possibly singular region
  - Uncertified reachable region



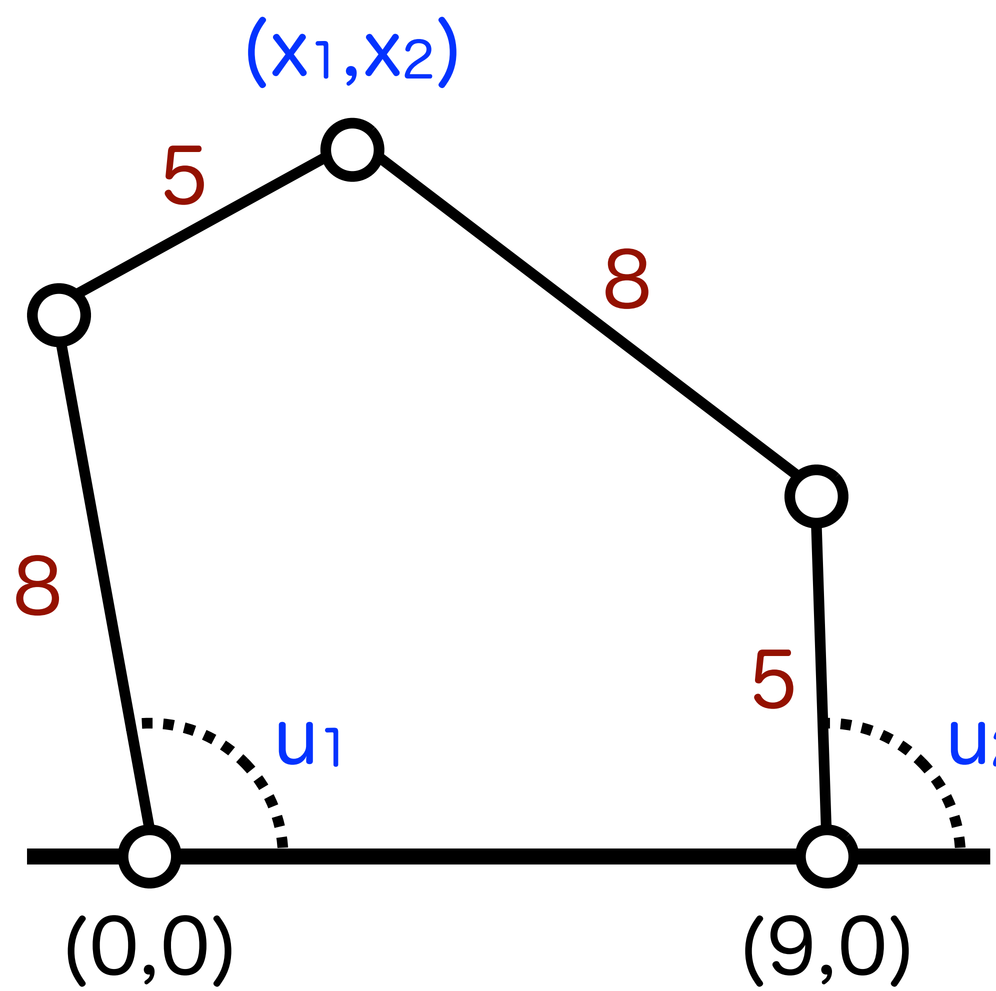


Example: RR-RRR

- 2-dimensional variables
- Initial domain:
  $u_i \in$ [-pi, pi],

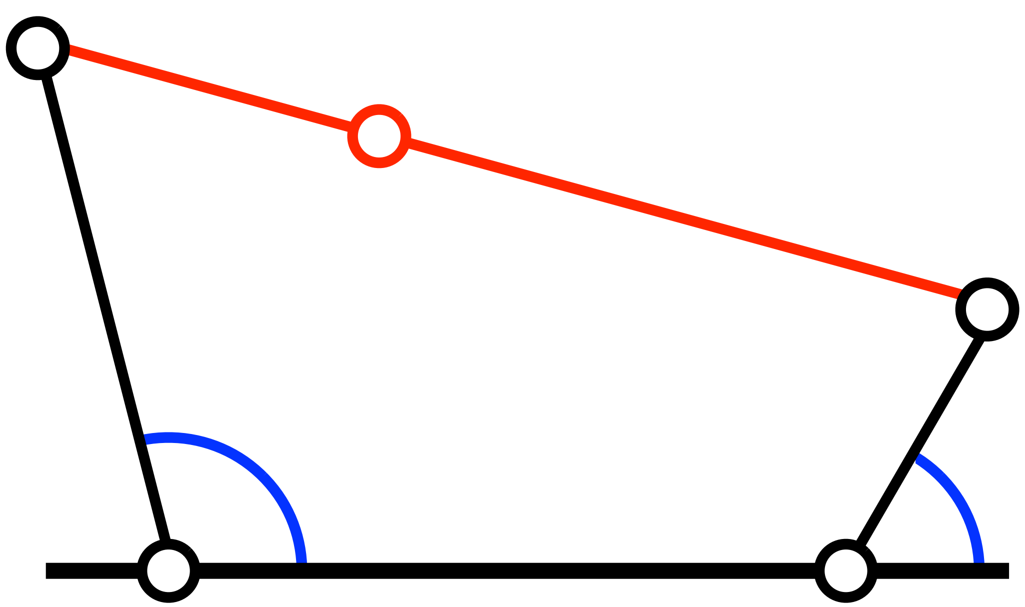
  $x_i \in [-20, 20]$
- Model:

$$\begin{pmatrix} (x_1 - 8\cos u_1)^2 \\ +(x_2 - 8\sin u_1)^2 - 5^2 \\ (x_1 - 5\cos u_2 - 9)^2 \\ +(x_2 - 5\sin u_1)^2 - 8^2 \end{pmatrix} = 0$$



Example: RR-RRR

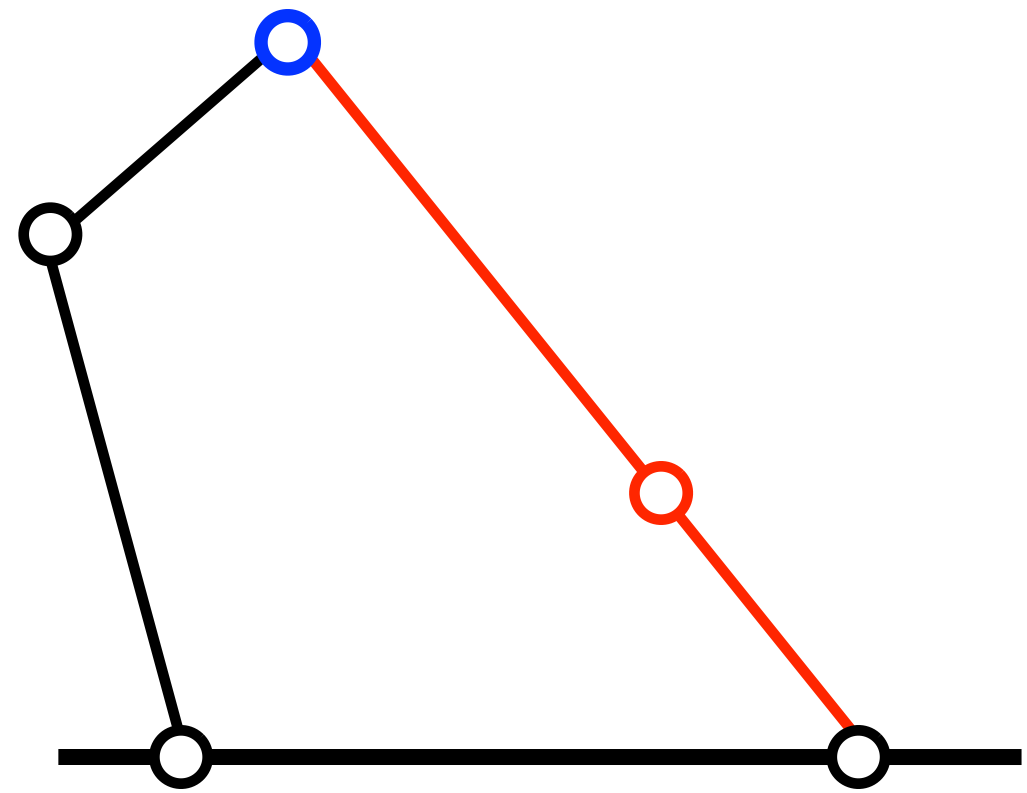
- Example of parallel singularity



Change of control variables
⇒ robot breakdown

$\det D_x f(u,x) = 0$

- Example of serial singularity

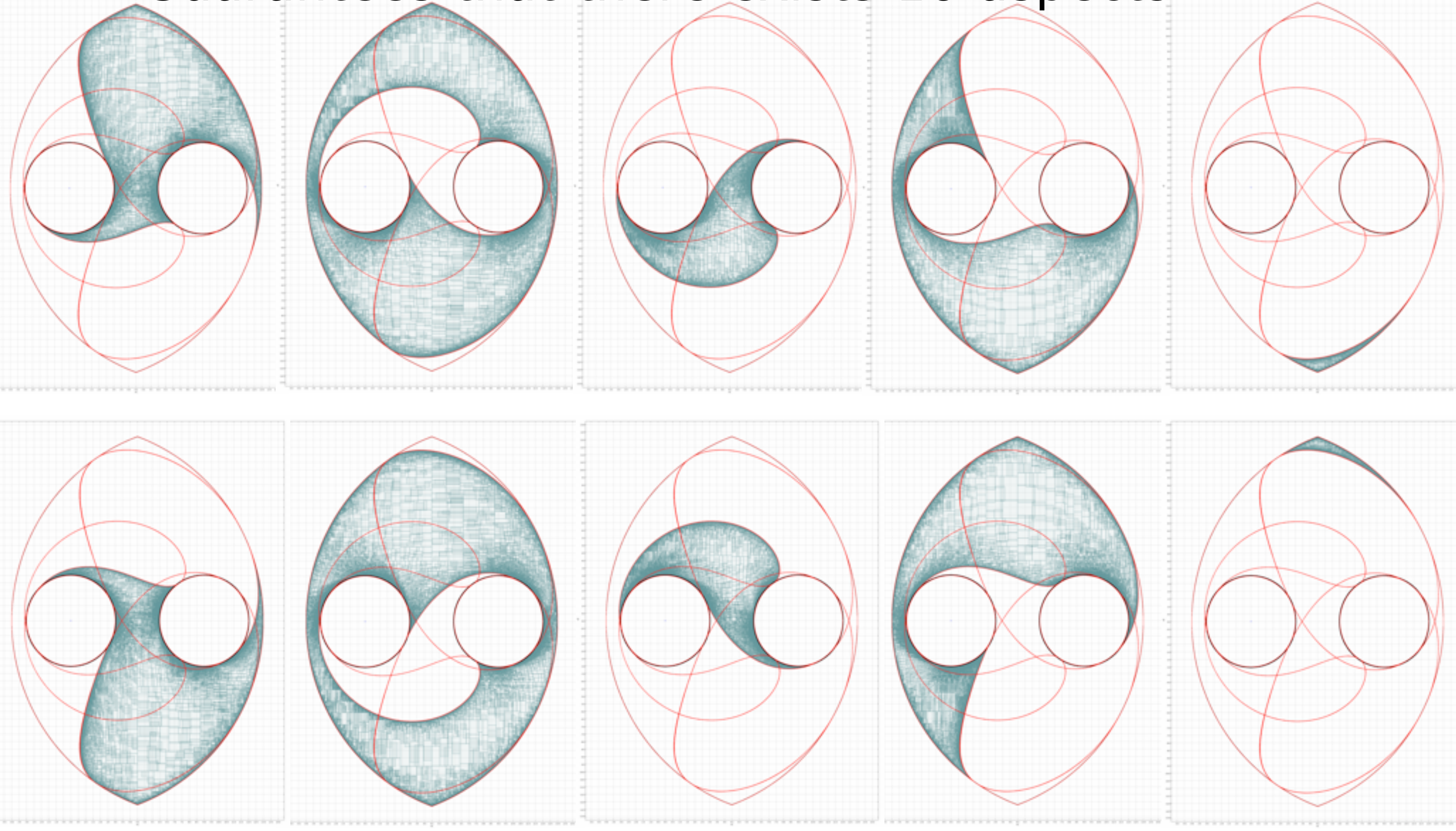


Change of pose variables
⇒ workspace limit

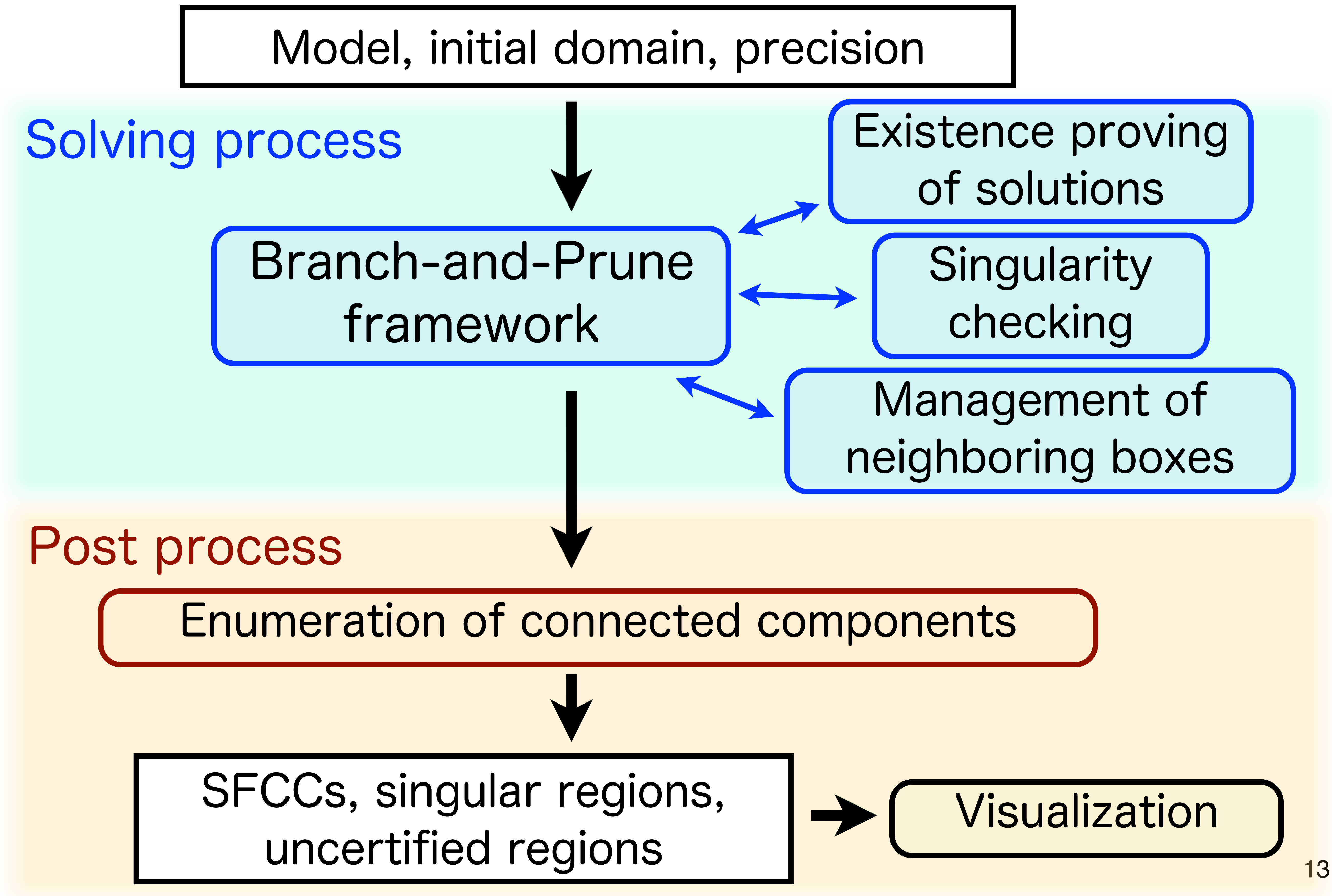
$\det D_u f(u,x) = 0$

Example: RR-RRR

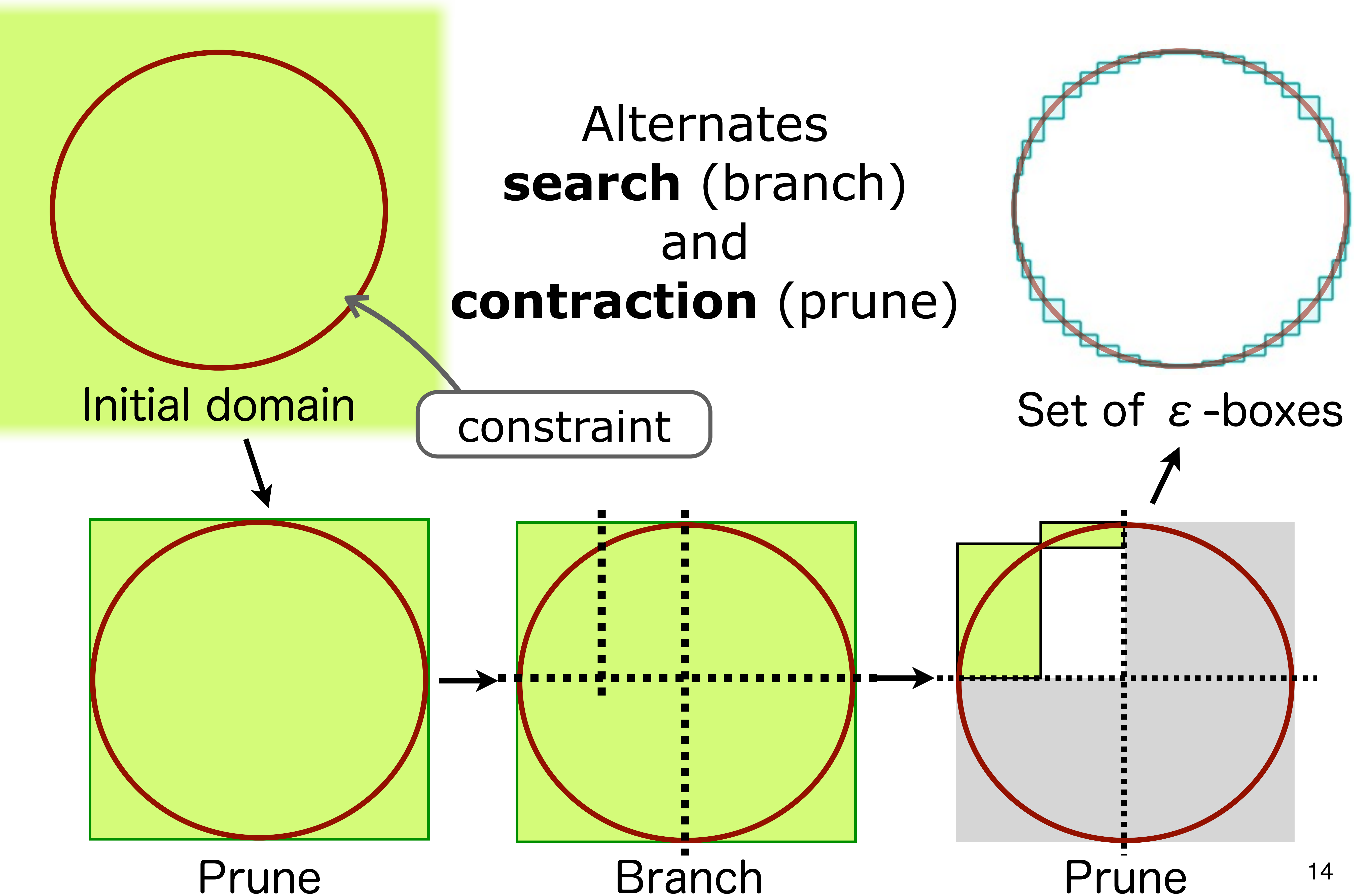
- Computed 10 SFCCs:
  - Guarantees that there exists 10 aspects



Overview of the Proposed Method



Branch-and-Prune Framework



Existence Proof of a Configuration

- Consider boxes $[u]\times[x] \in \mathbf{IR}^{2n}$,

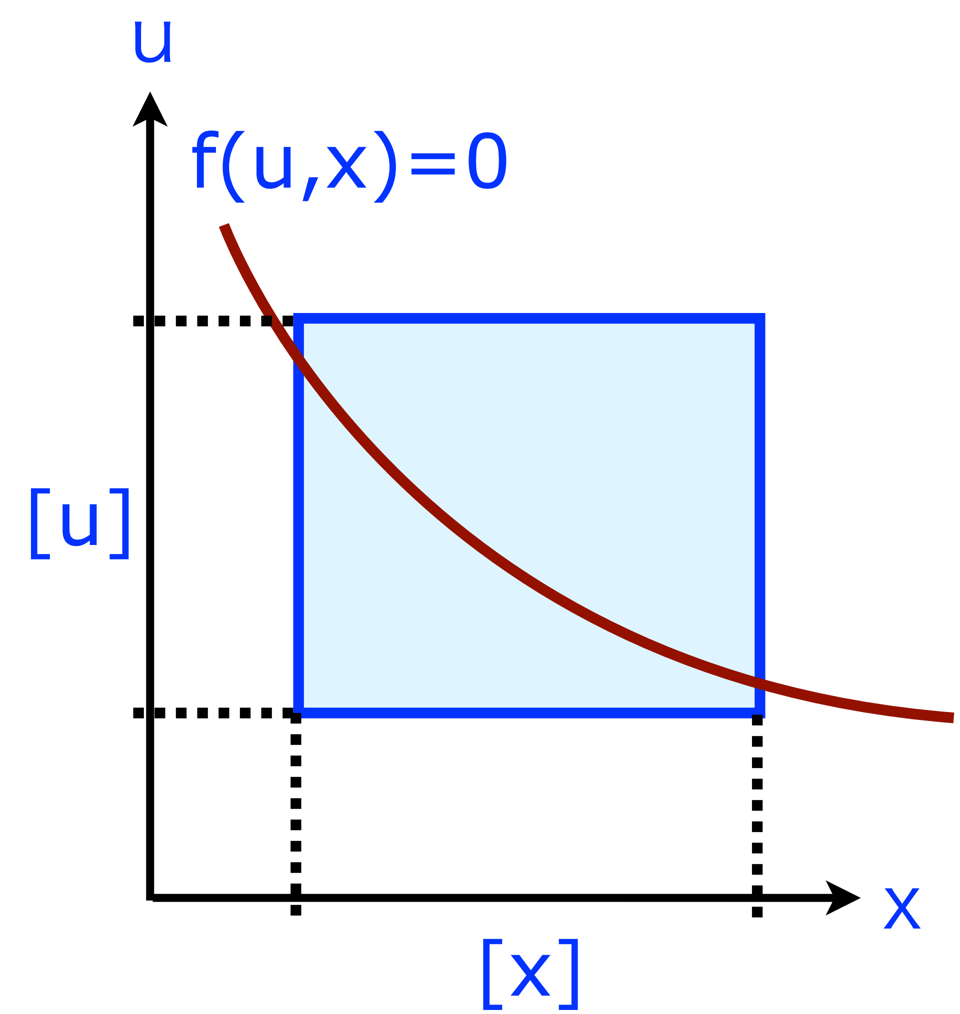
  a continuously differentiable function $f : \mathbf{R}^{2n} \to \mathbf{R}^{n}$,

  a real vector $\hat{u} \in [u]$, and

  an interval Jacobian matrix $[J_u] \in \mathbf{IR}^{n\times n}$ that contains all

  $D_u f(u,x)$ for $(u,x) \in [u]\times[x]$

  derivative w.r.t. u



- Then, $\forall x\in[x]\ \exists u\in[u]\ (f(u, x)=0)$, whenever

$$\hat{u} + \Gamma([J], ([u] - \hat{u}), f(\hat{u}, [x])) \subseteq \mathrm{int}[u]$$

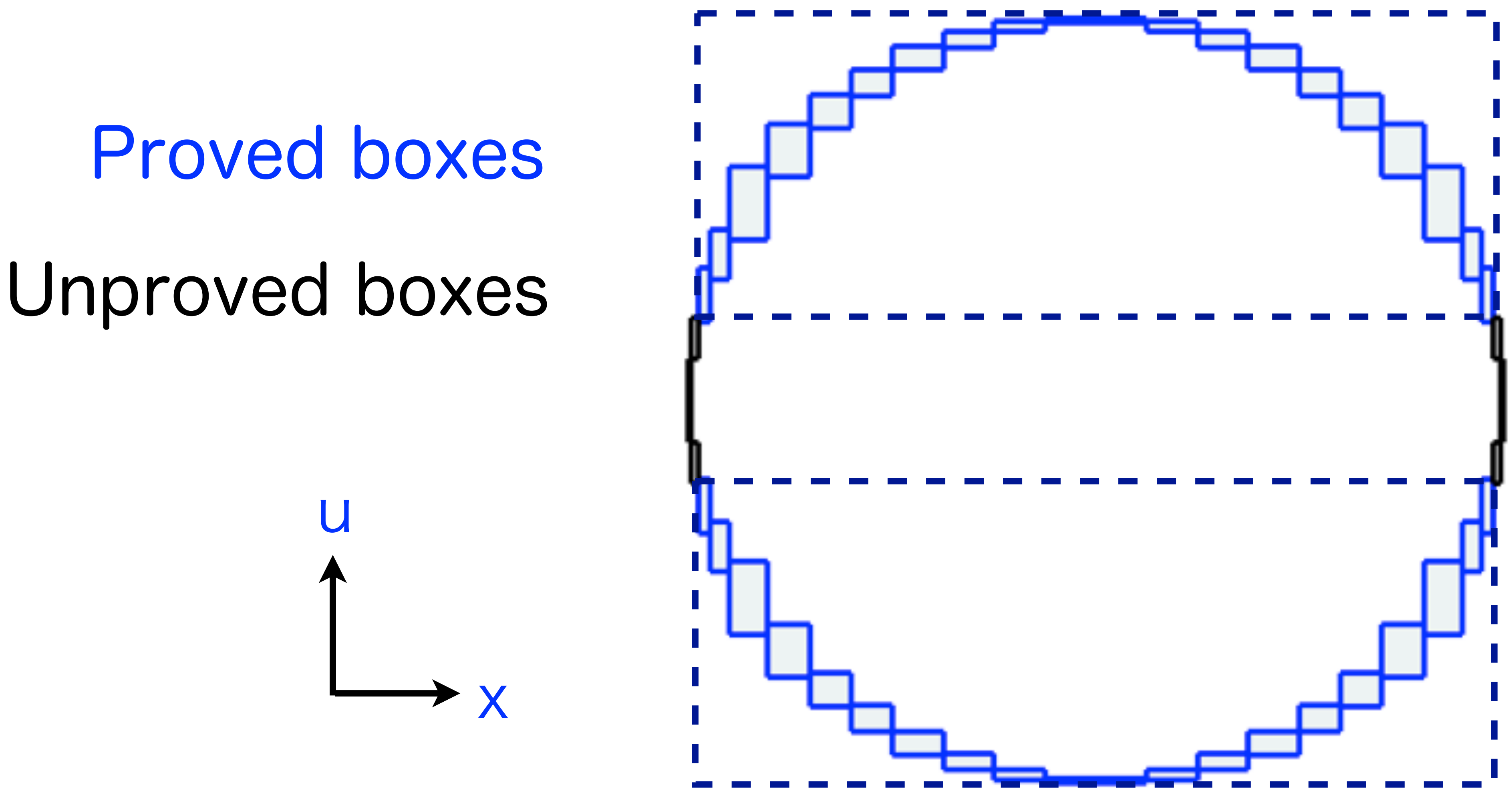
  where $\Gamma([A],[v],[b])$ is the Gauss-Seidel operator

Existence Proof of a Configuration

- Example:
  - Model:

$$u^2 + x^2 - 1 = 0$$

  - Computation result (ε=0.2):



Singularity Checking

- Consider a manipulator modeled by $f(u,x)=0$, where $f : \mathbf{R}^{2n} \rightarrow \mathbf{R}^n$

- A configuration $(u,x) \in \mathbf{R}^{2n}$ exhibits
  - **serial singularity** iff $\det J_u = 0$
  - **parallel singularity** iff $\det J_x = 0$

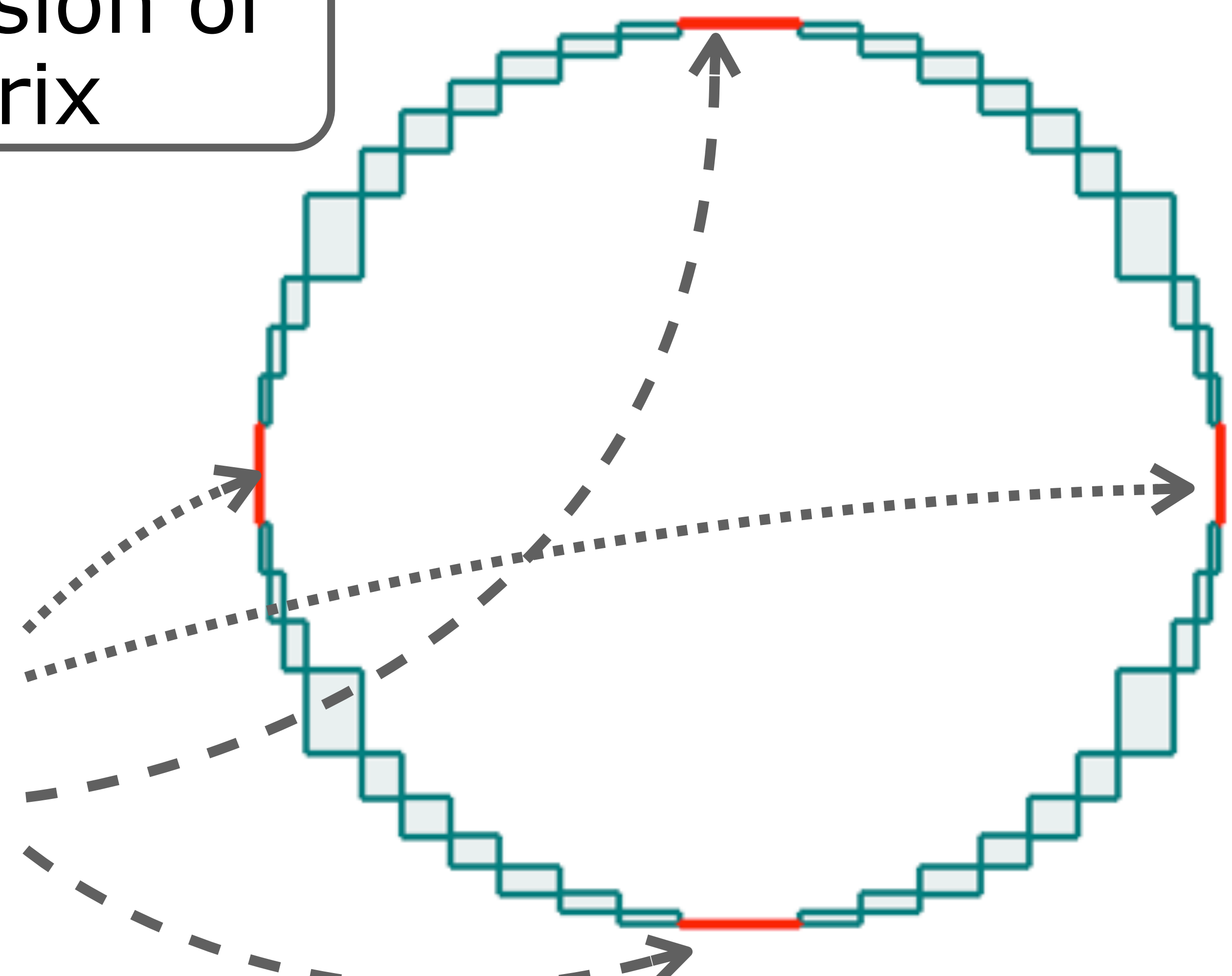
  where $J_u \in \mathbf{R}^{n \times n}$ is Jacobian w.r.t. u and

  $J_x \in \mathbf{R}^{n \times n}$ is Jacobian w.r.t. x

Guaranteeing Regularity

- Interval of configurations $[u]\times[x] \in \mathbf{IR}^{2n}$ is **singularity free** if $0 \notin [\det \mathbf{J}_u]$ and $0 \notin [\det \mathbf{J}_x]$ hold

computed from
an interval extension of
Jacobian matrix



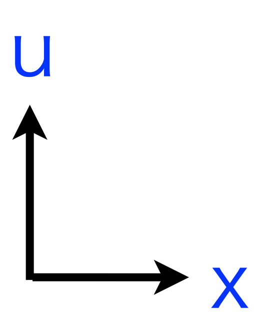
Inner Testing

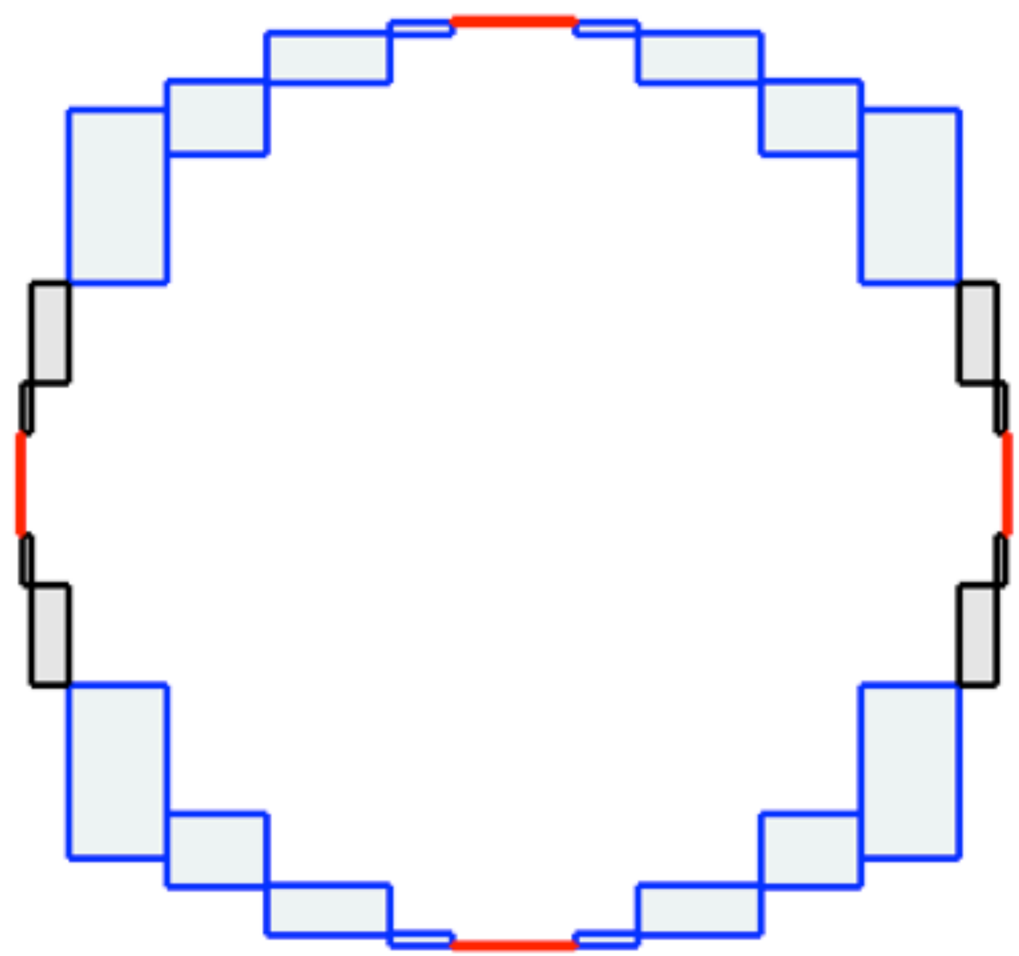
- A box [u]×[x] ∈ $\mathbf{IR}^{2n}$ is contained in an aspect
  ⇔ existence proving succeeded
  and singularity free check succeeded
- Using inner test as a search termination criterion
- Example: $x^2 + u^2 - 1 = 0$

Proved & SF boxes

Unproved & SF boxes

Possibly singular boxes





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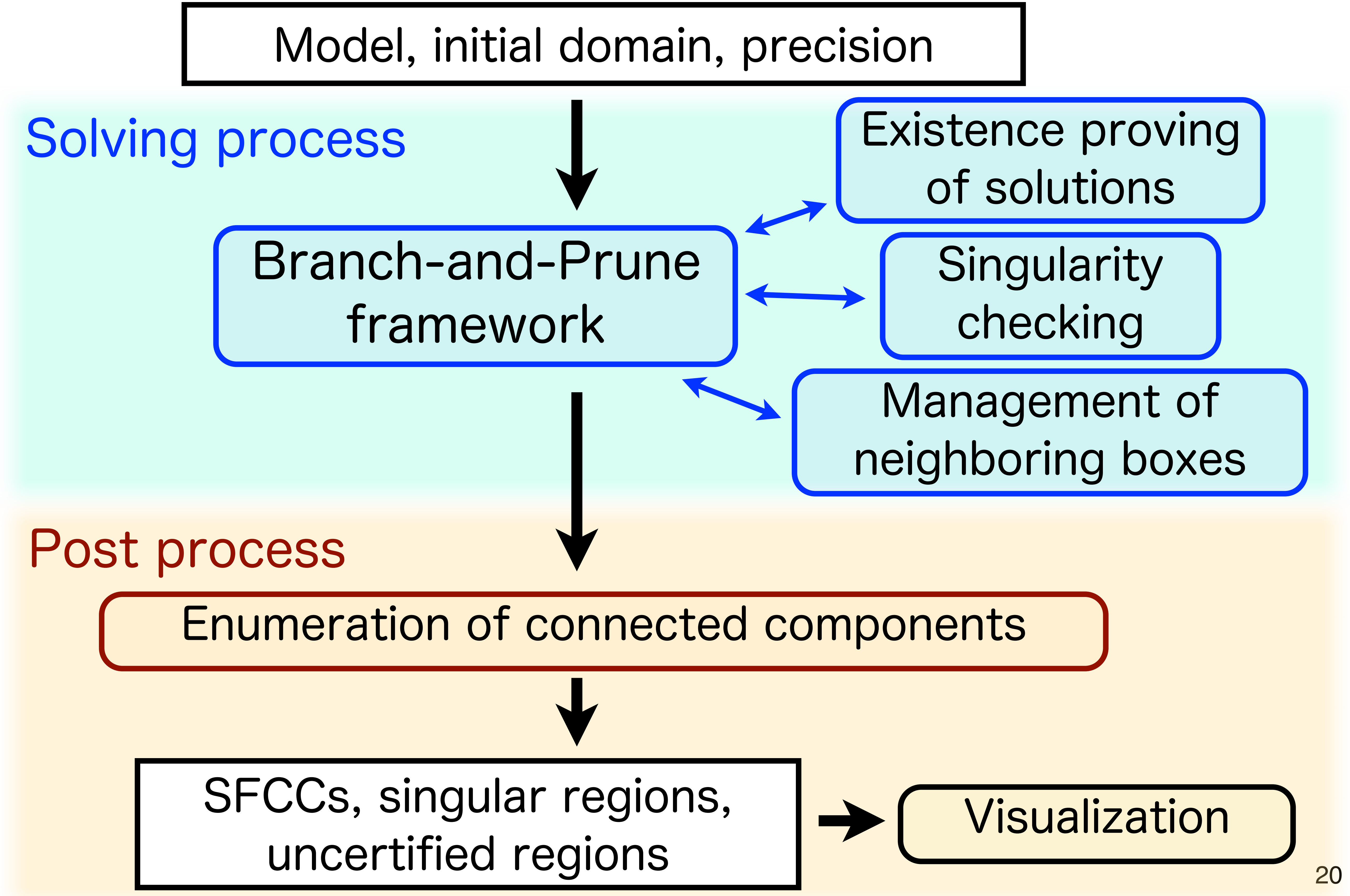
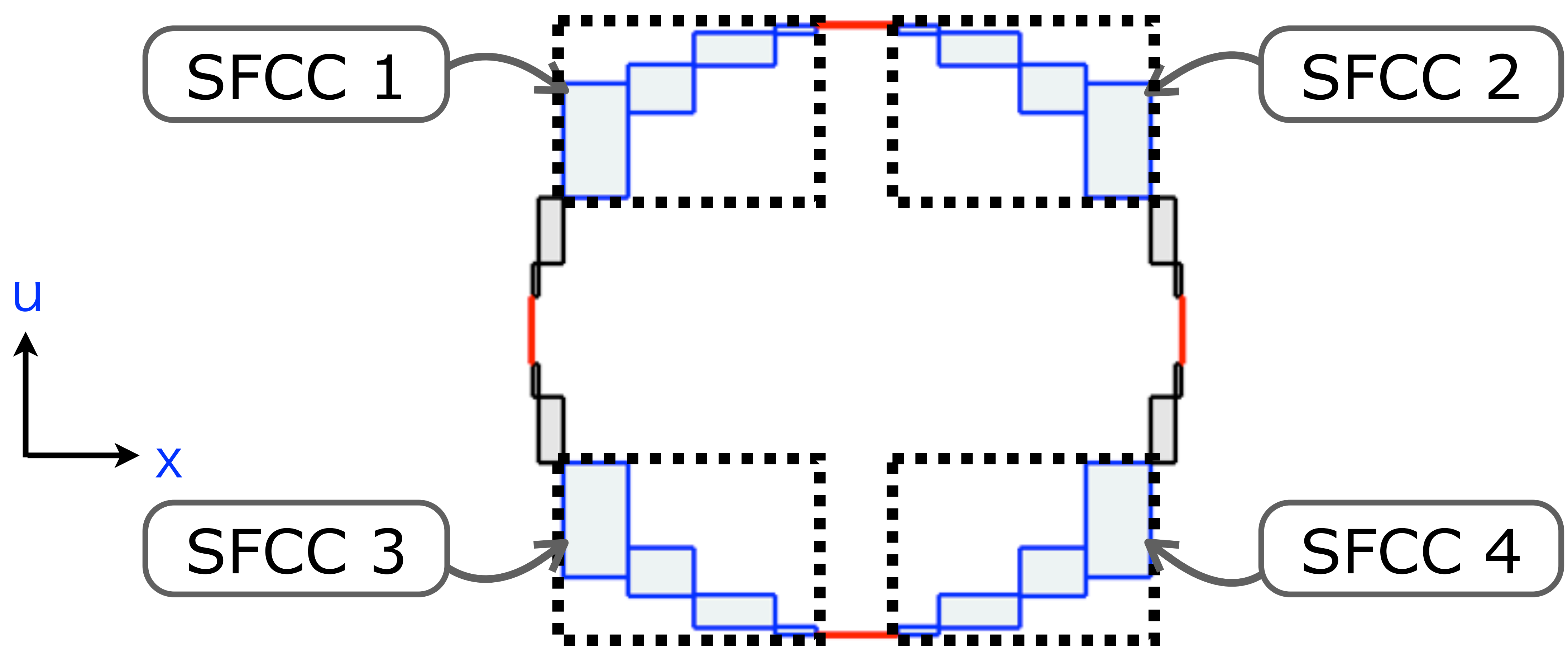


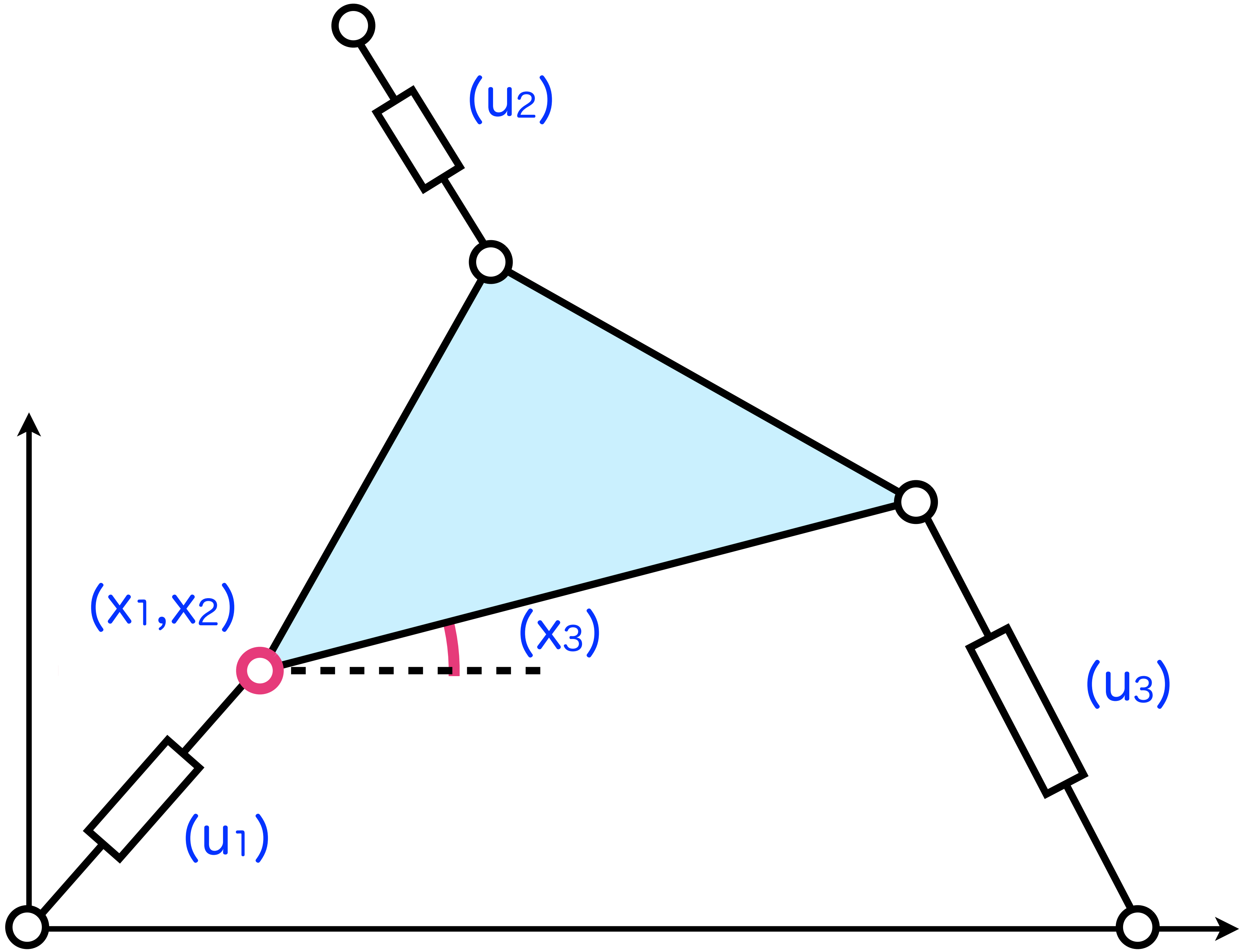
Enumeration of SFCCs

- Management of neighboring boxes during the search by Branch-and-Prune
- After the search, we apply a graph enumeration method to the set of inner boxes



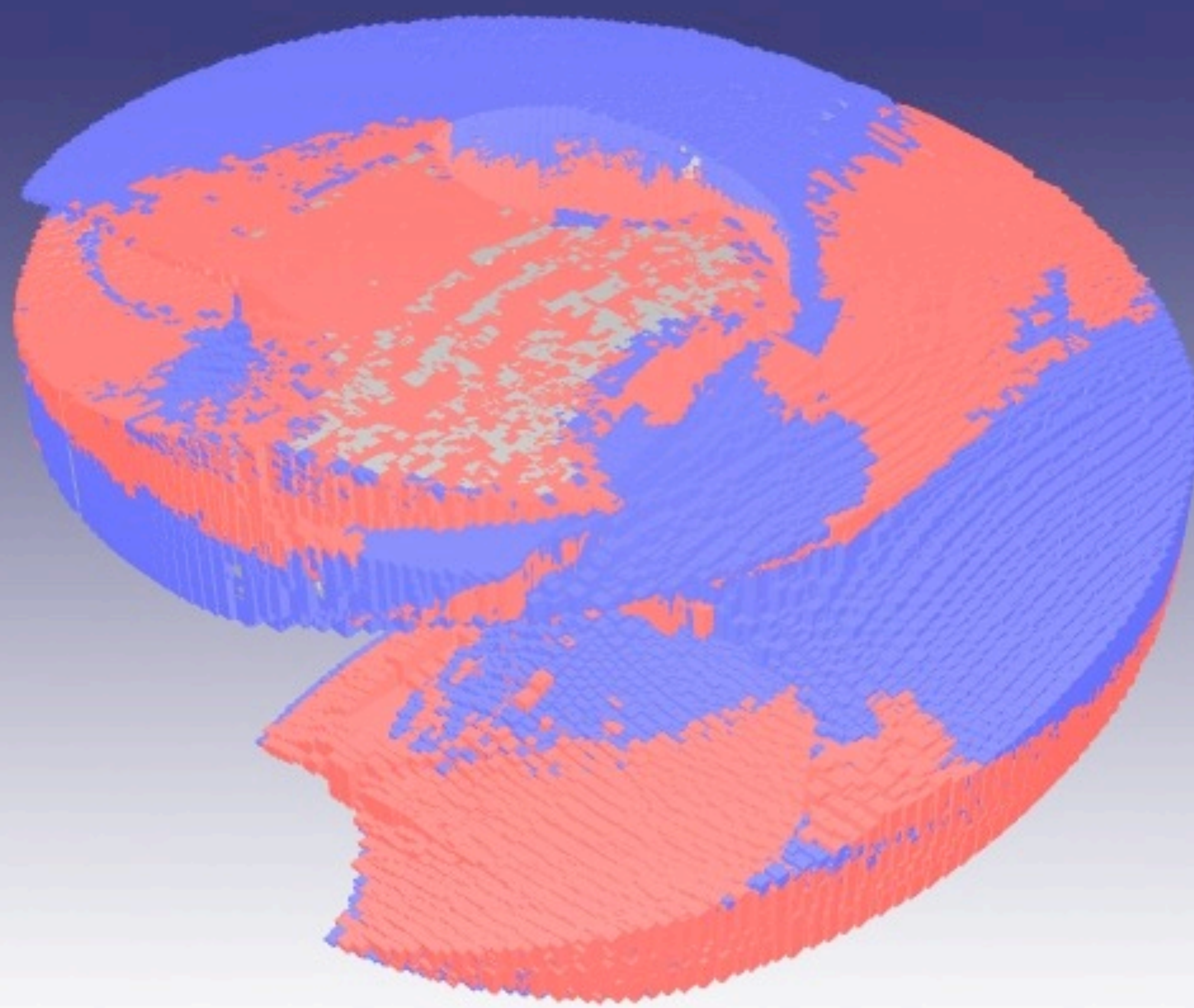
Example: 3-RPR

- 3 dimensional planer manipulator



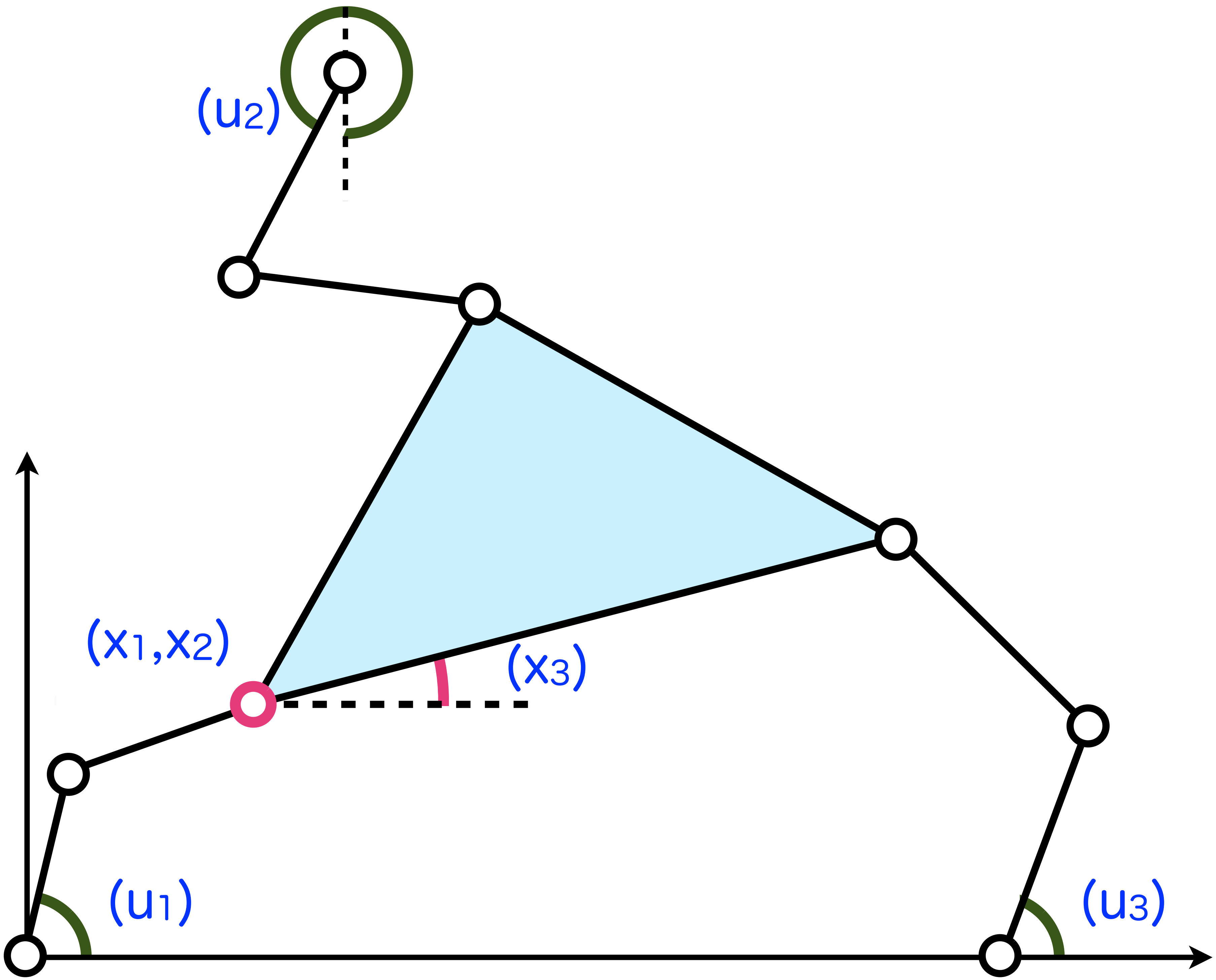
Example: 3-RPR

- Computed spiral workspace:



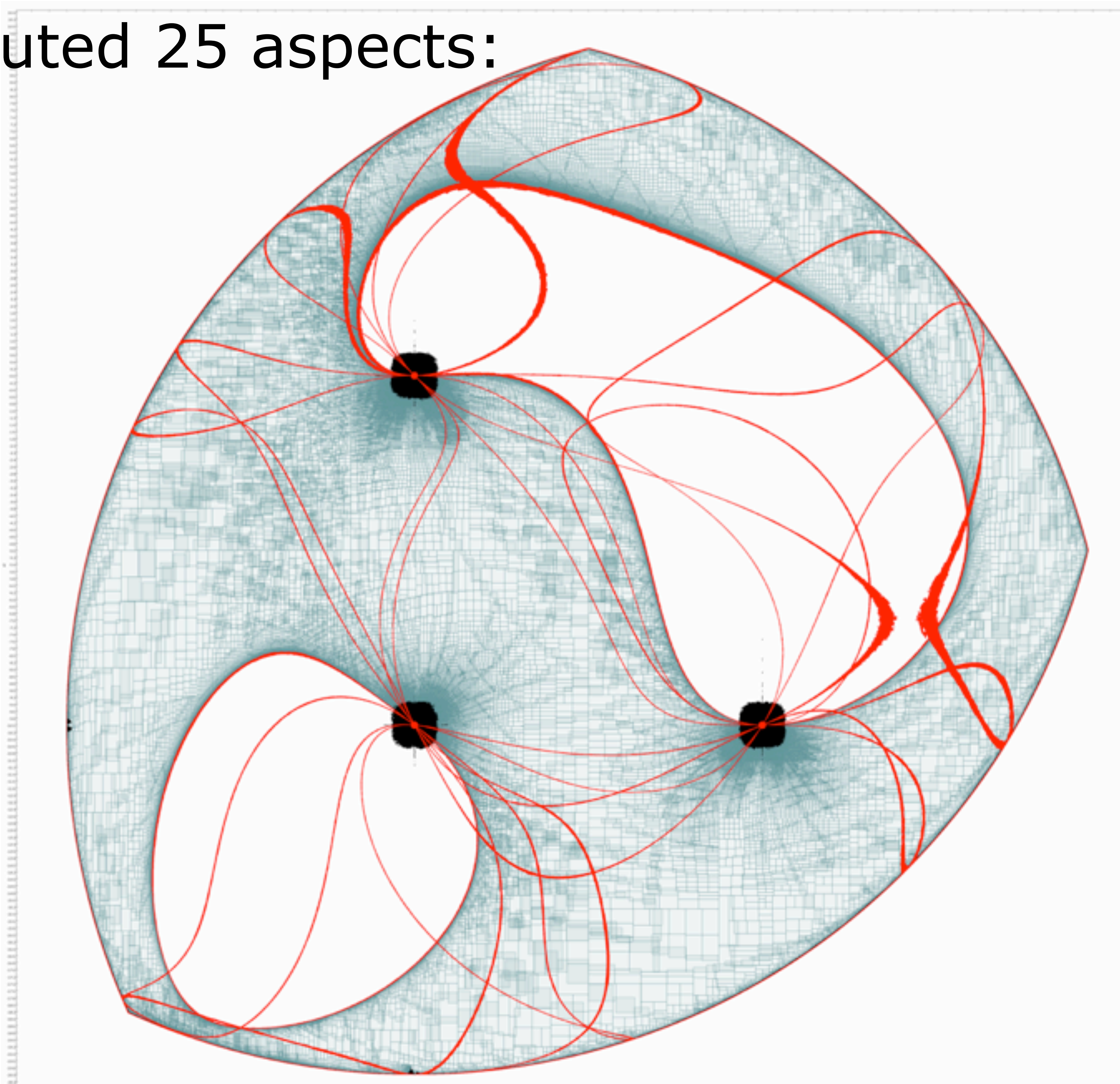
Example: 3-RRR

- Computed with fixed orientation: $x_3 = 0$



Example: 3-RRR

- Computed 25 aspects:



Experimental Results

	2-RPR	RRR	RR-RRR	3-RPR	3-RRR (x_3=0)
theoretical # aspects	2	2	10	2	?
prec	0.1	0.1	0.1	0.1	0.01
# SFCCs (filtered)	2	562 (2)	1456 (10)	49882 (2)	51901 (25)
# boxes	1240	31590	87584	13677836	6081438
time (s)	0.206	13.264	35.784	7913	5050

Conclusion

- We present a tool that supports
  - **simple modeling** of parallel manipulators
  - **validated computation** and **visualization** of workspace, working modes, and generalized aspects
- Experimental results indicate the correct number of generalized aspects

References

- D. Chablat and P. Wenger: **Working Modes and Aspects in Fully Parallel Manipulators**, ICRA'98, pp. 1964-1969, 1998.
- D. Chablat and P. Wenger: **The Kinematic Analysis of a Symmetrical Three-Degree-of-Freedom Planar Parallel Manipulator**, Symp. on Robot Design, Dynamics and Control, pp. 1-7, 2004.
- A. Goldsztejn and L. Jaulin: **Inner Approximation of the Range of Vector-Valued Functions**, Reliable Computing, vol. 14, pp. 1-23, 2010.
- A. Goldsztejn and L. Jaulin: **Inner and Outer Approximations of Existentially Quantified Equality Constraints**, CP'06, pp. 198-212, 2006.