From Singularities in Parallel Mechanisms to Singularities in Visual Servoing

Towards the discovery of common issues and potential methods for solving them

Sébastien Briot
Laboratoire des Sciences du Numérique de Nantes (LS2N)

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Introduction

Serial robots vs. Parallel robots
Introduction

Singularities of serial robots

TRAVERSEE Type 2
Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - “Usual” Leg (or Type 1) Singularities
- Platform singularities:
  - Type 2 singularities
  - Constraint singularities
  - Other (not detailed because extremely rare)
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  - Leg Active Joint Twist System Singularities (LAJTS)
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Introduction

Special types of singularities

In Type 2, constraint and LPJTS singularities
- Loss of stiffness (uncontrollable / gained motions)
- Considerable decrease of performance (deformation, vibration, effort transmission, dynamics, positioning error, etc.)
- Singularities located IN the workspace (not on the boundaries)
Type 2 (parallel) singularities of PKM

Probably, the most important drawback of PKM

Type 2 Singularities of a 3–RRR planar robot [Bonev 2001]
Type 2 (parallel) singularities of PKM

 Normally, impossible to cross these singularities  
  Because near these singularities, the input torques tend to infinity
Type 2 (parallel) singularities of *PKM*

**But...**

By proper trajectory planning respecting a dynamics criterion [Briot et Arakelian 2008] and an adequate controller [Pagis et al, 2015]
Singularities of parallel robots

How to find Type 2 or constraint singularities?

In the late 80’s

- Type 2 singularities
  - Compute the I/O kinematic relationship:

\[
A(q_a, x)^0 t_p + B(q_a, x) \dot{q}_a = 0
\]  

- Compute the determinant of \( A \) and find the conditions for which it is equal to 0

\[ \Rightarrow \text{Limited to simple cases} \]
Singularities of parallel robots

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In the late 80’s

• Type 2 singularities
  ○ Compute the I/O kinematic relationship:

\[
A(q_a, x)^0 t_p + B(q_a, x) \dot{q}_a = 0
\]  \hspace{1cm} (1)

  ○ Compute the determinant of \( A \) and find the conditions for which it is equal to 0
  \( \Rightarrow \) Limited to simple cases

• Constraint singularities:
  ○ Discovered at the early 2000’s
  ○ Cannot be found using the previous method
How to find Type 2 or constraint singularities?

In the late 80’s / early 90’s, a method based on the Grassmann geometry

Type 2 or constraint sing. ≡ singularities of the system of (static) wrenches applied by the legs on the platform
How to find Type 2 or constraint singularities?

In the late 80’s / early 90’s, a method based on the Grassmann geometry

Type 2 or constraint sing. ≡ singularities of the system of (static) wrenches applied by the legs on the platform

- Find the system of wrenches applied by the legs on the platform using the Screw Theory
- Analyze the degeneracy of this system of wrenches using the Grassmann geometry
Determination of the system of wrenches
Determination of the system of wrenches

For serial leg (the \( i \)th leg of the parallel robot)

\[
\mathbf{t}_p = \mathbf{J}_i(q_i)\dot{q}_i \quad \text{where} \quad \mathbf{J}_i = \begin{bmatrix} \$_{i1} & \cdots & \$_{im_i} \end{bmatrix}
\]

\( \$_{ij} \) is unit a twist representing the twist of the platform when joint \( ij \) is moving only.
Determination of the system of wrenches

We group, for the leg $i$,

- in a sub-matrix $^0s_{ia}$ the unit twists corresponding to the active joints of velocities $\dot{q}_{ai}$,
- in a sub-matrix $^0s_{id}$ the unit twists corresponding to the passive joints of velocities $\dot{q}_{di}$

and we express all equations in the base frame $\mathcal{F}_0$ (superscript “0” before the variables)
Determination of the system of wrenches

We group, for the leg $i$,

- in a sub-matrix $0^i s_{ia}$ the unit twists corresponding to the active joints of velocities $\dot{q}_{ai}$,
- in a sub-matrix $0^i s_{id}$ the unit twists corresponding to the passive joints of velocities $\dot{q}_{di}$

and we express all equations in the base frame $\mathcal{F}_0$ (superscript “0” before the variables)

Thus

$$0^t p = \begin{bmatrix} 0^i s_{ia} & 0^i s_{id} \end{bmatrix} \begin{bmatrix} \dot{q}_{ai} \\ \dot{q}_{di} \end{bmatrix} = 0^i s_{ia} \dot{q}_{ai} + 0^i s_{id} \dot{q}_{di}. \quad (3)$$
Determination of the system of wrenches

For the leg $i$,

- The constraint wrenches (i.e. the wrenches applied by the leg even if it is not actuated) are the wrenches $\zeta_{id}$ which are reciprocal to both $0\$i$ and $0\$id$, i.e. they are defined such that

$$\zeta_{id} \circ 0\$i = 0, \quad \zeta_{id} \circ 0\$id = 0$$

(4)

- The actuation wrenches (i.e. the wrenches applied by the leg because of the presence of the actuator) are the wrenches $\zeta_{ia}$ which are reciprocal to $0\$id$ and are not included in the system of constraint wrenches $\zeta_{id}$, i.e. they are defined such that

$$\zeta_{ia} \circ 0\$id = 0, \quad \zeta_{ia} \not\subset \zeta_{id}$$

(5)
Determination of the system of wrenches

Example of a RR leg with R axes along $z_0$

- Motion is represented by two unit twists:

$$0^\circ R_1 = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$ (6)

$$0^\circ R_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$ (7)
Determination of the system of wrenches

Example of a RR leg with R axes along $z_0$

- Motion is represented by two unit twists:

$$0\overset{\circ}{\mathbf{s}}_{R1} = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$  \hspace{1cm} (6)

$$0\overset{\circ}{\mathbf{s}}_{R2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$  \hspace{1cm} (7)

- If both joints are passive:

$$\mathbf{\zeta}_{d1} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } \overrightarrow{O_1O_2} \hspace{1cm} (8)$$

$$\mathbf{\zeta}_{d2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } z_0 \hspace{1cm} (9)$$
Determination of the system of wrenches

Example of a RR leg with R axes along $z_0$

- Motion is represented by two unit twists:

\[
0\mathcal{R}_1 = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T
\]

\[
0\mathcal{R}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T
\]

- If both joints are passive:

\[
\zeta_{d3} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } x_0
\]

\[
\zeta_{d4} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } y_0
\]
Determination of the system of wrenches

Then,

- Stack all constraint wrenches $\zeta_{id}$ in a matrix $\zeta_d$
- Stack all actuation wrenches $\zeta_{ia}$ in a matrix $\zeta_a$
- Analyze the degeneracy of $\zeta_a$ and $\zeta_d$ thanks to the Grassmann geometry
Singularities of parallel robots

Grassmann geometry

- Gives conditions on degeneracy of systems of lines
- Plücker representation of a line $\mathcal{L} : [\mathbf{u}^T \cdot (\overrightarrow{PQ} \times \mathbf{u})]^T$
  - A direction $\mathbf{u}$
  - Moment of the direction $\mathbf{u}$ wrt a given point $P$
Singularities of parallel robots

A pure force wrench is given by (at point $P$, if $f$ is applied at point $Q$)

$$\zeta_i = \begin{bmatrix} f \\ PQ \times f \end{bmatrix}$$

(10)
Singularities of parallel robots

A pure force wrench is given by (at point $P$, if $f$ is applied at point $Q$)

$$\zeta_i = \begin{bmatrix} f \\ PQ \times f \end{bmatrix}$$  \hspace{1cm} (10)$$

A pure moment wrench is given by, for any application point

$$\zeta_i = \begin{bmatrix} 0 \\ m \end{bmatrix}$$ \hspace{1cm} (11)$$
Singularities of parallel robots

A pure force wrench is given by (at point $P$, if $f$ is applied at point $Q$)

$$\zeta_i = \left[ \begin{array}{c} f \\ PQ \times f \end{array} \right]$$  \hspace{1cm} (10)

A pure moment wrench is given by, for any application point

$$\zeta_i = \left[ \begin{array}{c} 0 \\ m \end{array} \right]$$  \hspace{1cm} (11)

These expressions are Plücker representations of lines

• the pure force wrench: a line of direction $f$ passing through point $P$
• the pure moment wrench: a line of direction $m$ but in the projective plane at infinity
Singularities of parallel robots

Thanks to Grassmann geometry

Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines
Singularities of parallel robots

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Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

It is still quite complicated
However for 2 and 3DOF planar robots, the conditions are quite simple to analyze
Singularities of parallel robots

Thanks to Grassmann geometry
Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

It is still quite complicated
However for 2 and 3DOF planar robots, the conditions are quite simple to analyze

For planar robots
• 2 DOF: degeneracy if the two lines are parallel
• 3 DOF: degeneracy if the three (coplanar) lines intersect in the same point (that may be at infinity) \(\Rightarrow\) instantaneous center of rotation
Singularities of parallel robots

A few notations

- \( a, b \): two points located at the position \( a \) and \( b \) in the Cartesian space (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to \( w \neq 0 \))
- \( A, B \): two points located at the position \( A \) and \( B \) in the projective plane at infinity (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to \( w = 0 \))
- \( ab \), the line passing through points \( a \) and \( b \)
- \( abc \), the plane passing through points \( a, b \) and \( c \)
- \([abcd]\): the determinant of the \((4 \times 4)\) matrix whose columns are the expressions of the points \( a, b, c \) and \( d \) (in other words, the volume of the tetrahedron)
- \( \wedge \): the “meet operator”
Singularities of parallel robots

Superbracket decomposition

\[
\begin{bmatrix} ab, cd, ef, gh, ij, kl \end{bmatrix} = \sum_{i=1}^{24} y_i
\]  

(12)

where

\[
\begin{align*}
y_1 &= -[abcd][efgi][hjkl] \\
y_2 &= [abcd][efhi][gjkl] \\
y_3 &= [abcd][efgj][hikl] \\
y_4 &= -[abcd][efhj][gikl] \\
y_5 &= [abce][dfgh][ijkl] \\
y_6 &= -[abde][cfgh][ijkl] \\
y_7 &= -[abcd][degh][ijkl] \\
y_8 &= [abdf][cegh][ijkl] \\
y_9 &= -[abce][dghi][fjkl] \\
y_{10} &= [abde][cghi][fjkl] \\
y_{11} &= [abcf][dghi][ejkl] \\
y_{12} &= [abce][dghj][fikl] \\
y_{13} &= -[abdf][cghi][ejkl] \\
y_{14} &= -[abde][cghj][fikl] \\
y_{15} &= -[abcf][dghj][eikl] \\
y_{16} &= [abdf][cghj][eikl] \\
y_{17} &= [abcg][defi][hjkl] \\
y_{18} &= -[abcd][cefj][hikl] \\
y_{19} &= -[abch][defi][gjkl] \\
y_{20} &= -[abcg][defj][hjkl] \\
y_{21} &= [abdh][cefj][gjkl] \\
y_{22} &= [abdg][cefj][hikl] \\
y_{23} &= [abch][defj][gikl] \\
y_{24} &= -[abdh][cefj][gikl]
\end{align*}
\]  

(13)
Determination of the system of wrenches

By an adequate choice of the points for representing the lines (intersection points, points are infinity, etc)

Many monomials $y_i$ can be deleted

Example [Ben Horin and Shoham 2006]

$$[ab, ac, de, df, gh, gi] = [adfg][abcd][igh] = edf \land igh \land abc \land adg$$

Geometric interpretation

Intersection of four planes
Singularities of parallel robots

Remarks

• These tools for singularity analysis are difficult to be used by non-expert.
• But a lot of scientific literature ⇒ If we know the general formulation of the system of wrenches, for instance
  ○ 3 forces + 3 moments
  ○ 6 forces, but only three points of applications, two forces by points
  geometric interpretation of results are already given (see the next slides)
• Sometimes, we still must do the analysis
• These tools were primarily used for singularities of PKM, we will show now that they can be used for other singularity analyses
What is visual servoing?

TRAVERSEEE Type 2
What is visual servoing?

- to 3D features observed ⇒ measures in the camera frame $s$
- we can set a kinematic relationship between the twist $\tau$ of the relative motion between the object and camera frames and the velocity of the measurements $s$:

$$\dot{s} = L \tau$$  \hspace{1cm} (14)

- $L = L(s, x)$ is called the interaction matrix, in which $x$: relative configuration between the object and camera frames
- standard controller (wishing an exponential decay $\dot{e} = -\lambda e$ of error $e = s - s^*$ ⇒ $\dot{s} = -\lambda e$):

$$\tau = L \tau = -\lambda L^+ e$$  \hspace{1cm} (15)
Introduction to singularities in visual servoing

- Singularities appearing when observing image features (e.g. with a camera) = a huge challenge in visual servoing
Introduction to singularities in visual servoing

- Singularities appearing when observing image features (e.g. with a camera) = a huge challenge in visual servoing
- To the best of our knowledge, only known for three 3-D image points (*singularity cylinder*)
- Issue with singularities: interaction matrix cannot be inverted anymore = loss of controllability
Introduction to singularities in visual servoing

In order to avoid singularities

Increased number of image features (redundancy):
  • Pb of local minima
  • Proof that there is no singularity?

Determining the singularity cases stays an open problem
Introduction to singularities in visual servoing

Recently, the “Hidden Robot Concept” was developed

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM
Introduction to singularities in visual servoing

Recently, the “Hidden Robot Concept” was developed

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea $\Rightarrow$ Interaction matrix $\equiv$ Inv. Jacobian matrix of a virtual PKM

For instance, when observing the leg directions of the GS platform

- Real robot = 6–UPS
Recently, the “Hidden Robot Concept” was developed

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM

For instance, when observing the leg directions of the GS platform

- Real robot = 6–\(UPS\)
- Virtual robot = 6–\(UPS\)
Introduction to singularities in visual servoing

Here
I show how we used the hidden robot concept in order to solve, for the first time, the singularities in
1. the observation of \( n \) image points (\( n \geq 3 \))
2. the observation of three lines
3. the leg-based visual servoing of parallel robots
Observation of an image point

$\mathcal{L}_1$

$M_1 (X,Y,Z)$

$m_1 (x,y)$

Image plane

$C$ Camera center
Observation of an image point
Observation of an image point
Observation of an image point

![Diagram showing an image plane with a point M1, a line L1, and a camera center C.](image-url)
Observation of an image point
Observation of an image point

A **UPS** kinematic chain which allows for the same motion of the point $M_i$
Observation of three image points
Observation of three image points

A 3–**UPS** robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

\[
\dot{s} = L\tau \quad / / \quad \dot{q} = J_{\text{inv}}\tau
\]
Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix =
singularities of the virtual parallel robot

Singularities of parallel robots
Can be studied by using several (complementary) tools

Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots
Can be studied by using several (complementary) tools

In our case (3 points), it can be proven that
The planes $P_i$ ($i = 1, 2, 3$) and $P_4$ (containing all 3-D points) have a non-null intersection
Singularities when observing 3 points
Singularities when observing 3 points
Singularities when observing 3 points
Singularities when observing 3 points
Singularities when observing \( n \) points \((n > 3)\)

Possible if and only if

- All singularity cylinders associated with any subset of 3 points have a common intersection
- AND all kernels of the interaction matrices are identical

After (more complex) mathematical derivations, we proved that

The conditions of singularity when \( n \) coplanar points are observed only appear if and only if all 3-D points and the optical center are located on the same circle
Singularities when observing $n$ points ($n > 3$)

Examples of undetermined configurations
Simulations

Circumcircle to $M_1$, $M_2$, $M_3$ and $M_4$
Simulations

\[
\frac{1}{\kappa} \text{ (inverse of the condition number)} \quad \text{parameter } s
\]

\[
0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \quad 1.2 \quad 1.4
\]

\[
0 \quad 2 \quad 4 \quad 6 \quad 8 \times 10^{-3}
\]
Observation of an image line
Observation of an image line
Observation of an image line
Observation of an image line
Observation of an image line
Observation of an image line

A **UPRC** kinematic chain which allows for the same motion of the line $\mathcal{L}_i$.
Observation of three image lines
Observation of three image lines

A 3–UPRC robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

\[
\dot{s} = L\tau \quad / \quad \dot{q} = J_{\text{inv}}\tau
\]
Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix =
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Singularities of parallel robots
Can be studied by using several (complementary) tools

Singularities

Thanks to the hidden robot analogy
Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots
Can be studied by using several (complementary) tools
  • Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]

In our case (3 lines), singu. cond. iff

\[ f_1 = f_{11}^T(f_{21} \times f_{31}) = 0 \]
\[ f_2 = m_{12}^T(m_{22} \times m_{32}) = 0 \]

where \( \xi_{ij} = [f_{ij}^T m_{ij}^T]^T \)
Singularities

In order to simplify the problem

- Consider the “zero” platform orientation
- General case obtained by a simple rotation

\[
\begin{bmatrix}
X & Y & Z
\end{bmatrix}^T = \mathcal{R} \begin{bmatrix}
X' & Y' & Z'
\end{bmatrix}^T
\]

(16)

where

- \(X, Y, Z\): position of the origin of the object frame \(\mathcal{F}_b\) in the camera frame when considering the “zero” platform orientation
- \(X', Y', Z'\): position of the origin of the object frame for the considered “non-zero” platform orientation
- \(\mathcal{R}\): the rotation matrix between the two cases
Three coplanar lines with no common intersection point

\[ f_1 = 0 \iff Z = 0 \implies \text{Lines + optical center in the same plane} \]

\[ f_2 = 0 \iff Z(X^2 + Y^2 - \rho^2) = 0 \implies \text{Singularity cylinder!} \]
Three lines in space with a common intersection point

\[ \overrightarrow{OQ} = [X \ Y \ Z]^T, \ U_1 = [1 \ 0 \ 0]^T, \]
\[ U_2 = [a \ b \ 0]^T, \ U_3 = [c \ d \ e]^T \] (18)

\[ f_1 = 0 \Rightarrow \text{For any object configuration} \]
\[ f_2 = 0 \iff b(ad + bcd + ae^2)Z \]
\[ + (ac - bd)eX)Y^2 - e(bcX^2 + (ad - bc)Z^2 \]
\[ + 2beXZ)Y + ((-ad^2 + bcd - ae^2)XZ \]
\[ + (bd + ac)eXZ^2)) = 0 \] (19)

\[ \Rightarrow \text{The origin of the body frame belongs to a cubic surface parameterized by } f_2 = 0. \]
Three orthogonal lines in space

\[
\begin{align*}
\mathcal{L}_1 &\quad \mathcal{L}_2 &\quad \mathcal{L}_3 \\
 x_b &\quad (X Y Z) &\quad x_b \\
 y_b &\quad \mathcal{U}_2 &\quad \mathcal{U}_3 \\
 z_b &\quad \mathcal{U}_3 &\quad \mathcal{U}_1
\end{align*}
\]

\[
f_1 = 0 \iff aXY + bYZ - cXZ - abc = 0
\]

\[
f_2 = 0 \iff acX - abY + bcZ - XYZ = 0
\]

⇒ Expression \( f_1 \) represents a quadric surface while expression \( f_2 \) is a cubic surface
Three lines, two of them being parallel

![Diagram of three lines](image)

\[ f_1 = 0 \iff Z(dZ - eY) = 0 \]
\[ f_2 = 0 \iff Z(X(d^2 + e^2) - cYd - cZe) = 0 \]  

- \( Z = 0 \), which occur when the plane \( \mathcal{P} \) containing \( L_1 \) and \( L_2 \) also contains the optical center,
- \( eY - dZ = 0 \) is the plane containing \( U_1 \), \( U_3 \) and the optical center,
- \( X(d^2 + e^2) - cdY - ceZ = 0 \) is the plane containing \( (U_1 \times U_3) \), \( U_3 \) and the optical center.
Three general lines in space

Condition $f_1 = 0$ provides the expression of a quadric surface while $f_2 = 0$ leads to a cubic surface.
Example for three general lines in space

\[ f_1 = 0 \]
\[ f_2 = 0 \]
\[ Z \text{[m]} \]
\[ X \text{[m]} \]
\[ Y \text{[m]} \]
Simulation 1 (general case)

location of the camera when $s = 0$  
⇒ singularity

location of the camera when $s = -0.1$
Simulation 1 (general case)
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Planar robots: Example of the 3–RRR robot
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Spatial robots: Example of the GS Platform
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Spatial robots: Example of the Quattro
Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Experimental validation

![Graph showing the error over time for different legs. The x-axis represents time (in seconds) ranging from 0 to 50, and the y-axis represents the error magnitude (|\|e_i\||) ranging from 0.05 to 0.4. The graph includes lines for different legs, denoted as Leg 1 to Leg 4, showing the decrease in error over time. The initial and final platform configurations are also marked.]
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 1: Robots which are uncontrollable with the observation of the leg directions
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

**Class 2:** Robots which are partially controllable (in their workspace) with the observation of the leg directions
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

**Class 3:** Robots which are fully controllable (in their workspace) with the observation of the leg directions
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

**Class 4:** Robots which are fully controllable (in their workspace) thanks to additional measurements

A *PRRRP* robot

Hidden robot: a *PRRRP* robot
Singularities appear in many systems

Fleets of agents

TRAVERSESEE Type 2
Singularities appear in many systems

UAVs, ROVs

dynamics singularity
Singularities appear in many systems

UAVs, ROVs

TRAVERSEE Type 2
Singularities appear in many systems

Reconfigurable drones

TRAVERSEE Type 2
Singularities appear in many systems

GG and AGC needs adaption
Because propellers apply force and torque which are linked (non zero and non infinite pitch screws)
Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot!
Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot! 😊

In this talk,

- I presented a tool named the “hidden robot concept” able to solve the determination of the singularity cases visual servoing based on the observation of geometric features
- we proved the conditions of singularity for $n$ coplanar points and 3 lines
- we discussed about the generalization of the “hidden robot concept” to other case studies
Conclusions

The hidden robot concept

• a tangible visualization of the mapping between the observation space and the Cartesian space
• allowed to change the way we defined the problem (control community / mechanical engineering community ⇒ dual problems)
Conclusions

The hidden robot concept

• a tangible visualization of the mapping between the observation space and the Cartesian space
• allowed to change the way we defined the problem (control community / mechanical engineering community ⇒ dual problems)

Tools used here

• Easily extendable to the rigidity-based control theory
• And maybe other problems
• But useful for you?
Concluding remarks