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Simultaneous inertia force/moment balancing and torque compensation of slider-crank mechanisms

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ABSTRACT

This paper proposes a design concept which allows the simultaneous shaking force/shaking moment balancing and torque compensation in slider-crank mechanisms. At first, the shaking force and shaking moment are cancelled via a cam mechanism carrying a counterweight. Then, the spring designed for maintaining contact in this balancing cam mechanism is used for torque minimization. For this purpose, the spring is jointed with a second cam mounted on the input crank. The proposed design concept allows the development of only one device for solving the both mentioned problems. The suggested solution is illustrated by numerical example carried out by using ADAMS software.

1. Introduction

The slider-crank mechanisms are common elements in high-speed machines and many methods (Arakelian et al., 2000; Arakelian and Smith, 2005; Lowen et al., 1983) have been developed for their balancing. The known methods can be arranged into the following groups:

(a) Balancing by counterweights mounted on the links (Artobolevskii, 1968; Campbell, 1979; Berkof, 1979a; Gheronimus, 1968). The balancing methods based on the redistribution of mass of the mechanism by adding counterweights to links are well known. However, in the case of complete shaking force balancing, this approach is generally limited to simple mechanisms having only revolute joints. It is difficult to apply to mechanisms with a slider because the conditions for complete shaking force balancing bring about serious increase in the total mass of the balanced mechanism.

(b) Harmonic balancing by counter-rotating masses (Artobolevskii, 1968; Lanchester, 1914; Shchepetilnikov, 1982). These solutions are based on harmonic analysis. The reduction of inertia effects is primarily accomplished by the balancing of certain harmonics of the shaking forces and shaking moments. Unbalanced forces and moments are approximated by Fourier series (or Gaussian least-square formulation) and then each frequency component is studied. This solution has found wide application as it may be accomplished by attaching balancing elements to the crank. This approach has been used successfully for engine balancing. However, it is not applied on the off-set slider-crank mechanisms.

(c) Self-balancing via a double mechanism (Artobolevskii, 1968; Arakelian, 1998, 2006; Davies, 1968; Dresig and Holzweißig, 2004; Filonov and Petrikovetz, 1987; Koropetz, 1979; Turbin et al., 1978). The addition of an axially symmetric duplicate mechanism to any given mechanism will make the new combined centre of mass stationary and thus balances the shaking force. This approach involves building self-balanced mechanical systems, in which two identical mechanisms execute similar but opposite movements. In this case the shaking force is cancelled together with the shaking moment. A partial balancing is also possible by this approach.

(d) Balancing by added dyad (Arakelian, 1998; Arakelian and Smith, 1999; Doronin and Pospelov, 1991; Frolov, 1987). The parallelogram loop, consisting of the initial links of the mechanism and the added dyad, transfers the motion of the coupler link to a shaft on the frame, where it is connected to a counterweight of considerably reduced mass. Partial balancing may be achieved by the generation of an approximate straight-line movement of a counterweight mounted on the added dyad. Among several works may be distinguished also the studies in which pantograph mechanism properties are used. The aim of this approach is to balance the mechanism by using the copying properties of the pantograph formed from the links of the initial mechanism and added links. The pantograph carries a counterweight that achieves the condition necessary for shaking force and shaking moment balancing.

(e) Balancing by planetary systems (Arakelian and Smith, 1999; Berestov, 1978; Gao, 1990; Ye and Smith, 1994). The application of planetary systems allows the cancellation of the shaking moment

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of mechanisms. However, such a balancing can only be reached by a considerably complicated design of the initial mechanism.

(f) Balancing by using a cable and pulley arrangement (Berkof, 1979b). In this case the opposite motions of the balancing counterweight and the slider is achieved via a cable and pulley arrangement.

(g) Balancing by using a cam mechanism (Kamenski, 1968; Kato, 1995, 1997; Krause, 1987; Schrick and Hanula, 1995). In this approach the reduction of inertia forces has been performed by means of a cam carrying a counterweight and it was shown how cam-driven masses may be used to keep the total centre of mass of a mechanism stationary.

It is known that the inertia force balancing can be only achieved by adding complementary masses and it brings an increase in the input torque. The input torque may be reduced by optimal redistribution of moving masses (Arakelian, 2007; Berkof, 1979b; Chaudhary, 2007; Soong, 2001; Yan and Soong, 2001) or by using non-circular gears (Yao and Yan, 2003). One of the more efficient methods used to solve the problem of input torque balancing is creating a cam-spring mechanism, in which the spring is used to absorb the energy from the system when the torque is low, and release energy to the system when the required torque is high. It allows reducing the fluctuation of the periodic torque in the high-speed mechanical systems (Angeles and Wu, 2001; Arakawa et al., 1997; Benedict et al., 1971; Benedict and Tesar, 1970; Funk and Han, 1996; Guilan et al., 1999; Nishioka, 1999; Nishioka and Yoshizawa, 1995; Poludov, 1979).

In mechanical design, these two problems are considered separately, i.e. the mechanism can be balanced by mentioned methods and, after, its input torque can be compensated by a cam-spring mechanism.

In this paper, a new design approach is developed, which proposes simultaneous inertia force balancing and torque compensation in slider-crank mechanisms.

2. Design of the inertia force/moment balanced and torque compensated slider-crank mechanism

Fig. 1 shows an off-set slider-crank mechanism, which contains an initial slider-mechanism *OAB* with crank 1 mounted on the frame, rod 2 and slider 3, as well as cams 4, 5 with followers 6, 7 and a compression spring 8.

Let us first consider the inertia force and moment balancing of the slider-crank mechanism. For this purpose, we consider that rod 2 is a "physical pendulum" link (Berkof, 1973) (see also Arakelian, 2007), i.e. its mass distribution allows the dynamic substitution of the rod's mass by two point masses, also,

$$\begin{bmatrix} 1 & 1 & 1 \\ l_{AS2} & 0 & -l_{BS2} \\ l_{AS2}^2 & 0 & l_{BS2}^2 \end{bmatrix} \begin{bmatrix} m_A \\ 0 \\ m_B \end{bmatrix} = \begin{bmatrix} m_2 \\ 0 \\ I_{S2} \end{bmatrix}, \tag{1}$$

where m_A and m_B are point masses; m_2 is the mass of rod 2; I_{S2} is the axial moment inertia of the rod about the centre of mass S_2 of the link; l_{AS2} and l_{BS2} are the distances between the centres of the joints *A* and *B* and the centre of mass S_2 of link 2, respectively.

Thus, the dynamic model of the rod represents a weightless link with two point masses m_A and m_B situated in the centres of corresponding joints. This dynamic model is fully equivalent to the real rod.

We now require rotating masses to be balanced about point *O*. Therefore, the displacement of the counterweight mounted on follower 6 is selected in such a manner that the inertia force of the follower 6 with counterweight will be opposite to the inertia force of the masses m_3 and m_B carried out reciprocating motion:

$$\ddot{x}_{S6} = -\frac{m_B + m_3}{m_6}\ddot{x}_B, \tag{2}$$

where m_3 is the mass of the slider 3, m_6 the mass of follower 6 with counterweight, $\ddot{x}_{S6}$ the acceleration of the follower 6 and $\ddot{x}_B$ the acceleration of the slider.

Therefore, in order to generate a prescribed reciprocating motion of follower 6 with acceleration $\ddot{x}_{S6}$, the cam 4 is used. Note please that cams 4 and 5 are balanced about the centre of rotation *O*, i.e. the centres of mass of these cams are situated on the axis of rotation *O*. With regard to the crank 1, it is balanced with the substituted mass m_A by means of the counterweight mounted on the input crank.

After such a redistribution of moving masses, the shaking force and shaking moment are cancelled and the slider-crank mechanism transmits no inertia loads to surrounding. However, it is known that the added masses destined to balance the inertia force increase the input torque of the mechanism. For minimization of the input torque of mechanism, one uses another technique which consists in adding a cam-spring compensation device.

The above-mentioned literature review shows that these two problems, i.e. inertia force balancing and torque compensations, are studied separately and it is considered that they are not coupled. Thus, according with the known design approaches, two devices will be developed and coupled with the slider-crank mechanism.

It will be show that the design of these two problems can be considered together and the spring used for maintaining the contact between the counterweight and the cam may also be used for balancing the input torque. For this purpose, we use the elastic force of spring 8 designed for maintaining contact between follower 6 and cam 4. In other words, we will use the spring 8 for the generation of a complementary moment on the input crank.

Let us now consider the input torque compensation.

The input torque τ of the dynamically balanced mechanism with the spring can be written under the form:

$$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta}, \tag{3}$$

where $L = T - V$ is the Lagrangian of the system, T is kinetic energy and V its potential energy. Neglecting the mass of the spring and follower 7, T and V can be written as:

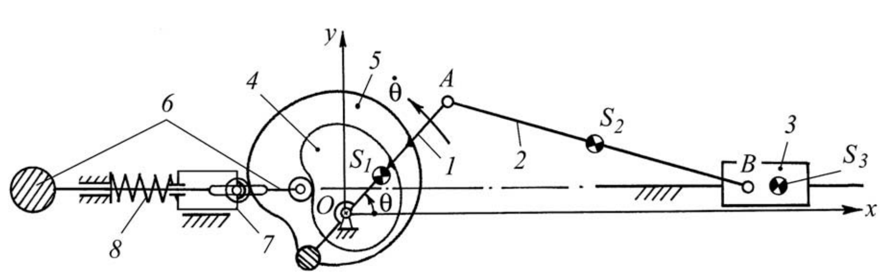


Fig. 1. Balanced and torque compensated off-set slider-crank mechanism.

$$T = 0.5(I_{S1}\dot{\theta}^2 + m_1\dot{x}_{S1}^2 + m_{cp}\dot{x}_{cp}^2) + 0.5(I_{S2}\dot{\theta}_2^2 + m_2\dot{x}_{S2}^2) + 0.5m_3\dot{x}_B^2 + 0.5m_6\dot{x}_{S6}^2$$
$$= 0.5(I_{S1} + (m_{cp}r_{cp}^2 + m_1r_1^2 + m_A)L_{OA}^2)\omega^2 + 0.5(m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\dot{x}_B^2, \quad (4)$$

$$V = 0.5k\delta^2, \quad (5)$$

where

- $\omega = \dot{\theta}$ is a constant for steady-state conditions,
- $\dot{x}_{S2}$ is the velocity of the centre of masses of element 2 and $\dot{\theta}_2$ its angular velocity
- k is the spring constant,
- δ is the displacement of the end of the spring from its equilibrium position,
- L_{OA} is the distance between joint centres O and A,
- m_{cp} is the mass of the counterweight mounted on the element 1,
- r_{cp} is the dimensionless position of this counterweight ($r_{cp} = (m_1 l_{OS1}/l_{OA} + m_A)/m_{cp}$),
- m_1 is the mass of the element 1 plus the cam 4 and I_{S1} their global axial moment of inertia,
- r_1 is the dimensionless position of the global mass centre of these elements ($r_1 = l_{OS1}/l_{OA}$).

Thus, the first and second terms of the Lagrange equation (3) become:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2(m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)\left(\frac{\partial^2 x_B}{\partial \theta^2}\right)\omega^2, \quad (6)$$

$$\frac{\partial L}{\partial \theta} = (m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)\left(\frac{\partial^2 x_B}{\partial \theta^2}\right)\omega^2 - k\frac{\partial \delta}{\partial \theta}\delta \quad (7)$$

taking into account that

$$x_B = L_{OA}\cos\theta \pm \sqrt{L_{AB-}^2(y_B - L_{OA}\sin\theta)^2} \quad (8)$$

where L_{AB} is the distance between joint centres A and B.

Therefore, the input torque can be deduced:

$$\tau = (m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)\left(\frac{\partial^2 x_B}{\partial \theta^2}\right)\omega^2 + k\frac{\partial \delta}{\partial \theta}\delta. \quad (9)$$

In order to avoid torque fluctuation, the spring has to create a force that makes the input torque constant. In many cases, when a mean value of the torque moment is equal to zero, as in the case of unloaded slider-crank mechanism, this constant is equal to zero. Thus, in such a case, the compensation of the input torque is equivalent to its cancellation. From the point of view of the energy fluctuation, after such a compensation, the periodic variations of the input torque is cancelled and the required input torque is equal to zero, i.e. in the stationary operating mode, the generation of motion may be accomplished by only a very small input torque, which is needed for overcoming friction.

Thus, under the condition that the input torque is equal to zero with the cam-follower system, Eq. (9) admits the integral:

$$(m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)^2\omega^2 + k\delta^2 = A, \quad (10)$$

where A is an integration constant. Note that the value of A represents two times the sum of the kinetic energy stored in the mechanism (without rod 1 and the cam 5) plus the potential energy in the spring.

For the continuity of contact between the cam and the follower, the force created by the spring has to be always superior to the inertia force of the follower 6 with the counterweight, i.e.

$$k\delta \geqslant (m_3 + m_B)\ddot{x}_B. \quad (11)$$

Also,

$$\delta^2 \geqslant \frac{(m_3 + m_B)^2}{k^2}\ddot{x}_B^2. \quad (12)$$

From expression (10), the value of δ may also be computed:

$$\delta^2 = \frac{A - (m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)^2\omega^2}{k}. \quad (13)$$

Combining Eqs. (12) and (13) leads to,

$$A \geqslant (m_B + m_3)\left(\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)^2 + \frac{m_3 + m_B}{k}\frac{\partial^2 x_B}{\partial \theta^2}\right)\omega^2. \quad (14)$$

In order Eq. (14) to be valuable for any value of θ, constant A should be equal to:

$$A = \max_{\theta\in[0,2\pi]}\left((m_B + m_3)\left(\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)^2 + \frac{m_3 + m_B}{k}\frac{\partial^2 x_B}{\partial \theta^2}\right)\omega^2\right). \quad (15)$$

To avoid resonance, the spring should be stiff enough so that the lowest natural frequency of the system is considerably higher than the highest significant harmonic of the output motion of the follower (Angeles and Wu, 2001). After the appropriate spring constant is selected, the displacement of the spring can be determined from Eqs. (13) and (15), namely:

$$\delta = \sqrt{\frac{A - (m_B + m_3)\left(1 + \frac{m_B + m_3}{m_6}\right)\left(\frac{\partial x_B}{\partial \theta}\right)^2\omega^2}{k}}. \quad (16)$$

Thus here we find the cam profile for torque compensation.

The next part presents an illustrative example of the proposed approach. The simulations have been carried out by using ADAMS software.

3. Illustrative example

3.1. Shaking force and shaking moment balancing

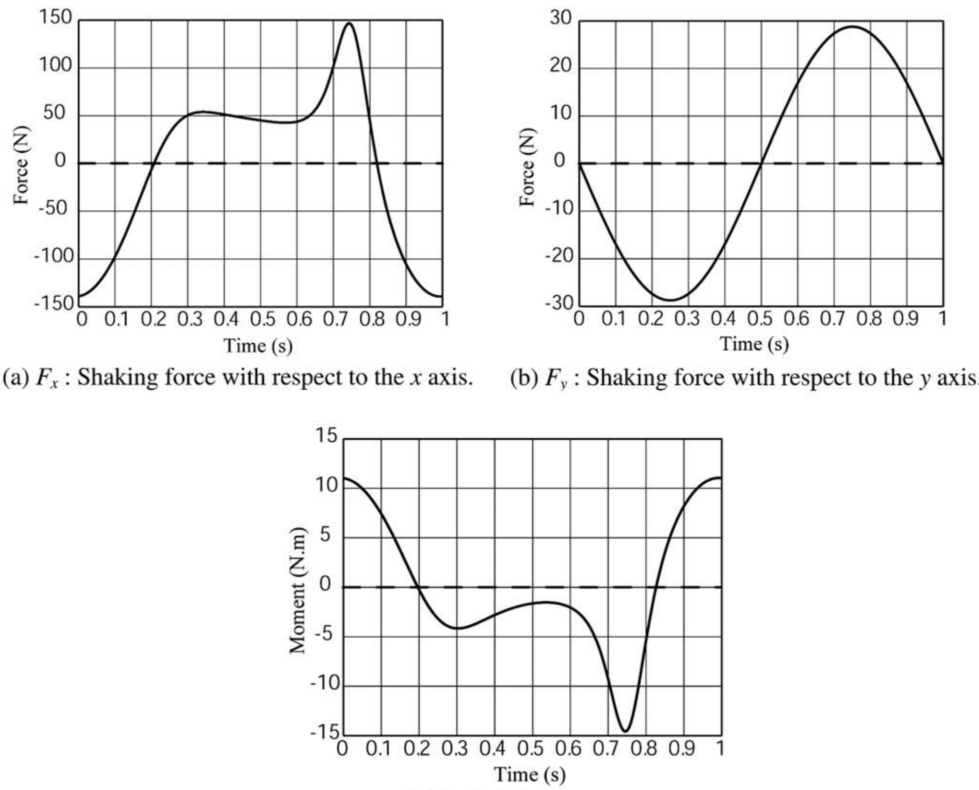
The following parameters of mechanism's links are specified for the simulations: $L_{OA} = 0.292$ m; $L_{AB} = 0.427$ m; $r_1 = 0.5$; $y_B = 0.1$ m; $m_1 = 2$ kg; $m_2 = 3$ kg; $m_3 = 4$ kg; $I_{S1} = 0.03$ kg/m^2, $I_{S2} = 0.14$ kg/m^2. The period of the mechanism is fixed to 1 s.

The shaking forces and shaking moment of the above-mentioned mechanism are represented in Fig. 2 (full line).

By selecting $L_{cp} = r_{cp}L_{OA} = 0.2$ m, we obtain $m_{cp} = 3.25$ kg. This counterweight will be mounted on the input crank. The second counterweight is carried out the reciprocating motion. In order to reduce the size of the cam, the displacement of the centre of mass S_6 of link 6 is three times smaller than the displacement of point B. Therefore $m_6 = 13.5$ kg. Fig. 3 shows the obtained cam profile. The variations of the shaking forces and shaking moment of the mechanisms with redistributed moving masses are given in Fig. 2 by dashed lines. The simulation results show that after balancing the shaking forces and moment are cancelled.

3.2. Input torque compensation

The parameters are identical to those used in the previous part. The constant spring is fixed to $k = 10$ kN/m. The joint between the cam and the follower has been modeled by a contact between two solid bodies. The linear spring 8 is designed in such a manner that it ensures a permanent contact in the cam mechanisms. Thus, the



(a) F_x : Shaking force with respect to the x axis.

(b) F_y : Shaking force with respect to the y axis.

(c) M : Shaking moment

Fig. 2. Variations of the shaking forces and shaking moment before (full line) and after (dashed line) balancing.

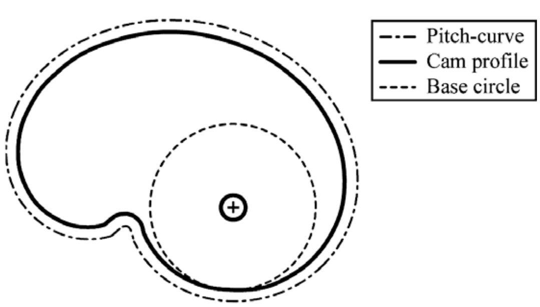


Fig. 3. Profile of the cam for the displacement of link 6 assuming the inertia forces balancing.

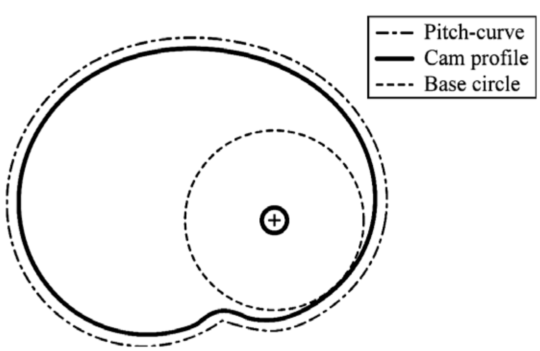


Fig. 4. Profile of the cam for the displacement of link 7 assuming the torque compensation.

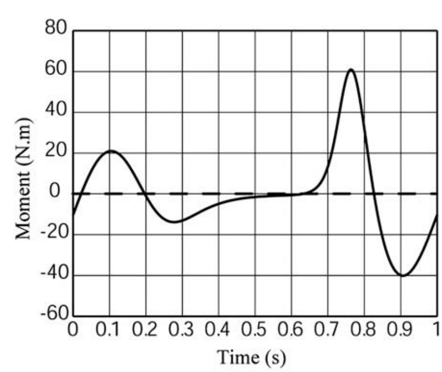


Fig. 5. Input torque before (full line) and after (dashed line) compensation.

displacements of the follower 7 are determined from Eq. (16) and the obtained cam profile is shown in Fig. 4.

The input torques before and after compensation are shown in Fig. 5.

Thus we can note that the suggested approach allows not only to carry out complete shaking force and shaking moment balancing of the of-set slider-crank mechanism but also it assumes its input torque compensation.

4. Conclusions

Fast-moving machinery with rotating and reciprocating masses is a significant source of variable dynamic loads. A major theme in machine dynamics and machine design is seeking to minimize the fluctuating forces that such machinery applies to its environment via its mount. Another theme, which is also very important in machine dynamics, is the minimization of the input torque fluctuation caused by the variable dynamic loads. These two problems are known and many methods have been developed and documented. However, these themes are considered separately, as two decoupled problems.

In this paper, for the first time, simultaneous shaking force/shaking moment balancing and torque compensation in slider-crank mechanisms is considered. The shaking force and shaking moment are cancelled via a cam mechanism carrying a counterweight. Then, the spring designed for maintaining contact in this balancing cam mechanism is used for torque compensation. For this purpose, the spring is jointed with a second cam mounted on the input crank. The proposed design concept allows the development only one devise for solution of the both mentioned problems.

The suggested solution is illustrated by simulations carried out for an off-set slider-crank mechanism.

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