

AUTOMATION AND CONTROL IN MACHINERY MANUFACTURE

Singular Positions of a Paminsa Parallel Manipulator

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Abstract—The singular positions of the PAMINSA parallel manipulator, developed by I.N.S.A (Institut National des Sciences Appliquées, Rennes, France), are considered analytically on the basis of the properties of Jacobi matrices and screw theory. The formulas obtained show all the possible manipulator configurations in singular positions. The conditions for the appearance of each manipulator position are presented. Approaches to removing the manipulator from the region of singular positions are proposed.

1. DISTINGUISHING FEATURES OF THE PAMINSA MANIPULATOR

We know that parallel manipulators offer several benefits in relation to traditional industrial robot mechanisms [1, 2]. The closed kinematic chain ensures higher structural rigidity. This reduces the size and mass of the mobile elements. With specified rated load capacity, the moving elements of a parallel manipulator are lighter than those of the analogous robot with an open kinematic chain. Thus, a rigid structure with light elements permits considerable increase in speed and positional accuracy of the manipulator.

Despite these benefits, parallel manipulators have certain disadvantages: limited working volume and dynamic interconnection of the drives. In the initial stage of parallel-manipulator development, it was assumed that the interconnection of the drives is beneficial since the load is distributed between the drives. However, their practical introduction showed that the dynamic interconnection of the drives complicates control, especially at large acceleration.

Accordingly, it is necessary to create manipulators with disconnected motions. Control is simpler for a manipulator with a linear relationship between the input and output parameters, rather than a nonlinear relationship. To this end, various structures characterized by linear input and output equations and by complete independence of the manipulator motions have been proposed [3, 4].

Note, however, that such uncoupling of the motions reduces the rigidity of the system or entails the installation of guidepieces. Evidently, it is difficult to reconcile complete uncoupling of the motions with retention of the significant benefits of parallel manipulators. Therefore, it is expedient to partially uncouple the motions, while retaining some important benefits of parallel manipulators.

We now consider the interrelation of the drives in parallel manipulators and propose an approach to partial uncoupling of the motions. The Delta robot with four degrees of freedom [5] (Fig. 1) consists of a base 1 connected to platform 7 by three kinematic chains. The three translational displacements of the platform in directions x , y , and z are controlled by three drives 2 mounted on the base. The clamp (output unit) 8 is capable of free rotation around vertical axis 9 and is connected to a fourth drive 10 through telescopic shaft 12. Two universal joints at the ends of the telescopic shaft permit clamp rotation around the vertical axis in any position of the platform. Such clamp rotation is not produced by displacement of the elements of the basic manipulator structure (3-6, 13, 14) but is an independent motion. In some versions of the Delta robot used in surgery, the technological shaft is replaced by a special device mounted on the platform. This ensures rotation of the clamp relative to three axes and permits six degrees of freedom of the robot. Thus, the drives ensuring translational motion of the platform are coupled. Motion of the platform along the x , y , and z axes calls for

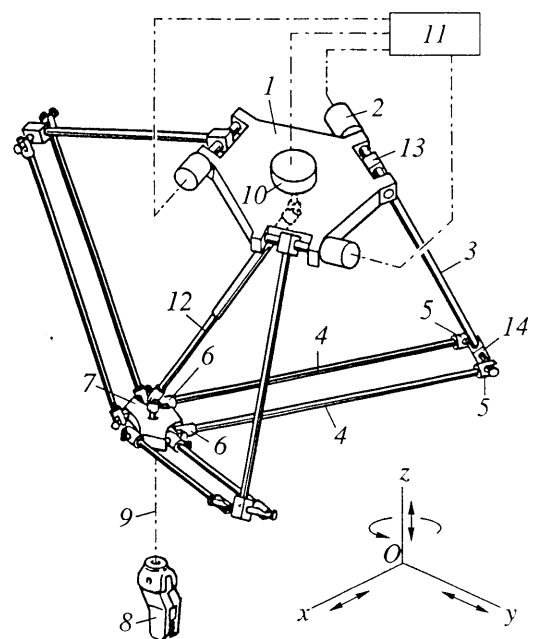


Fig. 1.

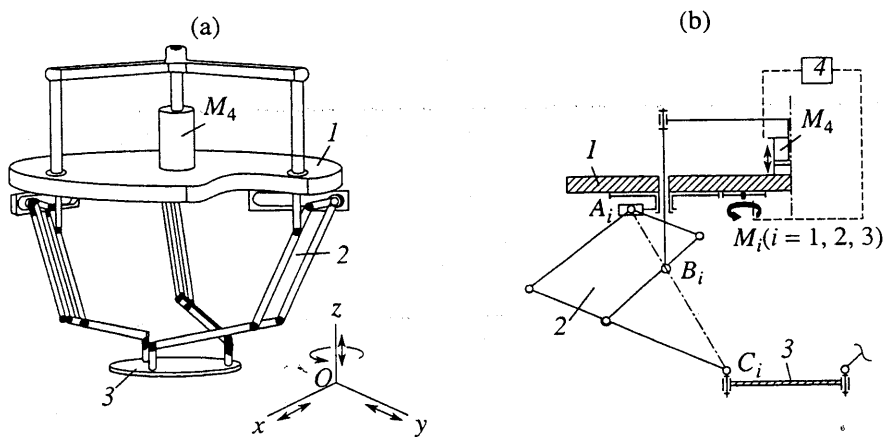


Fig. 2.

synchronized control of the drives 11 . The coupling of the motions has geometric, kinematic, and dynamic aspects, which are directly reflected in the positional accuracy of the platform. If the platform must be established at some constant height z and must move in the horizontal plane xOy , the motion of the three drives must be synchronized. In dynamic operation, with motion of the loaded platform, the height z varies with respect to its rated value by some amount $\pm\Delta z$, as a consequence of the errors in synchronous drive control.

This deviation depends on the trajectory, velocity, and acceleration of the platform, the load on the platform, and the manipulator position within the working volume. Even in static loading, the loads on the drives due to the weight of the manipulator components are described by nonlinear equations. Algorithms exist to solve static and dynamic nonlinear control problems, ensuring relatively reliable manipulator operation. However, for general use, a manipulator with a linear relation between the input and output parameters is preferable to the solution of nonlinear dynamic problems by means of complex algorithms included in the control system.

We now consider a new type of motion uncoupling in parallel manipulators. Suppose that the manipulator structure permits resolution of the motions into two components: along the vertical axis and in the horizontal plane. In such uncoupling, the work of the gravitational forces in the horizontal motion of any object is zero, since the force is perpendicular to the motion. The motion of gravitational forces in vertical motion is nonzero, since the force is parallel to the motion. This approach has been successfully used in balanced manipulators [6, 7], in which the object being manipulated is moved manually in the horizontal plane, where little effort is required, while vertical motion is mechanized.

This provides the basis for the design of a new parallel manipulator: the PAMINSA manipulator, developed by I.N.S.A (Institut National des Sciences Appliquées, Rennes, France). Its kinematic system (Fig. 2a) consists of an immobile base 1 and a platform 3 connected by three identical kinematic chains. Each chain is a pantograph mechanism 2 (Fig. 2b). This structure permits translational motion of platform 3 with respect to three axes x, y, z and its rotation around vertical axis z (element 4 is the control system). Rotary drives M_1, M_2 , and M_3 are responsible for translation in the xOy plane and rotation around the z axis, while linear drive M_4 is responsible for translation along the z axis.

In this structure, all the motions of the platform in the horizontal plane are independent of its vertical displacement, i.e., specified platform height may be established by drive M_4 , with horizontal motion by drives M_1, M_2 , and M_3 .

An obvious benefit of this design is more precise vertical positioning of the platform, since the locking mechanism does not permit change in z with horizontal motion of the plane. (This results in a linear dynamic model with vertical motion of the platform and considerably simplifies manipulator control.) Another benefit is greater positional accuracy in the horizontal plane, since low-power drives M_1, M_2 , and M_3 may be employed. The energy costs of drives M_1, M_2 , and M_3 are minimal, since the load due to gravitational forces is eliminated [8]. Obviously, drives with small loads are capable of more precise positioning than drives with relatively large loads that vary periodically.

Currently, a PAMINSA prototype is under assembly.

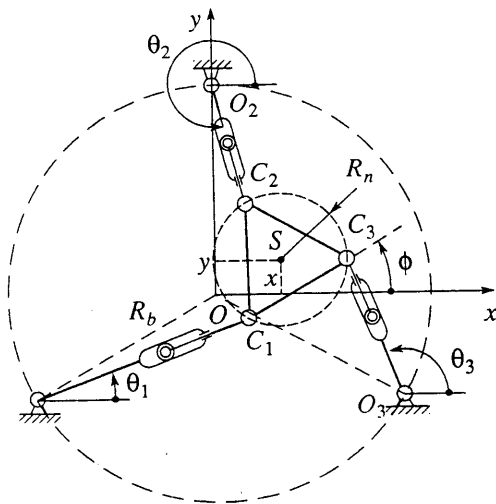


Fig. 3.

2. ANALYSIS OF SINGULAR POSITIONS OF THE PAMINSA MANIPULATOR ON THE BASIS OF JACOBI MATRICES

The physical interpretation of singular manipulations reduces to the existence of configurations where the number of degrees of freedom instantaneously changes. This is associated either with uncontrollable motion or with the loss of some degrees of mobility. An effective method of analysis of singular positions is to study the properties of the Jacobi matrix composed of input and output velocities. The manipulator is in a singular position when the determinant of the Jacobi matrix is zero or indeterminate [9]. Geometrically, such positions may be illustrated by means of screw theory. Singular positions arise with a linear dependence of the kinematic screws present in each kinematic chain or the force screws transmitted to the output element [10].

The PAMINSA manipulator has a parallel structure. Loss of controllability and loss of degrees of freedom may occur here. The pantograph mechanism constituting each kinematic chain of the manipulator has singular positions characteristic of an open chain, while the motion in the horizontal plane corresponds to a parallel structure of 3-VPV type. However, these are separate positions, since the horizontal motion is vertically uncoupled.

Methods have been developed for investigating the singular positions of manipulators with an open structure [1, 11, 12]. Using the results obtained, mechanisms with open kinematic chains in which singular positions are excluded may be designed. Attention is confined to the singular positions of the PAMINSA manipulator arising in horizontal motion of the platform.

Consider singular positions of a PAMINSA manipulator with three kinematic chains. The projection of the elements of this manipulator (Fig. 2) onto the horizontal plane is shown in Fig. 3. In this case, the horizontal motion of the platform is produced by three rotary drives M_1 , M_2 , and M_3 . The position and orientation of the mobile coordinate system of the platform in the immobile coordinate system xOy may be represented by the matrix $\mathbf{x} = (x, y, \phi)^T$, where x , y are the coordinates of the center of the isosceles triangle $C_1C_2C_3$; ϕ is the angle of rotation around this center. The input parameters are expressed by the matrix $\mathbf{q} = (\theta_1, \theta_2, \theta_3)^T$ (Fig. 3).

The relations between the input $\mathbf{q} = (\theta_1, \theta_2, \theta_3)^T$ and output $\mathbf{x} = (x, y, \phi)^T$ parameters are

$$\cos(\theta_1) \left(-R_n \cos\left(\phi + \frac{2\pi}{3}\right) - R_b/2 - y \right) - \sin(\theta_1) \left(R_n \sin\left(\phi + \frac{2\pi}{3}\right) - \sqrt{3}R_b/2 - x \right) = 0, \quad (1)$$

$$\cos(\theta_2) \left(-R_n \cos\left(\phi - \frac{2\pi}{3}\right) - R_b/2 - y \right) - \sin(\theta_2) \left(-R_n \sin\left(\phi - \frac{2\pi}{3}\right) + \sqrt{3}R_b/2 - x \right) = 0, \quad (2)$$

$$\cos(\theta_3) \left(-R_n \cos(\phi) + R_b - y \right) - \sin(\theta_3) \left(R_n \sin(\phi) - x \right) = 0, \quad (3)$$

where $R_b = l_{OC1} = l_{OC2} = l_{OC3}$ and $R_n = l_{SC1} = l_{SC2} = l_{SC3}$.

Differentiating Eqs. (1-3) with respect to the time, we obtain $\mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{q}} = 0$, where

$$\mathbf{A} = \begin{bmatrix} \sin\theta_1 & -\cos\theta_1 & R_n \sin\left(\phi + \frac{2\pi}{3} - \theta_1\right) \\ \sin\theta_2 & -\cos\theta_2 & R_n \sin\left(\phi - \frac{2\pi}{3} - \theta_2\right) \\ \sin\theta_3 & -\cos\theta_3 & R_n \sin(\phi - \theta_3) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix}.$$

Three types of possible singular positions will be considered [9].

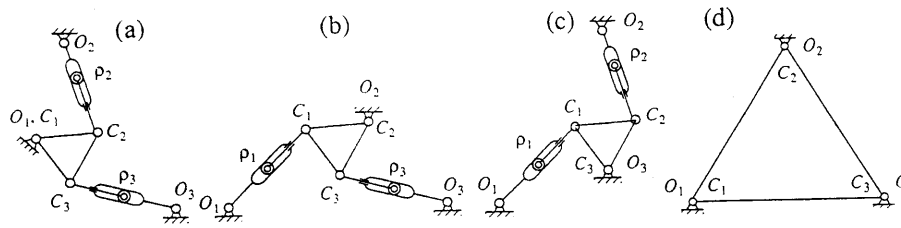


Fig. 4.

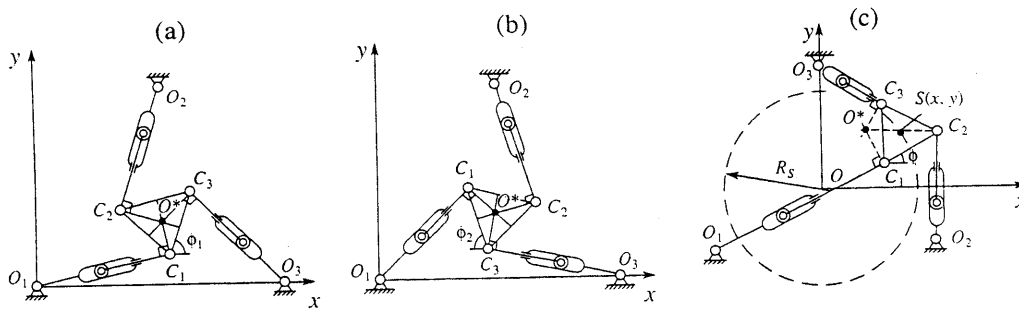


Fig. 5.

In type 1

$$\det(B) = 0. \tag{4}$$

Equation (4) is satisfied when $\rho_1\rho_2\rho_3 = 0$. These singular positions correspond to manipulator configurations in which the C_1 axis is aligned with the O_1 axis (Fig. 4a), the C_2 axis is aligned with the O_2 axis (Fig. 4b), and the C_3 axis is aligned with the O_3 axis (Fig. 4c), while all the C_1 , C_2 , and C_3 axes are aligned simultaneously with the O_1 , O_2 , and O_3 axes when $R_b = R_a$ (Fig. 4d). In these manipulator configurations, unlimited values of the input parameters θ_i ($i = 1, 2, 3$) correspond to the specified platform displacement, i.e., the motion of the input element has no influence on the change in position of the output element.

In type 2

$$\det(A) = 0. \tag{5}$$

Given that $R_n \neq 0$, Eq. (5) is satisfied when

$$R_n - R_b \cos \phi = 0 \tag{6}$$

or

$$x^2 + y^2 - (R_b^2 + R_n^2 - 2R_bR_n \cos \phi) = 0. \tag{7}$$

Equation (6) is satisfied when $\phi_{1,2} = \pm \arccos(R_n/R_b)$. The manipulator configurations corresponding to this singular position are shown in Fig. 5a ($\phi = \phi_1 = 73.4^\circ$) and in Fig. 5b ($\phi = \phi_2 = -73.4^\circ$). The angles ϕ_1 and ϕ_2 are calculated for $R_n = 100$ mm and $R_b = 350$ mm. In these positions, infinitesimal rotation around the center O^* may occur with motionless input elements of the platform.

Equation (7) may be written in the form $l_{OS} = \sqrt{x^2 + y^2} = \sqrt{R_b^2 + R_n^2 - 2R_bR_n \cos \phi}$. The manipulator configuration corresponding to this position is shown in Fig. 5c. In this configuration, the center S of triangle $C_1C_2C_3$ is on a circle of radius $\sqrt{R_b^2 + R_n^2 - 2R_bR_n \cos \phi} = R_s$. Correspondingly, with motionless input elements, the platform may perform infinitesimal rotation around center O^* .

In type 3, $\det(A) = 0/0$ and $\det(B) = 0$. When $\det(B) = 0$ and Eq. (6) or Eq. (7) applies, $\det(A)$ becomes indeterminate, i.e., $\det(A) = 0/0$. The manipulator configuration corresponding to this position is characterized by properties of types 1 and 2.

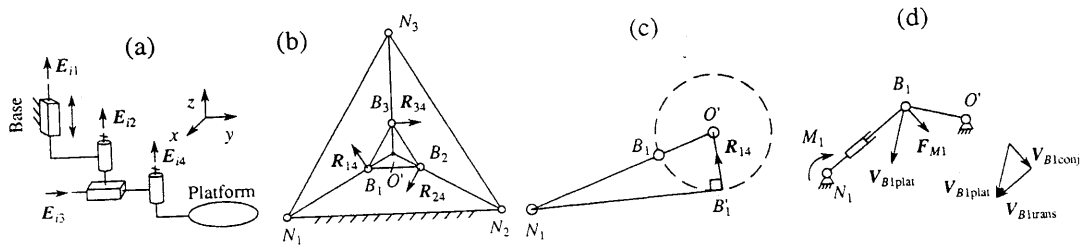


Fig. 6.

Thus, the singular positions of the PAMINSA manipulator have been analyzed on the basis of the properties of Jacobi matrices. This problem may also be illustrated using screw theory.

3. GEOMETRIC INTERPRETATION OF THE SINGULAR POSITIONS OF THE PAMINSA MANIPULATOR ON THE BASIS OF SCREW THEORY

To establish the singular positions, we will consider the kinematic and force screws corresponding to each kinematic chain. The connecting chain contains four equivalent kinematic pairs characterized by unit screws E_{i1}, \dots, E_{i4} (Fig. 6a), where i ($i = 1, 2, 3$) is the number of the kinematic chain. Screw E_{i1} corresponds to a translational pair with a drive; screw E_{i2} to a driven rotational pair; and screws E_{i3} and E_{i4} to passive translational and rotational pairs. In any manipulator position, the Plucker coordinates of the screws are as follows: $E_{i1}(0, 0, 0, 0, 0, 1)$, $E_{i2}(0, 0, 1, e_{i2x}^0, e_{i2y}^0, 0)$, $E_{i3}(0, 0, 0, e_{i3x}^0, e_{i3y}^0, 0)$, $E_{i4}(0, 0, 1, e_{i4x}^0, e_{i4y}^0, 0)$, where $i = 1, 2, 3$.

The system of force screws corresponding to the unit vectors of the kinematic-pair axes may now be determined: $R_{i1}(0, 0, 0, r_{i1x}^0, r_{i1y}^0, 0)$, $R_{i2}(0, 0, 0, r_{i2x}^0, r_{i2y}^0, 0)$ ($i = 1, 2, 3$). Without loss of generality, the screws may be expressed in the form: $R_{i1}(0, 0, 0, 1, 0, 0)$, $R_{i2}(0, 0, 0, 0, 1, 0)$. This means that each connecting kinematic chain corresponds to two force screws with an infinitely large parameter (torque). They are both perpendicular to the z axis. Thus, the constant system of force screws determining the couplings at the platform and also its possible motion has the Plucker coordinates

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (8)$$

In this case, all the force screws are pairs of forces, and the rank of the matrix in Eq. (8) is two. Therefore, there are only two independent force screws. Hence, the platform may move with four degrees of freedom. This agrees with the Kotel'nikov theorem that only closed groups of screws form motion groups [10]. All four kinematic screws of the connecting kinematic chains form a four-component closed group of screws, as may be confirmed by considering all the screw products of the basic screws in the group. This means that, for any kinematic chains, the number of degrees of freedom of the platform will always be four.

In the given mechanism, all four unit screws $E_{i1}(0, 0, 0, 0, 0, 1)$, are the same. Hence, all the displacements in these kinematic pairs must be consistent. In the given manipulator, this is the case, since the displacements are produced by the same drive. If all these pairs are fixed, each kinematic chain has another mutual force screw $R_{i3}(0, 0, 1, r_{i3x}^0, r_{i3y}^0, 0)$. If two mutual force screws are perpendicular to the z axis, then: $R_{i3}(0, 0, 1, 0, 0, 0)$. In that case, the system of force screws determining the mobility of the platform has the Plucker coordinates

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

The rank of this matrix is three, and there are only three independent coupling screws. This corresponds to a plane mechanism.

In considering the singular positions, we must analyze the linear relationship of all the kinematic screws in each kinematic chain and also all the force screws corresponding to the unit vectors of the axis of the undriven kinematic pairs [10]. The first type of linear dependence corresponds to the loss of one degree of freedom of the platform. This is possible if the unit vectors of axes \mathbf{E}_{i2} and \mathbf{E}_{i4} are aligned, but this case is ruled out in the design process.

In considering the linear relation of the force screws corresponding to the unit vectors of the axes of passive pairs, two drives in each kinematic chain must be specified. In that case, we add to Eq. (9) another mutual force screw $\mathbf{R}_{i4}(r_{i4x}, r_{i4y}, 0, r_{i4x}^0, r_{i4y}^0, r_{i4z}^0)$, whose axis is perpendicular to, and intersects, the \mathbf{E}_{i3} axis. In this case, all the screws \mathbf{R}_{i4} may be resolved within a single plane. Hence, we may write their Plucker coordinates in the form: $\mathbf{R}_{i4}(r_{i4x}, r_{i4y}, 0, 0, 0, r_{i4z}^0)$. Thus, analysis of the singular positions of this type may be based on the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ r_{14x} & r_{14y} & 0 & 0 & 0 & r_{14z}^0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ r_{24x} & r_{24y} & 0 & 0 & 0 & r_{24z}^0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ r_{34x} & r_{34y} & 0 & 0 & 0 & r_{34z}^0 \end{bmatrix}$$

In general, this matrix is of rank six, but in singular positions its rank is five. Consequently, the screws $\mathbf{R}_{i4}(r_{i4x}, r_{i4y}, 0, 0, 0, r_{i4z}^0)$ are dependent. This corresponds to a plane parallel mechanism with three degrees of freedom, whose singular positions were described in [13]. A simpler geometric approach is used here.

Consider the system of screws \mathbf{R}_{i4} acting on the output element of the PAMINSA manipulator. We use an auxiliary plane mechanism, whose base and output element are represented by similar triangles $N_1N_2N_3$ and $B_1B_2B_3$, respectively (Fig. 6b).

If the rotational pairs \mathbf{E}_{i2} are passive, while the translational pairs \mathbf{E}_{i3} are driven, the singular configurations are characterized by intersecting axes of the translational pairs (at point O'). On the plane, these positions form a two-dimensional region with constant orientation of the output element. The sides of the base and platform triangles are parallel [14].

For a PAMINSA manipulator, the singular position corresponds to intersection of the axes of the screws \mathbf{R}_{14} , \mathbf{R}_{24} , \mathbf{R}_{34} perpendicular to the lines N_1B_1 , N_2B_2 , N_3B_3 , respectively. (The case in which these axes are parallel is not considered here.)

It is evident that, with any position of the platform, it may rotate through an angle $\pm \arccos(B_1B_2/N_1N_2)$ about the point O' . All the axes of the screws \mathbf{R}_{14} , \mathbf{R}_{24} , \mathbf{R}_{34} intersect at point O' (Fig. 6c). The point B_1 turns through an angle $\arccos(B_1B_2/N_1N_2)$ about center O' . Points B_2 and B_3 also turn about O' .

Thus, this orientation of the platform corresponds to singular positions. These positions form a two-dimensional region in the plane. Considering the translational motion of the whole plane, we may conclude that all the singular positions of the PAMINSA manipulator correspond to a three-dimensional region. This agrees with the general case of parallel manipulators, whose singular zones are of dimensionality $W - 1$, where W is the number of degrees of freedom of the manipulator [14, 15].

It is important to investigate not only the singular positions but also the pressure angles between the forces and displacements. Consider the pressure angle at point B_1 (Fig. 6d). Here point O' is the instantaneous center of rotation of the platform. It is defined as the point of intersection of lines N_2B_2 and N_3B_3 or, in other words, the axes of the unit vectors \mathbf{E}_{23} and \mathbf{E}_{33} . A velocity plan is shown in Fig. 6d, where $\mathbf{V}_{B_1\text{plat}}$ is the velocity of point B_1 on the platform, while $\mathbf{V}_{B_1\text{conj}}$ is the velocity of point B_1 of the intermediate element (parallel to force \mathbf{F}_{m1}). The difference between $\mathbf{V}_{B_1\text{conj}}$ and $\mathbf{V}_{B_1\text{plat}}$ corresponds to the relative velocity $\mathbf{V}_{B_1\text{trans}}$ of point B_1 in the translational kinematic pair.

The angle between these vectors is the pressure angle if the contact point is at point B_1 . Otherwise, the pressure angle is larger. Accordingly, the working space is bounded by positions where the pressure angle is larger than the frictional angle.

To avoid singular positions, it is expedient to use auxiliary drives in the translational kinematic pairs \mathbf{E}_{13} , \mathbf{E}_{23} , \mathbf{E}_{33} . These drives may be controlled by the algorithms in [14].

CONCLUSIONS

Thus, the PAMINSA parallel manipulator described here has very valuable properties, on account of the structural and functional uncoupling of vertical and horizontal motion. Analysis of the singular positions breaks down into two separate problems: singular positions of the connecting kinematic chain; and singular positions of the equivalent plane mechanism.

Two approaches have been described. One, based on Jacobi matrices, yields a general solution. The other approach employs a clear geometric interpretation based on screw theory. These approaches may expediently be applied to the analysis of other similar manipulators and the investigation of their working spaces on the basis of pressure angles, Jacobians, and systems of kinematic and force screws.

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