
Sébastien Briot, François Chaumette, and Philippe Martinet

FULL EXPRESSION OF THE NULLSPACE OF THE WRENCH SYSTEM $\xi$

The expression of the nullspace given in Eq. (19) is recalled here

$$ t_{s1} = [0 \ 0 \ -ZYf_{11}(X,Y) \ -Zf_{12}(X,Y) \ -Zf_{13}(X,Y) \ (X+1)f_{14}(X,Y)]^T $$

in which $f_{11}(X,Y)$ takes the form

$$ f_{11} = \alpha_1 ((a_{21}a_{31} - a_{22}a_{32})X^2 $$

$$ + (a_{21}a_{32} + a_{22}a_{31})XY - a_{21}a_{31}) $$

with

$$ \alpha_1 = a_{21}a_{32} - a_{22}a_{31} + a_{22}n_3 - a_{32}n_2 $$

The other functions $f_{12}(X,Y)$, $f_{13}(X,Y)$ and $f_{14}(X,Y)$ are given by

$$ f_{12} = \gamma_1 \alpha_1 X^2 - (a_{22}a_{32}\beta_1 - \gamma_2\alpha_1 Y)X $$

$$ - \gamma_1 \gamma_3 - a_{21}a_{31} \beta_1 + a_{22}a_{32} \beta_2 Y $$

$$ f_{13} = X(a_{22}a_{32}\beta_2 - \gamma_1\alpha_1 Y) - \gamma_3\gamma_2 $$

$$ + Y(-a_{32}n_3 a_{21}^2 + a_{22}n_2 a_{31}^2 + \gamma_3 n_2 n_3) $$

$$ + a_{21}a_{32} n_2 - a_{22}a_{31} n_3 + \gamma_2\alpha_1 X^2 $$

$$ f_{14} = -a_{32}n_3 a_{21}^2 + a_{22}n_2 a_{31}^2 + \gamma_1\alpha_1 X^2 $$

$$ + (\gamma_2\alpha_1 Y - \gamma_3 (\gamma_1 - \beta_3 + n_2 n_3))X $$

$$ + a_{22}a_{32} \gamma_3 + \gamma_3 n_2 n_3 $$

$$ + \gamma_3 Y(a_{22}n_3 - \gamma_2 + a_{32}n_2) $$

where

$$ \gamma_1 = a_{21}a_{31} - a_{22}a_{32} $$

$$ \gamma_2 = a_{21}a_{32} + a_{22}a_{31} $$

$$ \gamma_3 = a_{21}a_{32} - a_{22}a_{31} $$

$$ \beta_1 = a_{22}n_3 - a_{32}n_2 $$

$$ \beta_2 = a_{21}n_3 - a_{31}n_2 $$

$$ \beta_3 = a_{21}n_3 + a_{31}n_2 $$

These expressions were obtained thanks to the use of the Matlab Symbolic Toolbox using the following methodology (see the Matlab scripts associated with this technical report: ScriptNullSpace_R2015a.m (or ScriptNullSpace_R2013a.m when using versions of Matlab previous to R2015)):

- Compute the components $f_{ij}$ and $m_{ij}$ of the actuation wrenches $\xi_{ij}$ when the origin of the camera frame becomes to the circumcircle of the 3-D points as shown in Eqs. (15)–(18) in the paper (lines 21–46 in ScriptNullSpace_R2015a.m)
- Build the actuation wrench matrix $\xi$ composed of all actuation wrenches $\xi_{ij}$ (line 49 in ScriptNullSpace_R2015a.m)
- Compute the nullspace of the actuation wrench matrix $\xi$ (line 52 in ScriptNullSpace_R2015a.m)
- Simplify the expression of the nullspace in order to obtain the equations shown in this report (line 60–237 in ScriptNullSpace_R2015a.m)