Type Synthesis of Two DOF Translational Parallel Manipulators with Hybrid Legs

Latifah Nurahmi, Stéphane Caro, and Sébastien Briot

Abstract This paper introduces a new methodology for the type synthesis of two degrees of freedom translational parallel manipulators with hybrid and identical legs. The type synthesis method is based upon screw theory. Two types of two degrees of freedom translational manipulators with two identical hybrid legs are identified based upon their wrench decompositions. Each leg of the manipulators is composed of a proximal module and a distal module mounted in series. The assembly conditions and the validity of the actuation scheme are also defined. Finally, some novel two degrees of freedom translational parallel manipulators are synthesized thanks to the proposed procedure.

Key words: Type synthesis; Parallel manipulators; Hybrid Legs, Screw theory.

1 Introduction

At the conceptual design stage of manipulator architectures, the idea is to construct several design alternatives by following a systematic approach. However, the information at this stage is usually qualitative and not quantitative, which makes the design process quite difficult and challenging.

A manipulator is a mechanical system that aims at *manipulating* objects. Manipulating means to move something with one's hands, as the word derived from the Latin *manus*, meaning *hand* [1]. For simple task such as pick-and-place operations, the two degrees of freedom (*dof*) parallel manipulators may be sufficient. Several two-*dof* translational parallel manipulators (*TPM*) are composed of a planar architecture that yields their stiffness quite low along the normal to the plane of motion [8, 10, 6, 9, 2, 4]. Moreover, those manipulators are usually not composed of identical legs.

In order to increase the stiffness properties of the two-*dof TPM*, some researchers have proposed a new manipulator architecture named the Par2 [3]. This architecture

Institut de Recherche en Communications et Cybernétique de Nantes, France,
e-mails: {nurahmi, caro, briot}@irccyn.ec-nantes.fr

has the particularity to be spatial instead of planar and thus is stiffer along the normal to the plane of motion. However, it contains some inactive joints in each parallelogram joint. Two legs amongst the four legs of the manipulator are linked to each other with a rigid belt in order to constrain the rotation of the moving platform. As a consequence, it leads to a robot with a poor accuracy. To avoid the design issues of the Par2, a new robot with spatial architecture and two legs has been developed: the IRSBot-2 [5]. Each leg of the IRSBot-2 is hybrid, i.e., it is composed of a proximal module and a distal module mounted in series, each module containing two kinematic chains. This mechanism has exhibited interesting stiffness properties. Therefore, it is of interest to focus on the type synthesis of two-*dof TPM* by considering architectures with hybrid legs.

The subject of the paper is about the type synthesis of two-*dof TPM* with identical and hybrid legs. Each leg is composed of a proximal module and a distal module mounted in series. Those modules contain one or two kinematic chains. These kinematic chains are called sub legs. This research work has been carried out in the framework of a French National Project[1] that aims to develop some fast and accurate robots with a large operational workspace.

In this paper, the general approach for the type synthesis of the manipulators is developed based on the procedure described in [7]. However, note that the latter does not allow the designer to synthesize robot architectures containing hybrid legs. This proposed approach is decomposed into five steps: (*i*) Classification of the wrenches; (*ii*) Decomposition of the wrench of the proximal and distal modules; (*iii*) Type synthesis of the sub legs; (*iv*) Assembly of the sub legs and legs; (*v*) Selection of the actuated joints.

2 Two DOF Hybrid Manipulators with Two Identical Legs

The general approach for the type synthesis of two *dof* hybrid manipulators with two identical legs are presented using the following procedure.

2.1 Step 1: Classification of the Wrenches

The moving platform of two-*dof* hybrid manipulators is intended to have two translational motions in plane ($\mathbf{x}O\mathbf{z}$). Therefore, the twist system of the moving-platform amounts to a $2\xi_\infty-$system[2]. Therefore, the overall wrench system $\mathscr{W}$ is $1\zeta_0 - 3\zeta_\infty-$[3]. Such manipulators can be obtained by a combination of any leg-wrench system with order $c^i (0 \leq c^i \leq 4)$, decomposed as follows:

- $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system
- $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system, $3\zeta_\infty-$system
- $c^i = 2 \rightarrow 1\zeta_0 - 1\zeta_\infty-$system, $2\zeta_\infty-$system

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[2] ξ_∞ denotes an infinite-pitch twist, namely, a pure translation.

[3] ζ_0 and ζ_∞ denote a zero-pitch wrench (a pure force) and an infinite-pitch wrench (a pure moment), respectively.

- $c^i = 1 \rightarrow 1\zeta_0-$system, $1\zeta_\infty-$system

Let $m = 2$ be the number of legs. The leg combinations are shown in Tab. 1 where $\triangle$ denotes the over constraint degree. c^i denotes the order of leg-wrench system of leg i.

It can be seen from Tab. 1 that there are five types of mechanisms (colored in gray) with two identical leg-wrench systems. However, only two types of two-*dof* manipulators can be properly assembled with two similar legs, namely,

- **Type 1**: $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system
- **Type 2**: $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system

Table 1: Combination of Leg-wrench System with Two Identical Legs

m	c	$\triangle$	$1\zeta_0-3\zeta_\infty$ c=4	$1\zeta_0-2\zeta_\infty$ c=3	$3\zeta_\infty$ c=3	$1\zeta_0-1\zeta_\infty$ c=2	$2\zeta_\infty$ c=2	$1\zeta_0$ c=1	$1\zeta_\infty$ c=1
2	4	4	2						
		3	1	1					
			1		1				
		2	1			1			
			1				1		
				2					
				1	1				
					2				
		1	1					1	
			1						1
				1		1			
				1			1		
					1	1			
					1		1		
		0				2			
						1	1		
							2		
				1				1	
				1					1
					1			1	
					1				1

Certain combinations of $3\zeta_\infty-$system and $2\zeta_\infty-$system are not able to produce ζ_0 along **y**-axis in any configuration. Furthermore, the combination of $1\zeta_0 - 1\zeta_\infty-$system cannot generate three independent infinite-pitch wrenches ($3\zeta_\infty-$system). Hence, they are eliminated.

The overall wrench system of one leg is determined by the intersection of the wrench systems associated with the proximal and distal modules as they are mounted in series. Therefore, the wrench systems associated with the proximal and distal modules are decomposed as follows:

2.1.1 Type 1: $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system

The overall wrench system associated with a leg i of Type 1 is of order $c^i = 4$ and expressed as $1\zeta_0 - 3\zeta_\infty-$system. Thus, the feasible wrench systems for the proximal and distal modules are the following:

Proximal module:
1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$

Distal module:
1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$

2.1.2 Type 2: $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system

The overall wrench system associated with a leg i of Type 2 is of order $c^i = 3$ and expressed as $1\zeta_0 - 2\zeta_\infty-$system. Thus, the feasible wrench systems for the proximal and distal modules are defined as follows:

Proximal module:
1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
3. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2})$
4. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2})$

Distal module:
1. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2})$
2. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2})$

2.2 Step 2: Decomposition of the Wrench for Proximal and Distal Modules

The next step for type synthesis of two *dof* hybrid manipulators is the decomposition of the wrench. Type 1 and Type 2 have almost equivalent wrench systems for the proximal and distal modules:

1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
3. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2})$
4. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2})$

The previous four types of wrench systems correspond to the sub leg wrench systems. All potential sub leg combinations are presented in Tab. 2.

2.3 Step 3: Type Synthesis of Sub Legs

2.3.1 Step 3a: Type Synthesis of Single-loop Kinematic Chains

The following mobility criterion [7] is useful to define the types and number of joints in a sub leg:

$$f = F + (6 - c) \tag{1}$$

Table 2: Combination of Two Identical Sub Leg-wrench Systems for Proximal and Distal Modules

c	Wrench System for Proximal and Distal Modules	Sub Leg-wrench System
5	$2\zeta_0 - 3\zeta_\infty$	$2\zeta_0 - 3\zeta_\infty$
		$1\zeta_0 - 3\zeta_\infty$
		$2\zeta_0 - 2\zeta_\infty$
		$1\zeta_0 - 2\zeta_\infty$
		$2\zeta_0 - 1\zeta_\infty$
4	$1\zeta_0 - 3\zeta_\infty$	$1\zeta_0 - 3\zeta_\infty$
		$1\zeta_0 - 3\zeta_\infty$
		$1\zeta_0 - 3\zeta_\infty$
	$2\zeta_0 - 2\zeta_\infty$	$2\zeta_0 - 2\zeta_\infty$
		$1\zeta_0 - 2\zeta_\infty$
		$2\zeta_0 - 1\zeta_\infty$
		$1\zeta_0 - 1\zeta_\infty$
		$2\zeta_0$
3	$1\zeta_0 - 2\zeta_\infty$	$1\zeta_0 - 2\zeta_\infty$
		$1\zeta_0 - 1\zeta_\infty$

where f is the number of 1-*dof* joints, F is the mobility of a single-loop kinematic chain, and c is the order of the wrench system.

Type synthesis of single-loop kinematic chains is illustrated for $2\zeta_0 - 3\zeta_\infty-$system only due to space limitation. The number of joints that involves a P-virtual chain[4] and has a $2\zeta_0 - 3\zeta_\infty-$system is defined as follows:

$$c^i = 5 \quad , \quad f = F + (6-c) = 1 + (6-5) = 2 \ joints \tag{2}$$

This single-loop kinematic chain can only be formed by one P joint, which is perpendicular to the axes of $2\zeta_0$ as depicted in Fig. 1.

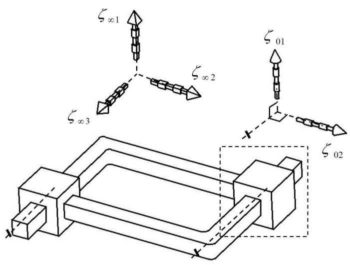

Fig. 1: Single-loop Kinematic Chain $c^i = 5$: P-Virtual Kinematic Chain

[4] P stands for a prismatic joint and R stands for a revolute joint.

2.3.2 Step 3b: Generation of Types of Sub Legs

The types of sub legs are derived by removing the P-virtual chain. It appears that there are limited types of sub legs that are free of inactive joint. Those types of sub legs are given in Tab. 3. Nonetheless, several types of sub legs that contain a non-invariant sub leg-wrench system are still kept as they can produce two-*dof* translational motions. All these types of sub legs are exhaustive and can be assembled to generate either proximal modules or distal modules.

2.4 Step 4: Assembly of Sub Legs and Legs

The assembly process of two-*dof* hybrid manipulators is performed both for Type 1 and Type 2. Each leg of the two-*dof TPM* are realized by connecting a proximal and a distal module in series. Then, each leg becomes a hybrid manipulator and is attached to the base at one end and to the moving-platform at the other end. Therefore, the assembly process consists of two steps as explained thereafter.

2.4.1 Step 4a: Assembly of Sub Legs → Proximal and a Distal Modules

The proximal and distal modules can be obtained by assembling some sub legs from Tab. 3. Nevertheless, the following conditions should be respected:

1. The overall wrench system of a module should constitute the desired wrench system, as explained in Step 1.
2. At least one translational twist generated by the module should lie in plane ($\mathbf{x}O\mathbf{z}$).

2.4.2 Step 4b: Assembly of Legs → Two-dof Hybrid Manipulator

The legs of the two-*dof TPM* are synthesized by mounting in series the proximal and distal modules derived in Step. 4a. However, the following conditions should be fulfilled:

1. The wrench system of the leg should of Type 1 or Type 2, namely, it should be a$1\zeta_0 - 3\zeta_\infty -$system or a $1\zeta_0 - 2\zeta_\infty -$system.
2. The linear combination of the wrench systems associated with the legs should be a $1\zeta_0 - 3\zeta_\infty -$system.

Figure 2 illustrates a novel two-*dof TPM* with identical and hybrid legs. This is a Type 1 mechanism that has been synthesized with the proposed approach. Each leg has a $1\zeta_0 - 3\zeta_\infty -$wrench system. Both proximal and distal modules have a $2\zeta_0 - 3\zeta_\infty -$wrench system and are composed of two RRR legs, known as *Sarrus Linkage*. This novel mechanism is named Q-Sarrus, Q standing for Quadruple.

Figure 3 depicts another mechanism synthesized with the proposed type-synthesis approach. This mechanism is named IRSBot-2 [5] and is of Type 2. Its proximal modules have a $2\zeta_0 - 3\zeta_\infty -$system and are made up of a Π joint. Its distal modules have a $2\zeta_0 - 2\zeta_\infty -$system and are composed of 2-UU kinematic chains.

Table 3: Types of Sub Legs for Proximal and Distal Modules Free of Inactive Joint

c	Wrench System for Proximal and Distal Modules	Sub Leg-wrench System	Type	Note
5	$2\zeta_0-3\zeta_\infty$	$2\zeta_0-3\zeta_\infty$	P	
		$2\zeta_0-2\zeta_\infty$	RR	Non-invariant
		$1\zeta_0-2\zeta_\infty$	RRR	
			PRR	
4	$1\zeta_0-3\zeta_\infty$	$1\zeta_0-3\zeta_\infty$	PP	
		$1\zeta_0-2\zeta_\infty$	RRR	
			PRR	
		$1\zeta_0-1\zeta_\infty$	RRŔŔ	Non-invariant
	$2\zeta_0-2\zeta_\infty$	$2\zeta_0-2\zeta_\infty$	PR	
			RR	Non-invariant
		$1\zeta_0-2\zeta_\infty$	RRR	
			PRR	
		$2\zeta_0-1\zeta_\infty$	RRŔ	Non-invariant
			RŔŔ	Non-invariant
		$1\zeta_0-1\zeta_\infty$	RRŔŔ	Non-invariant
		$2\zeta_0$	RRŔŘ	Non-invariant
			PRŔŘ	
3	$1\zeta_0-2\zeta_\infty$	$1\zeta_0-2\zeta_\infty$	RRR	
			PRR	
			PPR	
		$1\zeta_0-1\zeta_\infty$	RRŔŔ	Non-invariant

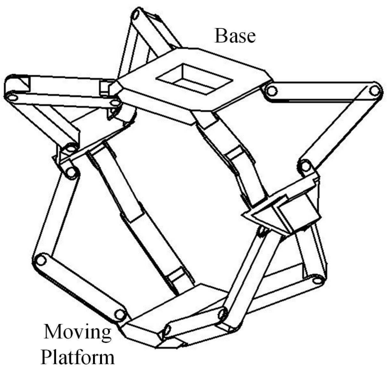

Fig. 2: Q-Sarrus

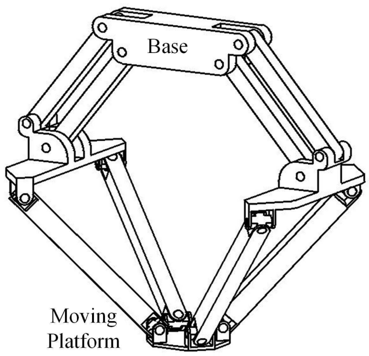

Fig. 3: IRSBot-2

2.5 Step 5: Selection of the Actuated Joints

Let assume that the condition of constraint wrench system is satisfied, namely, the assembly of legs applies a $1\zeta_0-3\zeta_\infty-$system. In a general configuration, a set of constraint wrench system, $\mathscr{W}^c$, together with an actuation wrench system, $\mathscr{W}^a$, constitute a 6-system. Ultimately, the selection of actuated joints for two-*dof TPM* can be made in such a way that a basis of the actuation wrench system $\mathscr{W}^a$ contains at least two actuation forces.

3 Conclusion

A general approach has been introduced in this paper for the type synthesis of two-*dof TPM* with hybrid and identical legs. The proposed approach is based on screw theory and is complementary to the method described in [7], the latter does not allow the designer to synthesize robot architectures containing hybrid legs. Two types of two-*dof TPM* have been highlighted with regard to their leg wrench system. Moreover, many novel parallel manipulators have been obtained and two of them have been illustrated, namely, the Q-Sarrus and the IRSBot-2. The comparison of the synthesized manipulators with regard to their complexity and intrinsic stiffness is part of the future work.

References

[1] Angeles, J.: Fundamentals of Robotic Mechanical System: Theory, Methods, and Algorithm. Springer (2007)
[2] Chung, Y.h., Lee, J.w.: Design of a New 2 DOF Parallel Mechanism. 2001 IEEE/ASME International Conference on Advanced Intelligent Mechatronics. Proceedings (Cat. No.01TH8556) **1**, 129–134 (2001)
[3] Company, O., Pierrot, F., Krut, S., Baradat, C., Nabat, V.: Par2: a Spatial Mechanism for Fast Planar Two-Degree-of-Freedom Pick-and-Place Applications. Meccanica **46**(1), 239–248 (2011)
[4] Gao, F., Li, W., Zhao, X., Jin, Z., Zhao, H.: New kinematic structures for 2-, 3-, 4-, and 5-DOF parallel manipulator designs. Mechanism and Machine Theory **37**(11), 1395–1411 (2002)
[5] Germain, C., Briot, S., Glazunov, V., Caro, S., Wenger, P.: IRSBOT-2: A Novel Two-DOF Parallel Robot for High-Speed Operations. Proceedings of the ASME 2011 International Design Engineering Technical Conference & Computers and Information in Engineering Conference pp. 1–11 (2011)
[6] Huang, T., Li, M., Li, Z., Chetwynd, D.G., Whitehouse, D.J.: Planar Parallel Robot Mechanism With Two Translational Degrees of Freedom, Patent US007090458B2 (2006)
[7] Kong, X., Gosselin, C.: Type Synthesis of Parallel Mechanism. Springer, Germany (2007)
[8] Liu, X.J., Tang, X., Wang, J.: A novel 2-DOF Parallel Mechanism Based Design of a New 5-axis Hybrid Machine Tool. 2003 IEEE International Conference on Robotics and Automation (Cat. No.03CH37422) **3**, 3990–3995 (2003)
[9] Peng, B., Zeng, L., Sun, Y., Chen, X.: A Novel High Rigid 2-DOF Parallel Translating Robot. 2009 Second International Conference on Intelligent Computation Technology and Automation pp. 375–379 (2009)
[10] Torgny Brogardh: Device for Relative Movement of Two Elements, Patent US006095011A (2000)