Shaking Force Minimization of High-Speed Robots via Optimal Trajectory Planning

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Abstract The shaking force balancing is mostly obtained via an optimal redistribution of movable masses. Therefore, the full or partial cancellation of the shaking force is a complicated task, which leads to a significant increase in mass and assembly complexity. In this paper an innovative solution is developed which is based on the optimal control of the robot links centre of masses. Such a solution allows the reduction of the acceleration of the total mass centre of moving links and, consequently, the considerable reduction in the shaking forces. The efficiency of the suggested method is illustrated by the numerical simulations carried out for the three links serial manipulator. This approach is also a more appealing alternative to conventional balancing methods because it allows the reduction of the shaking force without counterweights. As a result, the input torques are also decreased, which is shown via mentioned numerical simulations.

Key words: Shaking force, balancing, high-speed robots and optimal control.

1 Introduction

Different approaches and solutions devoted to the problem of mechanism balancing have been developed and documented for one degree of freedom mechanisms [1, 2]. A new field for their applications is the design of mechanical systems for fast manipulation, which is a typical problem in advanced robotics.

The balancing of a mechanism is generally carried out by two steps: (i) the cancellation (or reduction) of the shaking force and (ii) the cancellation (or reduction) of the shaking moment. Traditionally, the cancellation of the shaking force transmitted to the manipulator frame can be achieved via adding counterweights in order to keep the total centre of mass of moving links stationary [1], via additional structures [2] or by elastic components [3].

With regard to the shaking moment balancing of manipulators, the following approaches were developed: (i) balancing by counter-rotations [4], (ii) balancing by adding four-bar linkages [5], (iii) balancing by creating redundant mechanism which generates optimal trajectories of moving links [6], (vi) balancing by prescribed rotation of the end-effector [7, 8] and (vii) balancing by adding an inertia flywheel rotating with a prescribed angular velocity [9].

In the present paper we consider a simple and effective balancing method, which allows the considerable reduction of the shaking force of non-redundant manipulators without adding counterweights. It is based on the optimal motion planning of the acceleration of the total mass centre of moving links.

2 Minimization of the shaking forces via an optimal motion planning of the total mass centre of moving links

The shaking forces $\mathbf{f}^{\text{sh}}$ of a manipulator can be written in the form:

$$\mathbf{f}^{\text{sh}} = m\ddot{\mathbf{x}}_S \tag{1}$$

where m is the total mass of the moving links of the manipulator and $\ddot{\mathbf{x}}_S$ is the acceleration of the total mass centre. The classical balancing approach consists in adding counterweights in order to keep the total mass centre of moving links stationary. However, it leads to the increase of the total mass of the manipulator. Thus, in order to avoid this drawback, in the present study, a new approach is proposed, which consists of the optimal motion planning of the total mass centre of moving links.

Classically, manipulator displacements are defined considering either articular coordinates $\mathbf{q}$ or Cartesian variables $\mathbf{x}$. Knowing the initial and final manipulator configurations at time t_0 and t_f, denoted as $\mathbf{q_0} = \mathbf{q}(t_0)$ and $\mathbf{q_f} = \mathbf{q}(t_f)$, or $\mathbf{x_0} = \mathbf{x}(t_0)$ and $\mathbf{x_f} = \mathbf{x}(t_f)$, in the case of the control of the Cartesian variables, the classical displacement law may be written in the form:

$$\mathbf{q}(t) = s_q(t)\left(\mathbf{q_f} - \mathbf{q_0}\right) + \mathbf{q_0} \tag{2a}$$

or

$$\mathbf{x}(t) = s_x(t)\left(\mathbf{x_f} - \mathbf{x_0}\right) + \mathbf{x_0} \tag{2b}$$

where $s_q(t)$ and $s_x(t)$ may be polynomial (of orders 3, 5 and higher), sinusoidal, bang-bang, etc. laws.

From expression (1), we can see that the shaking force, in terms of norm, is minimized if the norm $\left\|\ddot{\mathbf{x}}_S\right\|$ of the masses centre acceleration is minimized along the trajectory. This means that if the displacement $\mathbf{x}_S$ of the manipulator centre of masses is optimally defined, the shaking force will be minimized. As a result, the first problem is to define the optimal trajectory for the displacement $\mathbf{x}_S$ of the manipulator centre of masses.

For this purpose, let us consider the displacement $\mathbf{x}_S$ of a point S in the Cartesian space. First, in order to minimize the masses centre acceleration, the length of the path followed by S should be minimized, i.e. point S should move along a straight line passing through its initial and final positions, denoted as $\mathbf{x}_{S0}$ and $\mathbf{x}_{S\mathbf{f}}$, respectively.

Then, the temporal law used on this path should be optimized. It is assumed that, at any moment during the displacement, the norm of the maximal admissible acceleration the point S can reach is constant and denoted as $\ddot{x}_S^{\max}$. Taking this maximal value for the acceleration into consideration, it is known that the displacement law that minimize the time interval (t_0, t_f) for going from position $\mathbf{x_{S0}} = \mathbf{x_S}(t_0)$ to position $\mathbf{x_{Sf}} = \mathbf{x_S}(t_f)$ is the "bang-bang" law:

$$\begin{cases} \mathbf{x}_S(t) = s(t)(\mathbf{x}_{S\mathbf{f}} - \mathbf{x}_{S\mathbf{0}}) + \mathbf{x}_{S\mathbf{0}} \\ \dot{\mathbf{x}}_S(t) = \dot{s}(t)(\mathbf{x}_{S\mathbf{f}} - \mathbf{x}_{S\mathbf{0}}) \\ \ddot{\mathbf{x}}_S(t) = \ddot{s}(t)(\mathbf{x}_{S\mathbf{f}} - \mathbf{x}_{S\mathbf{0}}) \end{cases} \tag{3}$$

with

$$\ddot{s}(t) = \frac{1}{\|\mathbf{x}_{S\mathbf{f}} - \mathbf{x}_{S\mathbf{0}}\|} \begin{cases} \ddot{x}_S^{\max} & for\ t \le (t_f - t_0)/2 \\ -\ddot{x}_S^{\max} & for\ t \ge (t_f - t_0)/2 \end{cases} \tag{4}$$

Consequently, if the time interval (t_0, t_f) for the displacement between positions $\mathbf{x_{S0}}$ and $\mathbf{x_{Sf}}$ is fixed, the "bang-bang" law is the trajectory that minimizes the value of the maximal acceleration $\ddot{x}_S^{\max}$. Thus, in order to minimize $\|\ddot{\mathbf{x}}_S\|$ for a displacement during the fixed time interval (t_0, t_f), the "bang-bang" law has to be applied on the displacement $\mathbf{x}_S$ on the manipulator total mass centre.

Once the displacement of the manipulator centre of masses is defined, the second problem is to find the articular (or Cartesian) coordinates corresponding to this displacement. For this purpose, let us consider a manipulator composed of n links. The mass of the link i is denoted as m_i ($i = 1, \ldots, n$) and the position of its centre of masses as $\mathbf{x}_{Si}$. Once the articular coordinates $\mathbf{q}$ or Cartesian variables $\mathbf{x}$ are known, the values of $\mathbf{x}_{Si}$ may easily be obtained using the manipulator kinematics relationships. As a result, the position of the manipulator centre of masses, defined as

$$\mathbf{x}_S = \frac{1}{m}\sum_{i=1}^{n} m_i \mathbf{x}_{Si} \tag{7}$$

may be expressed as a function of $\mathbf{x}$ or $\mathbf{q}$. But, in order to control the manipulator, the inverse problem should be solved, i.e. it is necessary to express variables $\mathbf{q}$ or $\mathbf{x}$ as a function of $\mathbf{x}_S$. Here, two cases should be distinguished:

(i) dim($\mathbf{x}_S$) = dim($\mathbf{q}$), i.e. the manipulator has got as many actuators as controlled variables for the displacements $\mathbf{x}_S$ of the centre of masses (two variables for planar cases, three variables for spatial problems). In such case, the variables $\mathbf{q}$ or $\mathbf{x}$ can be directly expressed as a function of $\mathbf{x}_S$ using (7), i.e. $\mathbf{q} = \mathbf{f}(\mathbf{x}_S)$.

(ii) dim($\mathbf{x}_S$) < dim($\mathbf{q}$), i.e. the manipulator has got more actuators than controlled variables. In this case, the some of parameters will be used for ensuring the necessity conditions of the optimal displacement of the manipulator centre of masses and the other parameters can be used in order to minimize some other performance criteria, by example, such as the shaking moment.

3 Illustrative example

The end-effector pose (x,y,ϕ) of the planar 3R serial manipulator (Fig. 1) is controlled using three input parameters q_1, q_2 and q_3. The manipulator parameters are the following: the lengths of links: l_{OA} = 0.5 m, l_{AB} = 0.3 m and l_{BC} = 0.1 m, the dispositions of the mass centres: $l_{OS1} = r_1$= 0.289 m, $l_{AS2} = r_2$= 0.098 m, $l_{BS3} = r_3$= 0.05 m. The masses and the axial inertia parameters are: m_1 = 24.4 kg, m_2 = 8.3 kg, m_3 = 2 kg, I_1 = 1.246 kg.m², I_2 = 0.057 kg.m², I_3 = 0.025 kg.m². The payload: m_p = 5 kg.

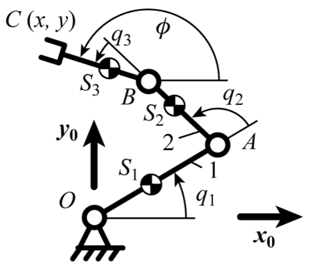

Fig. 1. The 3R serial manipulator.

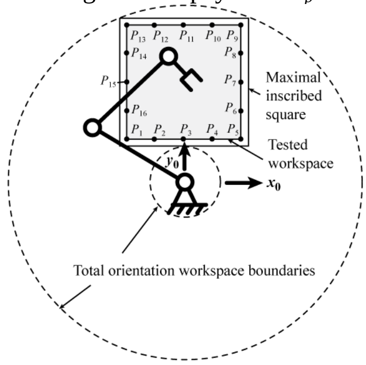

Fig. 2. The tested trajectories.

In the relationship $\mathbf{x}_S = [x_S, y_S]^T = f(q_1, q_2, q_3)$, there are three unknowns q_1, q_2, q_3 for two fixed parameters x_S and y_S. Therefore, as mentioned above, a way to solve this problem is to consider that one parameter, for example ϕ, is used to minimize some other objective function. In the present study, the angle ϕ is used for minimisation of the shaking moment m^{sh}. However, it should be noted that the same parameter can be used for minimisation, by example, the torques or another chosen objective function.

The angles q_1, q_2 and q_3, which are functions of x_S, y_S and ϕ, are determined from $\mathbf{x}_S = f(q_1, q_2, q_3)$:

$$q_1 = 2\tan^{-1}\left(\frac{-b \pm \sqrt{b^2 - c^2 + a^2}}{c - a}\right) \tag{8}$$

where $a = -2l_{eq1}(x_S - l_{eq3}\cos\phi)$, $b = -2l_{eq1}(y_S - l_{eq3}\sin\phi)$, $c = (x_S - l_{eq3}\cos\phi)^2 + (y_S - l_{eq3}\sin\phi)^2 + l_{eq1}^2 - l_{eq2}^2$, $l_{eq1} = (m_1 r_1 + m_2 + m_3 + m_p)l_{OA}/m$, $l_{eq2} = (m_2 r_2 + m_3 + m_p)l_{AB}/m$ and $l_{eq3} = (m_3 r_3 + m_p)l_{BC}/m$.

In expression (8), the sign ± stands for the two possible working modes of the manipulator (for simulations, the working mode with the "+" sign is used).

$$q_2 = \tan^{-1}\left(\frac{(y_S - l_{eq3}\sin\phi) - l_{eq1}\sin q_1}{(x_S - l_{eq3}\cos\phi) - l_{eq1}\cos q_1}\right) - q_1 \tag{9}$$

$$q_3 = \phi - q_1 - q_2 \tag{10}$$

Let us now test the proposed approach. The tested trajectories are defined as follows. First, the maximal inscribed square inside of the workspace, for any end-effector orientation, is found (Fig. 2). For this manipulator, it is a square of length 0.375 m, of which centre *E* is located at $x = 0$ m and $y = 0.487$ m. Then, in order to avoid problems due to the proximity of singular configuration, the tested zone is restricted to a square centered in *E* of edge length equal to 0.3 m (in grey on Fig. 2). Finally, each edge is discretized into four segments delimited by the points P_i ($i = 1$ to 16). The tested trajectories are the segments P_1P_{13}, P_2P_{12}, P_3P_{11}, P_4P_{10}, P_5P_9, $P_{15}P_7$, $P_{14}P_8$ and $P_{13}P_9$. For numerical simulations, the independent parameter ϕ is chosen to begin the tested trajectories with an end-effector orientation $\phi_0 = 0$ deg and to finish it at $\phi_f = 120$ deg.

The simultaneous minimization of the shaking force and the shaking moment cannot be done without using an optimization algorithm. Several laws for ϕ were tested. Our observation showed that the polynomial function that makes it possible to obtain optimal results is of degree 8.

For each trajectory three different kinds of law are applied: 1) a fifth order polynomial law is applied on the displacement (translation and rotation) of the manipulator end-effector; 2) a “bang-bang” law is applied on the displacement of the manipulator centre of masses and the angle ϕ is optimized in order to minimize the shaking moment; 3) a trapeze acceleration profile is applied on the displacement of the manipulator centre of masses, taking into account that, for each actuator, the input effort variation is limited by 3.10^4 Nm/s; the trajectory for angle ϕ optimized in the previous case is used in order to compute the actuator displacements.

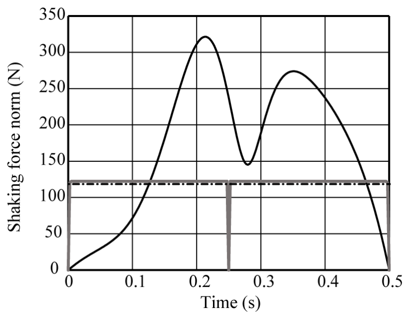

Fig. 3. Variations of the shaking forces: case 1 (black full line), case 2 (black dashed line) and case 3 (grey full line).

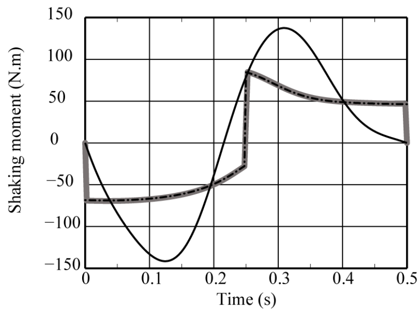

Fig. 4. Variations of the shaking moment: case 1 (black full line), case 2 (black dashed line) and case 3 (grey full line).

The obtained results showed that the optimal trajectory planning by bang-bang law allows the reduction of the shaking forces from 48 % up to 62.2 %. Moreover, with a simultaneous optimal definition of angle ϕ, the shaking moment can be reduced from 37.2 % up to 61 %. The obtained profit depends on the design parameters of the manipulator, as well as the given trajectory. It is clear that it will be variable. However, it is obvious, that the shaking force and shaking moment for any manipulator shall be decreased. Figures 3 and 4 show the variations of the shaking force and the shaking moment of the manipulator for the trajectory $P_{15}P_7$ (see Fig.

2). The software simulations showed that in comparison with mass balanced manipulator a significant reduction in input torque has been achieved.

4 Conclusions

In this paper, we have presented a new approach, based on an optimal trajectory planning, which allows the considerable reduction of the shaking force. This simple and effective balancing method is based on the optimal motion planning of the acceleration of the manipulator centre of masses. For this purpose, the "bang-bang" displacement law has been used. The aim of the suggested method consists in the fact that the manipulator is controlled not by applying end-effector trajectories but by planning the displacements of the total mass centre of moving links. Such an approach allows the reduction of the maximum value of the centre of mass acceleration and, consequently, the reduction in the shaking force. It should be mentioned that such a solution is also very favourable for reduction of input torques because it is carried out without adding counterweights. The proposed balancing method has been illustrated via a numerical example.

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