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Improvement of functional performance of spatial parallel manipulators using mechanisms of variable structure

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Abstract— *A procedure for the increase of singularity-free zones in the workspace of spatial parallel manipulators is presented in this paper. The procedure is based on the control of the pressure angles in the joints of the manipulator. The zones, which cannot be reached by the manipulator, are detected. For increase of the reachable workspace of the manipulator the legs of variable structure are proposed. The design of the optimal structure of the spatial parallel manipulator 3-RPS is illustrated by a numerical simulation.*

Keywords: Parallel manipulator, optimal control, singularity-free zones, pressure angle, force transmission

I. Introduction

It is well-known that the multiple closed chains in parallel manipulators often lead to difficulties in their design and control. One of the most important problems in the design of parallel mechanisms is the study of their singular configurations. The singular analysis has attracted the attention of several researchers and different studies have been published [1-11]. These studies are often developed for design of parallel mechanisms without singular configurations. It would be very suitable if such a result could be achieved by optimal legs assembly.

However, in the previous works, there is another trend which consists in the elimination of singular zones from the whole workspace of the manipulator by the limitation of the workspace. Thus, the workspace of the parallel manipulators which is less than the serial manipulators becomes smaller and limits their functional performances. This has led some researchers to the problem of the optimal control of the parallel manipulators with singular configurations. Alvan and Slousch [12] suggested a solution based on the following considerations: the well known Gough-Steward platform is modified and two legs are added, then the optimal control of the manipulator is carried out by six actuators chosen from eight. As an optimization criterion the algebraic value of the Jacobian matrices and the minimum sum of the root-mean-square value of the input torques are used.

The introduction in the initial system of complementary actuators, which make it possible to eliminate the singular configurations of the parallel manipulator by means of optimal control of the motion, can be exerted. However, it is an expensive solution because of the additional actuators (it is well known that the actuators are one of the most expensive components of manipulators) and the complicated control of the manipulator caused by actuation redundancy.

In our previous work [13] the optimal control of planar parallel manipulators was studied and a new solution of this problem was proposed. It carried out by using mechanisms of variable structure, i.e. a mechanism whose structure parameters can be altered. With regard to the determination of singularity zones inside the workspace of the manipulator we proposed a kinetostatic approach taking account of the force transmission.

In this paper a similar problem for spatial parallel manipulators is studied and a procedure for the increase of singularity-free zones in the workspace of spatial parallel manipulators is presented.

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II. Force transmissivity analysis

Let us consider a spatial parallel manipulator (Fig. 1) with 3 degrees of freedom (two orientations and one translation) [14], which consists of the base $A_1A_2A_3$, the output link $B_1B_2B_3$ and 3 identical legs composed of one revolute pair A_i, one prismatic pair C_i and one spherical pair B_i (i = 1,2,3).

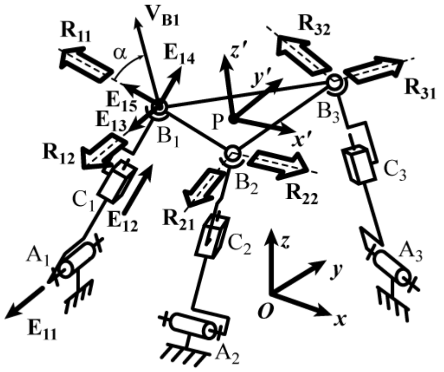


Fig. 1. Spatial parallel manipulator 3-RPS.

Let us examine the pressure angles of the considered manipulator [15]. Let the revolute pairs A_i be actuated and passive joints be located at B_i and C_i. Thus, each kinematic chain includes one actuated and two passive pairs. The wrench acting to the output link is reciprocal to the unit vectors situated along the axes of non-actuated pairs. Let $\mathbf{E}_{i1}$, $\mathbf{E}_{i2}$, $\mathbf{E}_{i3}$ $\mathbf{E}_{i4}$, $\mathbf{E}_{i5}$ (Fig. 1) be the unit vectors of the axes of kinematic pairs, where i (i=1,2,3) is the number of the chain. Here $\mathbf{E}_{i1}$ corresponds to rotating actuated pair, $\mathbf{E}_{i2}$ corresponds to sliding passive pair, $\mathbf{E}_{i3}$ $\mathbf{E}_{i4}$, $\mathbf{E}_{i5}$ correspond to the spherical passive pair. The Plücker co-ordinates of these unit screws can be described in the matrix $\mathbf{(E)}_i$.

$$
\mathbf{(E)}_i = \begin{pmatrix} e_{i1x} & e_{i1y} & e_{i1z} & e^{\circ}_{i1z} & e^{\circ}_{i1y} & e^{\circ}_{i1z} \\ 0 & 0 & 0 & e^{\circ}_{i2z} & e^{\circ}_{i2y} & e^{\circ}_{i2z} \\ e_{i3x} & e_{i3y} & e_{i3z} & e^{\circ}_{i3z} & e^{\circ}_{i3y} & e^{\circ}_{i3z} \\ e_{i4x} & e_{i4y} & e_{i4z} & e^{\circ}_{i4z} & e^{\circ}_{i4y} & e^{\circ}_{i4z} \\ e_{i5x} & e_{i5y} & e_{i5z} & e^{\circ}_{i5z} & e^{\circ}_{i5y} & e^{\circ}_{i5z} \end{pmatrix}
$$

Here $\mathbf{E}_{i1}$, $\mathbf{E}_{i3}$ $\mathbf{E}_{i4}$, $\mathbf{E}_{i5}$ are the unit screws of zero pitch, i.e. $e_{i1x} \cdot e^0_{i1x} + e_{i1y} \cdot e^0_{i1y} + e_{i1z} \cdot e^0_{i1z} = 0$, etc., $\mathbf{E}_{i2}$ is the unit screw of infinite pitch, $e^0_{i2x} = (x_{Bi} - x_{Ai})/l_i$, $e^0_{i2y} = (y_{Bi} - y_{Ai})/l_i$, $e^0_{i2z} = (z_{Bi} - z_{Ai})/l_i$, x_{Ai}, x_{Bi}, y_{Ai}, y_{Bi}, z_{Ai}, z_{Bi} are the coordinates of the points A_i and B_i, l_i is the distance between the points A_i and B_i, i=1,2,3. Without interruption of generality we can assume that $\mathbf{E}_{i3}$ is parallel to $\mathbf{E}_{i1}$, $\mathbf{E}_{i4}$ is parallel to $\mathbf{E}_{i2}$, and $\mathbf{E}_{i5}$ is perpendicular to $\mathbf{E}_{i4}$ and $\mathbf{E}_{i2}$.

The determinant of the matrix $\mathbf{(E)}_i$ vanishes if the axes $\mathbf{E}_{i1}$ and $\mathbf{E}_{i3}$ coincide. It means the occurrence of singularity when the actuator causes only rotation in the joint $\mathbf{E}_{i3}$.

We can obtain the wrenches which are reciprocal to the unit vectors of the axes of the passive kinematic pairs. The conditions of reciprocity are:

$$
\begin{aligned}
& e^0_{i2x} r_{ix} + e^0_{i2y} r_{iy} + e^0_{i2z} r_{iz} = 0, \\
& e^0_{i3x} r_{ix} + e^0_{i3y} r_{iy} + e^0_{i3z} r_{iz} + e_{i3x} r^0_{ix} + e_{i3y} r^0_{iy} + e_{i3z} r^0_{iz} = 0 \\
& e^0_{i4x} r_{ix} + e^0_{i4y} r_{iy} + e^0_{i4z} r_{iz} + e_{i4x} r^0_{ix} + e_{i4y} r^0_{iy} + e_{i4z} r^0_{iz} = 0 \\
& e^0_{i5x} r_{ix} + e^0_{i5y} r_{iy} + e^0_{i5z} r_{iz} + e_{i5x} r^0_{ix} + e_{i5y} r^0_{iy} + e_{i5z} r^0_{iz} = 0
\end{aligned} \qquad (1)
$$

Here r_{ix}, r_{iy}, r_{ix}, r^0_{ix}, r^0_{iy}, r^0_{iz} are the Plücker co-ordinates of the wrenches to be found. Equations (1) mean that each connecting kinematic chain determines two wrenches of zero pitch (vector) $\mathbf{R}_{i1}\left(r_{i1x} \;\; r_{i1y} \;\; r_{i1x} \;\; r^0_{i1x} \;\; r^0_{i1y} \;\; r^0_{i1z}\right)$ and $\mathbf{R}_{i2}\left(r_{i2x} \;\; r_{i2y} \;\; r_{i2x} \;\; r^0_{i2x} \;\; r^0_{i2y} \;\; r^0_{i2z}\right)$ (i=1,2,3). They are perpendicular to the axis $\mathbf{E}_{i2}$ and intersect the point B_i. Without the loss of generality we can assume that $\mathbf{R}_{i1}$ is perpendicular to $\mathbf{E}_{i1}$ and coincides with $\mathbf{E}_{i5}$, and $\mathbf{R}_{i2}$ is parallel to $\mathbf{E}_{i1}$ and coincides with $\mathbf{E}_{i3}$. The coordinates of wrenches in the form of the matrix **(R)** 6×6 are given by:

$$
\mathbf{(R)} = \begin{pmatrix} r_{11x} & r_{11y} & r_{11z} & r^{\circ}_{11x} & r^{\circ}_{11y} & r^{\circ}_{11z} \\ r_{12x} & r_{12y} & r_{12z} & r^{\circ}_{12x} & r^{\circ}_{12y} & r^{\circ}_{12z} \\ r_{21x} & r_{21y} & r_{21z} & r^{\circ}_{21x} & r^{\circ}_{21y} & r^{\circ}_{21z} \\ r_{22x} & r_{22y} & r_{22z} & r^{\circ}_{22x} & r^{\circ}_{22y} & r^{\circ}_{22z} \\ r_{31x} & r_{31y} & r_{31z} & r^{\circ}_{31x} & r^{\circ}_{31y} & r^{\circ}_{31z} \\ r_{32x} & r_{32y} & r_{32z} & r^{\circ}_{32x} & r^{\circ}_{32y} & r^{\circ}_{32z} \end{pmatrix}
$$

In singular configurations the system of the wrenches **(R)** degenerates.

To find the pressure angles we consider the wrenches $\mathbf{R}_{ij}$ and the directions of the velocities of the points B_i [11]. The velocity of the point B_1 is determined by the equations expressing the Plücker co-ordinates $\left(\omega_{1x} \;\; \omega_{1y} \;\; \omega_{1z} \;\; v_{1x} \;\; v_{1y} \;\; v_{1z}\right)$ of the twist $\mathbf{\Omega_1}$ existing by fixed actuated pairs A_2 and A_3.

$$
\begin{aligned}
& r^{\circ}_{11x}\omega_{1x} + r^{\circ}_{11y}\omega_{1y} + \ldots + r_{11z}v_{1z} = \omega_{11}\left(r^{\circ}_{11x}e_{11x} + \ldots + r_{11z}e^0_{11z}\right) \\
& r^{\circ}_{12x}\omega_{1x} + r^{\circ}_{12y}\omega_{1y} + \ldots + r_{12z}v_{1z} = 0 \\
& r^{\circ}_{21x}\omega_{1x} + r^{\circ}_{21y}\omega_{1y} + \ldots + r_{21z}v_{1z} = 0 \\
& r^{\circ}_{22x}\omega_{1x} + r^{\circ}_{22y}\omega_{1y} + \ldots + r_{22z}v_{1z} = 0 \\
& r^{\circ}_{31x}\omega_{1x} + r^{\circ}_{31y}\omega_{1y} + \ldots + r_{31z}v_{1z} = 0 \\
& r^{\circ}_{32x}\omega_{1x} + r^{\circ}_{32y}\omega_{1y} + \ldots + r_{32z}v_{1z} = 0
\end{aligned} \qquad (2)
$$

Here ω_{11} is the generalized velocity in the pair A_{11}. The left hand sides of the equations (2) express the reciprocal moments of the twist $\mathbf{\Omega_1}$ and the wrenches $\mathbf{R_{11}}$, $\mathbf{R_{12}}$, $\mathbf{R_{21}}$, $\mathbf{R_{22}}$, $\mathbf{R_{31}}$, $\mathbf{R_{32}}$. According to the last five equations the twist $\mathbf{\Omega_1}$ is reciprocal with the wrenches $\mathbf{R_{12}}$, $\mathbf{R_{21}}$, $\mathbf{R_{22}}$, $\mathbf{R_{31}}$, $\mathbf{R_{32}}$. The right hand sides of the first and second equations correspond to reciprocal moments of the twist $\mathbf{E}_{11}\omega_{11}$ and the wrenches $\mathbf{R_{11}}$ and $\mathbf{R_{12}}$.

As the wrench $\mathbf{R_{11}}$ is of zero pitch and parallel to $\mathbf{E}_{11}$ then the reciprocal moment of this wrench and the twist $\mathbf{E}_{11}\omega_{11}$ can be written as:

$$\omega_{11}\left(r^\circ_{11x}e_{11x}+...+r_{11z}e^0_{11z}\right)=\omega_{11}l_1\sqrt{r^2_{11x}+r^2_{11y}+r^2_{11z}}\ .$$

As the wrench $\mathbf{R_{12}}$ is perpendicular to the $\mathbf{E}_{11}$ then the reciprocal moment of this wrench and the twist $\mathbf{E}_{11}\omega_{11}$ is equal to zero:

$\omega_{11}\left(r^\circ_{12x}e_{11x}+...+r_{12z}e^0_{11z}\right)=0$. All the wrenches $\mathbf{R_{11}}$, $\mathbf{R_{12}}$, $\mathbf{R_{21}}$, $\mathbf{R_{22}}$, $\mathbf{R_{31}}$, $\mathbf{R_{32}}$ are of zero pitch therefore the reciprocal moment of the twist $\mathbf{\Omega_1}$ and the wrench $\mathbf{R_{11}}$ can be written as:

$$r^\circ_{11x}\omega_{1x}+r^\circ_{11y}\omega_{1y}+...+r_{11z}v_{1z}=V_{B1x}r_{11x}+V_{B1y}r_{11y}+V_{B1z}r_{11z}$$

where V_{B1x}, V_{B1y}, V_{B1z} are the co-ordinates of the velocity $\mathbf{V_{B1}}$ of the point B_1:

$$V_{B1x}=V_{1x}+\omega_{1y}z_{B1}-\omega_{1z}y_{B1},\quad V_{B1y}=V_{1y}+\omega_{1z}x_{B1}-\omega_{1x}z_{B1},$$

$$V_{B1z}=V_{1z}+\omega_{1x}y_{B1}-\omega_{1y}x_{B1},$$

Finally, the pressure angle of leg 1 can be written as:

$$\alpha_1=\left|\arccos\left[\frac{V_{B1x}r_{11x}+V_{B1y}r_{11y}+V_{B1z}r_{11z}}{\sqrt{V^2_{B1x}+V^2_{B1y}+V^2_{B1z}}\sqrt{r^2_{11x}+r^2_{11y}+r^2_{11z}}}\right]\right| \quad (3)$$

We can find two other pressure angles by a similar way.

It was noted that in the singular configurations all the pressure angles are equal to 90°. Indeed, in this case the determinant of the matrix **(R)** is equal to zero. Therefore the wrench $\mathbf{R_{11}}$ is reciprocal to the twist Ω_1 and $V_{B1x}r_{11x}+V_{B1y}r_{11y}+V_{B1z}r_{11z}=0$, i.e. $\cos\alpha_1=0$, $\alpha_1=\pm 90°$, the velocity $\mathbf{V}_{B1}$ is perpendicular to the axis of the wrench $\mathbf{R}_{11}$. In this case $\omega_{11}=0$.

Thus, the pressure angles can be determined at the joints of each kinematic chain. In this way we could map the maximum value of the pressure angles in the whole workspace of the parallel manipulator to detect the inaccessible zones with unfavourable values of the pressure angles.

If the prescribed path of the parallel manipulator intersects any unacceptable zone in which the pressure angle has an inadmissible value the transmission of the motion can be disrupted. In this case, it is necessary to change the structural parameters of the mechanism, i.e. the input motions.

Fig. 2 shows a schematic of the modified leg with the added articulated dyad which allows changing the input motion. The rotating actuators are mounted on the base and connected by electromagnetic clutches with the links A_iD_i and A_iC_i. The input motion can be transmitted either by the link A_iD_i or A_iC_i (i=1,2,3). In this way we can obtain the leg of the mechanisms with different structural parameters, which changes the direction of the wrench $\mathbf{R_{i1}}$ and allows increasing the singularity-free zones.

Let us consider the system of wrenches existing in this case. The link B_iC_i is constrained by two wrenches of zero pitch $\mathbf{T}_{i1}$ and $\mathbf{T}_{i2}$. The axis of the wrench $\mathbf{T}_{i1}$ is perpendicular to the line A_iB_i and the axis of the wrench $\mathbf{T}_{i2}$ coincides with the axis of the link C_iD_i. The unit screw $\mathbf{E'_{i1}}$ of the twist of the link B_iC_i is reciprocal to the wrenches $\mathbf{T}_{i1}$ and $\mathbf{T}_{i2}$. This twist is of zero pitch and is parallel to the axis $\mathbf{E_{i1}}$. The location of the axis $\mathbf{E'_{i1}}$ corresponds to the point of intersection of the wrenches $\mathbf{T}_{i1}$ and $\mathbf{T}_{i2}$. If the link C_iD_i is perpendicular to the link B_iC_i then the wrenches $\mathbf{T}_{i1}$ and $\mathbf{T}_{i2}$ are parallel and the instantaneous motion of the link B_iC_i is a translation. The wrench $\mathbf{R_{i1}}$ (i=1,2,3) can be determined using the equation analogous to (1). The pressure angle can be found using the equation (3).

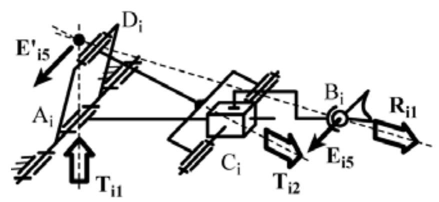


Fig. 2. Planar representation of the leg with variable structure.

This approach can be applied for mechanisms with different degrees of freedom and different structures of legs. Particularly at the point A_i can be situated a universal joint. Then each kinematic chain determines only one wrench $\mathbf{R_i}$ whose direction can be changed by choosing different input links. Thus, by such a way, we can determine the pressure angles corresponding to the different structures and obtain all possible workspace with singularity-free zones.

III. Optimal structural architecture of the manipulator taking account of pressure angle

In order to obtain the best structural architecture of the manipulator for a given trajectory, in this section we describe a procedure, which allows determining the optimal system of actuation. For this purpose, at the first time, we would like to show the singularity-free zones in the workspace of the 3-RPS spatial parallel manipulator with modified legs. These zones have been determined by using the maximum acceptable values of the pressure angles.

For numerical simulation we consider a manipulator in which the basis triangle $A_1A_2A_3$ is equilateral with radius 0.35 m (Fig. 1) and the platform $B_1B_2B_3$ also represents an equilateral triangle with radius 0.1 m. For added dyads A_iD_i=C_iD_i=0.25 m, the articulated dyads are always located on the top of the prismatic pairs as it is shown in Fig. 2 and the translations of the prismatic pairs are limited relative to the joints A_i and C_i by values $(A_iC_i)_{min}$=$(B_iC_i)_{min}$=0.05 m.

The origin of the fixed base frame (***Oxyz***) is located at the centre of the equilateral triangle $A_1A_2A_3$, the vertical axis **z** is orthogonal to this triangle and the screws $\mathbf{E_{i1}}$ are tangent to the circle passing through $A_1A_2A_3$.

This manipulator has three degrees of freedom and only three of the six position/orientation variables are independent. In this work, as the output parameters are defined two orientation angles β_1, β_2 and the vertical position z of the platform. The angles β_1, β_2 can be obtained by expressing the directional cosines in terms of ***x-y-x*** Euler angles β_1, β_2, β_3 (see the basic kinematic equations in [14]).

Taking into account that the modified manipulator can be actuated either by links A_iD_i or by links A_iC_i, for given output parameters (z,β_1,β_2) of the platform, we have 8 different combinations of actuation, i.e. we have 8 different combinations of input parameters presented below (underlined letters show the input pairs, "R" for input links A_iC_i with input angles θ_i and "P" for input links A_iD_i with input displacements ρ_i):

RRR: RPS-RPS-RPS: $\mathbf{q}_{(1)}=(\theta_1,\theta_2,\theta_3)$
RRP: RPS-RPS-RPS: $\mathbf{q}_{(2)}=(\theta_1,\theta_2,\rho_3)$
RPR: RPS-RPS-RPS: $\mathbf{q}_{(3)}=(\theta_1,\rho_2,\theta_3)$
RPP: RPS-RPS-RPS: $\mathbf{q}_{(4)}=(\theta_1,\rho_2,\rho_3)$
PRR: RPS-RPS-RPS: $\mathbf{q}_{(5)}=(\rho_1,\theta_2,\theta_3)$
PRP: RPS-RPS-RPS: $\mathbf{q}_{(6)}=(\rho_1,\theta_2,\rho_3)$
PPR: RPS-RPS-RPS: $\mathbf{q}_{(7)}=(\rho_1,\rho_2,\theta_3)$
PPP: RPS-RPS-RPS: $\mathbf{q}_{(8)}=(\rho_1,\rho_2,\rho_3)$

Table 1 shows the workspaces of each case of actuation for the altitude of the platform equal to 0.1 m. In these figures, several zones can be seen, which correspond to the variations of the maximum values of the pressure angle for given orientation (β_1,β_2) of the platform. The contrast intensity shows the variations of the pressure angle (see Fig. 3).

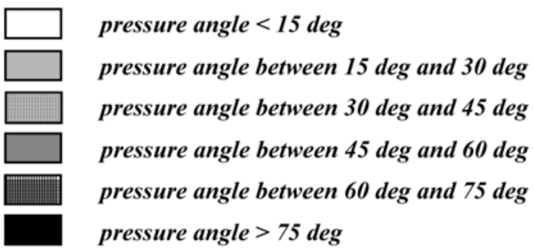


Fig. 3. The contrast intensity corresponding to the pressure angle values.

Thus, the black zones are the surfaces where the pressure angle has inadmissible values, and as a result, these are the zones which cannot be reached and crossed by the parallel mechanism. These zones separate the workspace into different aspects, what decrease the capacity of displacement of the platform.

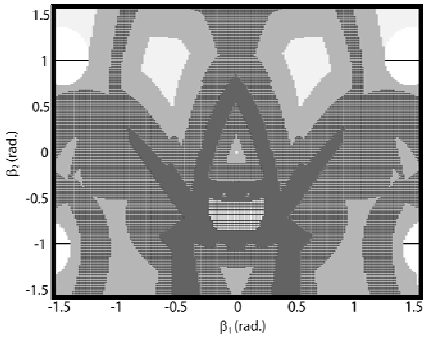


Fig. 4. The reachable workspace of the spatial parallel manipulator with modified legs (z = 0.1 m).

Fig. 4 shows the reachable workspace of the modified parallel mechanism with legs of variable structure. We can see that the workspace of the modified manipulator is only composed of singularity-free zones and the whole workspace of the manipulator is reachable (increase until 100%).

Now we would like to describe a procedure, which allows determining the optimal system of actuation. This algorithm is based on the control of the pressure angles in the joints of the manipulator along the given trajectory. It is similar to the procedure given in [13] for planar parallel manipulators.

At first the calculation of the pressure angles in the joints along the trajectory for all possible structures of the parallel mechanism with variable architecture must be accomplished, then the best structure must be chosen for which the maximum value of the pressure angle along the trajectory is always less than the limit value. If there is no structure satisfying this condition, the given trajectory must be decomposed in several parts and the generation of the motion must be carried out by different structures. It is obvious that in this case it would be desirable that the trajectory can be realized by minimal structural changes.

A numerical example is considered below in order to illustrate the application of the suggested design procedure.

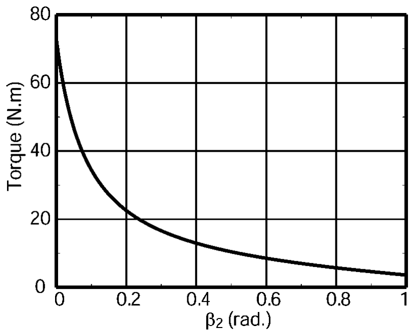
For given parallel manipulator (Fig. 1) with legs of variable structure (Fig. 2) generate the trajectory from the initial position P_1 (z=0.3 m, β_1=0 rad., β_2=0 rad.) to the final position P_2 (z=0.3 m, β_1=0 rad., β_2=1 rad.), keeping z and β_1 constant.

Estimation of the pressure angle along the given trajectory shows that the best structural solution for generation of motion is the RPS-RPS-RPS mechanism, i.e. when the first actuator is connected with the link A_1D_1 and two others with the links A_2C_2 and A_3C_3. In this case the maximum values of the pressure angles in the joints are always less than the limit value.

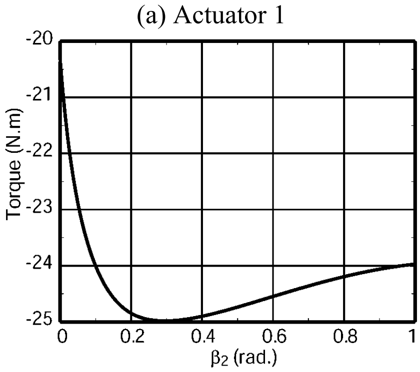
In order to illustrate the variations of torques for examined case we develop a model of the manipulator with the given trajectory using the ADAMS software.

A force parallel to the **z**-axis and equal to 100 N was applied to the platform and the friction coefficients in the prismatic pairs were equal to 0.01. The obtained torques are shown in Fig. 5. We can note that the torques have admissible values along the trajectory.

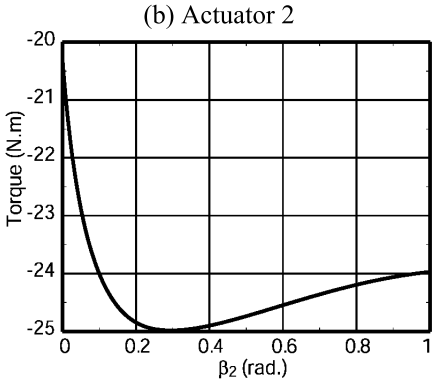
It is obvious that the similar mechanisms of variable structure can also be designed on the base of the screw or cam systems, the rhombic pantographs, etc.



(a) Actuator 1



(b) Actuator 2



(c) Actuator 3

Fig. 5. Torques of the actuators.

IV. Conclusion

A procedure for the improvement of functional performance of spatial parallel manipulators has been presented in this paper. The procedure is based on the control of the pressure angles in the joints of the manipulator along the given trajectory of the platform. The zones, which cannot be reached by the manipulator, were detected. For increase of the reachable workspace of the manipulator the legs of variable structure were proposed. Such a solution allows obtaining the best structural architecture of the manipulator for any trajectory. The design of the optimal structure of the planar parallel manipulator 3-RPS was illustrated by a numerical simulation. We believe that the suggested method is a useful tool for the improvement of the functional performance of parallel manipulators with singular zones.

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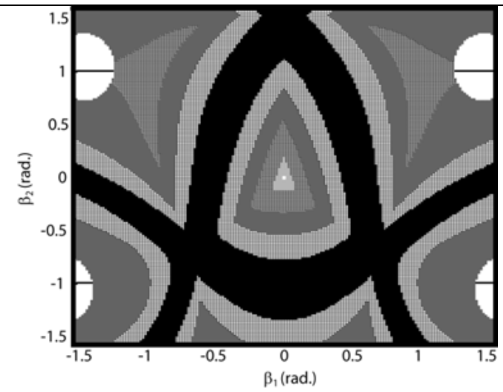
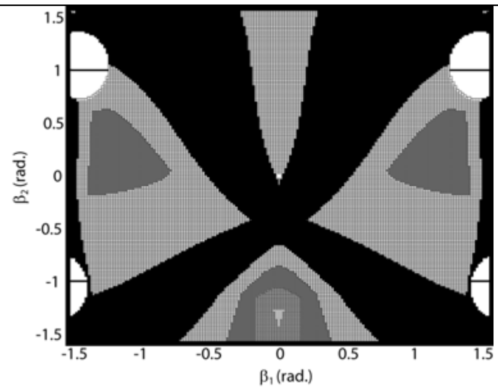
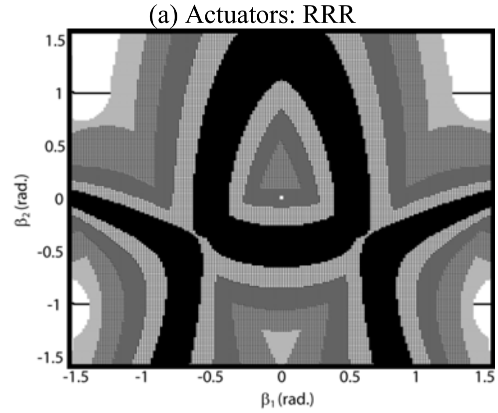


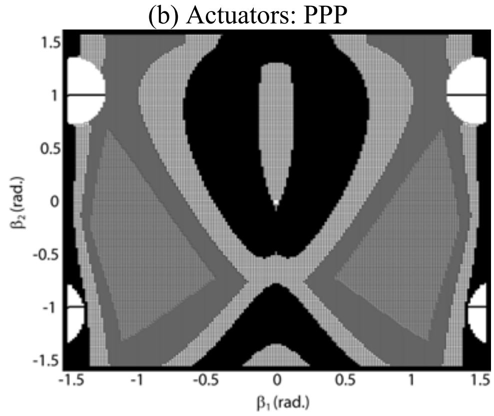
(a) Actuators: RRR



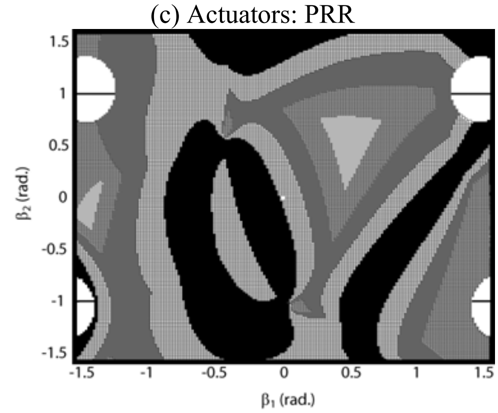
(b) Actuators: PPP



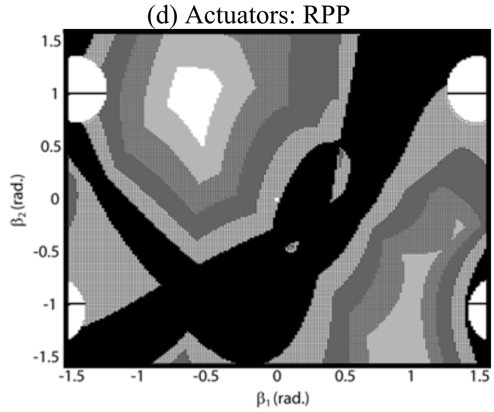
(c) Actuators: PRR



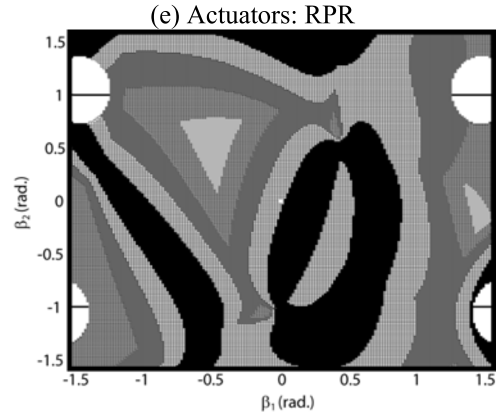
(d) Actuators: RPP



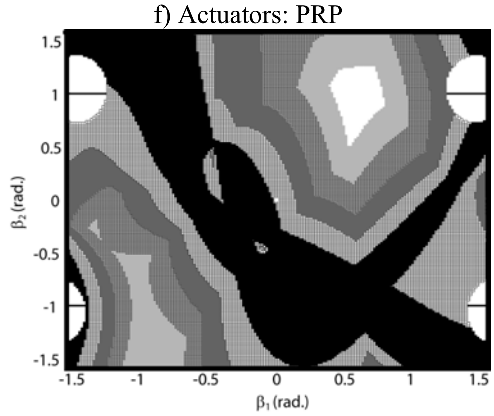
(e) Actuators: RPR



f) Actuators: PRP



(g) Actuators: RRP



(h) Actuators: PPR

TABLE I. Maximum values of the pressure angles (z = 0.1m)