Dynamic Parameter Identification of Actuation Redundant Parallel Robots using their Power Identification Model: Application to the DualV

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Abstract— Off-line robot dynamic identification methods are generally based on the use of the Inverse Dynamic Identification Model (*IDIM*), which calculates the joint forces/torques (estimated as the product of the known control signal - the input reference of the motor current loop - by the joint drive gains) that are linear in relation to the dynamic parameters, and on the use of linear least squares technique to calculate the parameters (*IDIM-LS* technique). However, as actuation redundant parallel robot are overconstrained, their *IDIM* has infinity of solutions for the force/torque prediction, depending of the value of the desired overconstraint that is *a priori* unknown in the identification process. As a result, the *IDIM* cannot be used for the identification procedure.

On the contrary the Power Identification Model (*PIM*) of any types of robot manipulator has a unique formulation and contains the same dynamic parameters as the *IDIM*. This paper proposes to use the *PIM* of actuation redundant robots for identification purpose. The identification of the inertial parameters of a planar parallel robot with actuation redundancy, the DualV, is then carried out using its *PIM*. Experimental results show the validity of the method.

I. INTRODUCTION

Parallel robots are increasingly being used since a few decades. This is due to their main advantages compared to their serial counterparts that are: (i) a higher intrinsic rigidity, (ii) a larger payload-to-weight ratio and (iii) higher velocity and acceleration capacities [1]. However, there main drawback is probably the presence of singularities in the workspace. In order to overcome this difficulty, actuation redundancy can be used [2], [3]. Actuation redundancy means that the robot has more actuators that degrees of freedom (*dof*) to control and is thus overconstrained. Overconstraints can be smartly used to improve the robot properties, such as increasing the acceleration or payload capacities [4] or even decreasing the backlash [5]. However, this involves the use of more complicated controllers.

Several control approaches could be envisaged [6], [7], but it appears that, for high-speed robots or when varying loads have to be compensated (e.g. in pick-and-place operations or

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machining), computed torque control is generally used [5], [8]. This approach requires an accurate identification of the dynamic model of the robot with the load [9], which can be obtained if two main conditions are satisfied:

1) a well-tuned derivative band-pass filtering of actuated joints position is used to calculate the actuated joints velocities and accelerations, and
2) the values of actuator drive gains g_τ are accurately known to calculate the actuator force/torque as the product of the known control signal computed by the numerical controller of the robot (the current references) by the drive gains

The usual identification procedure of the robot dynamic parameters requires the computation of the inverse dynamic identification model (*IDIM*) of the studied robot that gives the values of the input forces/torques as a function of the robot configuration, velocity and acceleration [10]. However, for actuation redundant parallel robots, the inverse dynamic model is not unique and depends on the overconstraint in the mechanism that cannot be *a priori* known in the identification process. Thus, *identification using the usual IDIM for redundant robots parallel cannot be carried out*.

There exist anothers type of model that contains exactly the same dynamic parameters as the *IDIM* but that has a unique formulation for any kind of robot manipulators: the power identification model (*PIM*). The *PIM* has been formely used by two of the authors of the present paper for the identification of the dynamic parameters of 2 degrees of freedom (*dof*) [11] and of a 6 *dof* serial industrial robots [12]. In [12], it is demonstrated that the identification of 6 *dof* serial robots with the *PIM* requires the definition of special trajectories that make it possible to decouple the observation matrix. This decoupled matrix is necessary for identifying the dynamic parameters of the wrist that have less significant contribution to the robot consumed power than those of the three first joints. In general, for serial robots, the *PIM* offers the advantage, with respect to the *IDIM*, of simplifying the identification procedure as it considerably reduces the complexity of computation for the identification model. This paper focuses on another newly discovered advantage of the *PIM*: for parallel robots with actuation redundancy, for which the usual *IDIM* cannot be computed, the unique formulation of the *PIM* makes it possible to achieve the identification of the dynamic parameters. To the best of our knowledge, the identification of the inertial parameters of

actuation redundant parallel robots had never been presented until now.

This paper is divided as follows. Section II presents some recalls on the computation of the *PIM* of robots. Section III briefly recalls the identification procedure and experimental results on the DualV are presented in Section IV. Finally, in Section V, conclusions are drawn.

II. COMPUTATION OF THE POWER IDENTIFICATION MODEL

A. Computation of the Usual PIM

The *PIM* of a rigid robot composed of m moving bodies, n_a active joints and n_p passive joints calculates the motors consumed power as a function of the generalized coordinates and their derivatives. It can be computed from the following relation:

$$P_{pm} = \frac{d}{dt}\left(H\left(\mathbf{q}_{tot}, \dot{\mathbf{q}}_{tot}, \mathbf{x}, \mathbf{v}\right)\right) + \dot{\mathbf{q}}_{tot}^T \tau_f \quad (1)$$

where $H\left(\mathbf{q}_{tot}, \dot{\mathbf{q}}_{tot}\right)$ is the total energy of the system calculted using the recursive equations proposed in [10], $\mathbf{q}_{tot}$ and $\dot{\mathbf{q}}_{tot}$ are respectively the $(n_a + n_p) \times 1$ vectors of the active and passive joint positions and velocities, $\mathbf{x}$ and $\mathbf{v}$ are the end-effector position and velocity vectors, and τ_f is the $(n_a + n_p) \times 1$ vector of the friction torques in active and passive joints, i.e.

$$\begin{aligned} \tau_f &= \left[\tau_{f_1} \cdots \tau_{f_{n_a+n_p}}\right]^T \\ \tau_{f_j} &= fv_j \dot{q}_j + fs_j sign(\dot{q}_j) + \tau_{off_j} \end{aligned} \quad (2)$$

where fv_j, fs_j are the viscous and Coulomb friction coefficients in the joint j, respectively, and $\tau_{off_j} = \tau_{offfs_j} + \tau_{off\tau_j}$ is an offset parameter which regroups the current amplifier offset $\tau_{off\tau_j}$ and the asymmetrical Coulomb friction coefficient τ_{offfs_j}.

It should be mentionned here that, for closed-loop robots, the passive joint coordinates and velocities (denoted as $\mathbf{q}_p$ and $\dot{\mathbf{q}}_p$, resp.) and the end-effector position and velocity $\mathbf{x}$ and $\mathbf{v}$ can be computed from the values of the active joint coordinates and velocities (denoted as $\mathbf{q}_a$ and $\dot{\mathbf{q}}_a$, resp.) via the use of the loop-closure equations [1]. For parallel robots with actuation redundancy, this computation is not straigthforward and will be detailed in the Section II-C.

It is known that the power model (1) of any manipulator can be expressed as a linear form with respect to the standard dynamic parameters χ_{st} of the robot:

$$P_{pm} = \frac{d}{dt}\left(\mathbf{h}\left(\mathbf{q}_{tot}, \dot{\mathbf{q}}_{tot}, \mathbf{x}, \mathbf{v}\right)\right) \chi_{st} = \mathbf{dh}_{st}\chi_{st} \quad (3)$$

where $\mathbf{dh}_{st}$ is the $(1 \times n_{st})$ Jacobian matrix of P_{pm} with respect to the $(n_{st} \times 1)$ vector χ_{st} of the standard parameters given by $\chi_{st}^T = [\chi_{st}^{1T}, \chi_{st}^{2T}, \ldots \chi_{st}^{mT}]$. For rigid robots, the vector χ_{st}^j of link j is composed of 14 standard parameters described as:

- $xx_j, xy_j, xz_j, yy_j, yz_j, zz_j$ are the 6 components of the inertia matrix of link j at the origin of frame j,
- mx_j, my_j, mz_j are the 3 components of the first moment of link j,

- m_j is its mass,
- ia_j is the total inertia moment for rotor and gears,
- fv_j, fs_j, τ_{off_j} are the friction parameters detailed above.

The model (3) can be simplified through the use of the identifiable parameters. The identifiable parameters are the base parameters which are the minimum number of dynamic parameters from which the dynamic model can be calculated [10]. The minimal dynamic model can be written using the n_b base dynamic parameters χ as follows:

$$P_{pm} = \mathbf{dh}\chi \quad (4)$$

where $\mathbf{dh}$ is a subset of independant columns in $\mathbf{dh}_{st}$ which defines the identifiable parameters.

Finally, because of perturbations due to noise measurement and modeling errors, the actual power P differs from P_{pm} by an error e, such that:

$$P = P_{pm} + e = \mathbf{dh}\chi + e \quad (5)$$

where P is calculated from

$$P = \dot{\mathbf{q}}_a^T \tau \quad (6)$$

with τ the actual motor torques/forces computed with the drive chain relation:

$$\tau = \mathbf{v}_\tau \mathbf{g}_\tau = \mathrm{diag}(v_\tau^j) \begin{bmatrix} g_\tau^1 & \cdots & g_\tau^{n_a} \end{bmatrix}^T \quad (7)$$

$\mathbf{v}_\tau$ is the $(n_a \times n_a)$ matrix of the actual motor current references of the current amplifiers (v_{τ_j} corresponds to actuator j) and $\mathbf{g}_\tau$ is the $(n_a \times 1)$ vector of the joint drive gains (g_{τ_j} corresponds to actuator j) that is given by a priori manufacturer's data or using some special procedures [13], [14]. Equation (5) represents the Power Identification Model (*PIM*).

B. PIM Including the Payload

The payload is considered as an additional link (denoted as link l) fixed to the robot platform [9]. The model (5) becomes:

$$P = \begin{bmatrix} \mathbf{dh} & \mathbf{dh}_l \end{bmatrix} \begin{bmatrix} \chi \\ \chi l \end{bmatrix} + \mathbf{e} = \mathbf{dh}_{tot}\chi_{tot} + \mathbf{e} \quad (8)$$

where:

- χ_l is the (10×1) vector of the inertial parameters of the payload;
- $\mathbf{dh}_l$ is the $(r \times 10)$ Jacobian matrix of P, with respect to the vector χ_l.

C. Computation of the Passive Joint and Platform Coordinates and Velocities

As mentionned above, for closed-loop robots, the passive joint coordinates and velocities $\mathbf{q}_p$ and $\dot{\mathbf{q}}_p$ and the end-effector position and velocity $\mathbf{x}$ and $\mathbf{v}$ can be computed from the values of the active joint coordinates and velocities (denoted as $\mathbf{q}_a$ and $\dot{\mathbf{q}}_a$, resp.) via the use of the loop-closure equations [1],

$$\mathbf{f}_t(\mathbf{q}_a, \mathbf{q}_p) = \mathbf{0}, \; \mathbf{f}_p(\mathbf{q}_a, \mathbf{x}) = \mathbf{0} \quad (9)$$
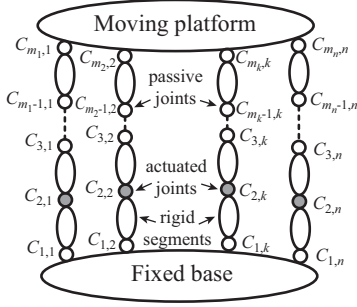
Fig. 1. A general parallel robot.

Differentiating (9) with respect to time, all joint and platform velocities can be computed using the following expressions:

$$\mathbf{A}_t\dot{\mathbf{q}}_p + \mathbf{B}_t\dot{\mathbf{q}}_a = \mathbf{0} \Rightarrow \dot{\mathbf{q}}_p = -\mathbf{A}_t^{-1}\mathbf{B}_t\dot{\mathbf{q}}_a = \mathbf{J}_t\dot{\mathbf{q}}_a, \quad (10)$$

$$\mathbf{A}_p\mathbf{v} + \mathbf{B}_p\dot{\mathbf{q}}_a = \mathbf{0} \Rightarrow \dot{\mathbf{q}}_a = -\mathbf{B}_p^{-1}\mathbf{A}_p\mathbf{v} = \mathbf{J}_p^{inv}\mathbf{v}, \quad (11)$$

or also, as for parallel robots with actuation redundancy, the matrix $\mathbf{A}_p$ is rectangular with more rows than columns,

$$\dot{\mathbf{q}}_a = \mathbf{J}_p^{inv\,+}\mathbf{v},\ \mathbf{J}_p^{inv\,+} = \left(\mathbf{J}_p^{inv\,T}\mathbf{J}_p^{inv}\right)^{-1}\mathbf{J}_p^{inv\,T} \quad (12)$$

where matrices $\mathbf{A}_t$, $\mathbf{A}_p$ ($\mathbf{B}_t$, $\mathbf{B}_p$, resp.) can be obtained through the differentiation of the loop-closure equations (9) with respect to all passive joint coordinates $\mathbf{q}_p$ and the platform coordinates (actuated joints positions, resp.), i.e.

$$\mathbf{A}_t = \left[\frac{\partial \mathbf{f}_t}{\partial \mathbf{q}_p}\right],\ \mathbf{B}_t = \left[\frac{\partial \mathbf{f}_t}{\partial \mathbf{q}_a}\right]$$
$$\mathbf{A}_p = \left[\frac{\partial \mathbf{f}_p}{\partial \mathbf{x}}\right],\ \mathbf{B}_p = \left[\frac{\partial \mathbf{f}_p}{\partial \mathbf{q}_a}\right] \quad (13)$$

It should be mentioned that in the case of parallel robots, the computation of matrices $\mathbf{A}_t$ and $\mathbf{B}_t$ is generally not straightforward. Therefore, it is preferable to:

1) express the kinematic relation between the independent coordinates $\mathbf{v}_{tk}$ of the twists for all leg extremities $C_{m_k,k}$ (Fig. 1) and the velocities of all joints $\dot{\mathbf{q}}_{tot}$, $\mathbf{v}_{tk} = \mathbf{J}_k\dot{\mathbf{q}}_{tot}$ (the matrix $\mathbf{J}_k$ stackles all Jacobian matrices corresponding to the displacements of the last joint for each serial legs,
2) express the kinematic relation between the platform velocities $\mathbf{v}$ and the velocities $\mathbf{v}_{tk}$ of all leg extremities $C_{m_k,k}$, $\mathbf{v}_{tk} = \mathbf{J}_{tk}\mathbf{v}$ ($\mathbf{J}_{tk}$ is a matrix that can be obtained by considering the rigid body displacement of any point of the robot platform),
3) combine these two relations with (12) in order to obtain

$$\mathbf{J}_k\dot{\mathbf{q}}_p = \mathbf{J}_{tk}\mathbf{J}_p^{inv\,+}\dot{\mathbf{q}} \Rightarrow \dot{\mathbf{q}}_t = \mathbf{J}_t\dot{\mathbf{q}},\ \mathbf{J}_t = \mathbf{J}_k^{-1}\mathbf{J}_{tk}\mathbf{J}_p^{inv\,+} \quad (14)$$

All the previous expressions are valuable as long as the robot does not meet any singularity and as long as there are the same number of actuators as the number of platform *dof* to control. The singularity avoidance or crossing is not the main topic of this paper, and the reader should refer to [15], [16] for further developments.

III. USUAL IDENTIFICATION PROCEDURE

This part presents some necessary recalls on the identification procedure.

A. Recalls on Least Squares Identification of the Dynamic Parameters

The off-line identification of the base dynamic parameters χ is considered, given measured or estimated off-line data for τ and $(\mathbf{q}_a, \dot{\mathbf{q}}_a)$, collected while the robot is tracking some planned trajectories. The model (5) is sampled at frequency f_m in order to get an over-determined linear system of r_{f_m} equations and n_b unknowns:

$$\mathbf{Y}_{f_m}(\tau) = \mathbf{W}_{f_m}(\hat{\mathbf{q}}_a, \hat{\dot{\mathbf{q}}}_a, \hat{\mathbf{dh}})\chi + \rho_{f_m} \quad (15)$$

where $(\hat{\mathbf{q}}_a, \hat{\dot{\mathbf{q}}}_a, \hat{\mathbf{dh}})$ is an estimation of $(\mathbf{q}_a, \dot{\mathbf{q}}_a, \mathbf{dh})$, respectively, obtained by sampling and band-pass filtering the measure of $\mathbf{q}_a$ and values of $\mathbf{h}$ [11], ρ_{f_m} is the $(r_{f_m} \times 1)$ vector of errors, $\mathbf{Y}_{f_m}$ is the $(r_{f_m} \times 1)$ vector of the inputs (computed using the relation $\dot{\mathbf{q}}_a^T\tau$ from (6) and (7)), sampled at frequency f_m and $\mathbf{W}_{f_m}(\hat{\mathbf{q}}_a, \hat{\dot{\mathbf{q}}}_a)$ is the $(r_{f_m} \times n_b)$ observation matrix.

The force/torque τ is perturbed by high frequency unmodelled friction and flexibility force/torque of the joint drive chain which is rejected by the closed loop control. These force/torque ripples are eliminated with a parallel decimation procedure which low pass filters in parallel $\mathbf{Y}_{f_m}$ and each column of $\mathbf{W}_{f_m}$ and resamples them at a lower rate, keeping one sample over n_d. This parallel decimation can be carried out with the MATLAB *decimate* function, where the low pass filter cutoff frequency, $\omega_{fp} = 2\pi 0.8 f_m / (2n_d)$, is chosen in order to keep $\mathbf{Y}_{f_m}$ and $\mathbf{W}_{f_m}$ in the same frequency range of the model dynamics. After the data acquisition procedure and the parallel decimation of (15), we obtain the over-determined linear system

$$\mathbf{Y}(\tau) = \mathbf{W}(\hat{\mathbf{q}}_a, \hat{\dot{\mathbf{q}}}_a)\chi + \rho \quad (16)$$

where ρ is the $(r \times 1)$ vector of errors, $\mathbf{Y}$ is the $(r \times 1)$ vector of the input torques/force and $\mathbf{W}(\hat{\mathbf{q}}_a, \hat{\dot{\mathbf{q}}}_a)$ is the $(r \times n_b)$ observation matrix.

It is to be noted that no error is introduced by the parallel filtering process in the linear relation (16) compared with (15). In [11], practical rules for tuning this filter are given.

Using the base parameters and tracking 'exciting' reference trajectories, i.e. *optimized trajectories* that can be computed by nonlinear minimization of a criterion function of the condition number of the $\mathbf{W}$ matrix [17], a well-conditioned matrix $\mathbf{W}$ can be obtained. The *LS* solution $\hat{\chi}$ of (16) is given by:

$$\hat{\chi} = \mathbf{W}^+\mathbf{Y}, \text{ where } \mathbf{W}^+ = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T \quad (17)$$

It is computed using the *QR* factorization of $\mathbf{W}$.

Standard deviations $\sigma_{\hat{\chi}_i}$ can be estimated assuming that $\mathbf{W}$ is a deterministic matrix and ρ is a zero mean additive independent noise [11], with a covariance matrix $\mathbf{C}_{\rho\rho}$ such that

$$\mathbf{C}_{\rho\rho} = E\left(\rho\rho^T\right) = \sigma_\rho^2 \mathbf{I}_r \quad (18)$$

E is the expectation operator and $\mathbf{I}_r$, the $r \times r$ identity matrix. An unbiased estimation of the standard deviation σ_ρ is:

$$\sigma_\rho^2 = \left\| \mathbf{Y} - \mathbf{W}\hat{\chi} \right\|^2 / (r - n_b) \qquad (19)$$

The covariance matrix of the estimation error is given by:

$$\mathbf{C}_{\hat{\chi}\hat{\chi}} = E\left[(\chi - \hat{\chi})(\chi - \hat{\chi})^T \right] = \sigma_\rho^2 \left(\mathbf{W}^T \mathbf{W} \right)^{-1} \qquad (20)$$

$\sigma_{\hat{\chi}i}^2 = \mathbf{C}_{\hat{\chi}\hat{\chi}}(i,i)$ is the i-th diagonal coefficient of $\mathbf{C}_{\hat{\chi}\hat{\chi}}$.

The ordinary *LS* can be improved by taking into account different standard deviations on actuated joint j equations errors [11]. Data in $\mathbf{Y}$ and $\mathbf{W}$ of (16) are weighted with the inverse of the standard deviation of the error calculated from ordinary *LS* solution of the equations of joint j [11]

$$\mathbf{Y}^j = \mathbf{W}^j \chi + \rho^j \qquad (21)$$

This weighting operation normalizes the errors in (16) and gives the weighted *LS* estimation of the parameters (*PIM-WLS*).

B. Payload Identification

In order to identify both the robot and the payload dynamic parameters, using the model (8), it is necessary that the robot carried out two types of trajectories [18]:

1) trajectories without the payload, and
2) trajectories with the payload fixed to the end-effector.

The sampling and filtering of the model *IDIM* (8) can be then written as:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{W}_a & \mathbf{0} \\ \mathbf{W}_b & \mathbf{W}_l \end{bmatrix} \begin{bmatrix} \chi \\ \chi_l \end{bmatrix} + \rho \qquad (22)$$

where

- $\mathbf{W}_a$ is the observation matrix of the robot in the unloaded case,
- $\mathbf{W}_b$ is the observation matrix of the robot in the loaded case,
- $\mathbf{W}_l$ is the observation matrix of the robot corresponding to the payload inertial parameters.

Thus, these two types of trajectories avoid the regrouping of the payload parameters with those of the platform and allow their independent identification. Next section presents experimental results on a prototype of actuation redundant parallel robot.

IV. CASE STUDY

A. Description of the DualV

The DualV (Fig. 2) is a prototype of planar parallel robot with actuation redundancy developped at the LIRMM [19]. This robot has 3 controlled *dof* (two translations in the plane (xOy) and one rotation around the z axis) but 4 legs, with one actuator by leg. Thus, its degree of redundancy is equal to 1. Each leg is composed of one proximal and one distal link. The proximal link is attached to the base by one actuated revolute joint and to the distal link by one passive revolute joint. The distal link is also attached to the moving platform by one passive revolute joint. It should be mentionned here that *all proximal links are identical*.
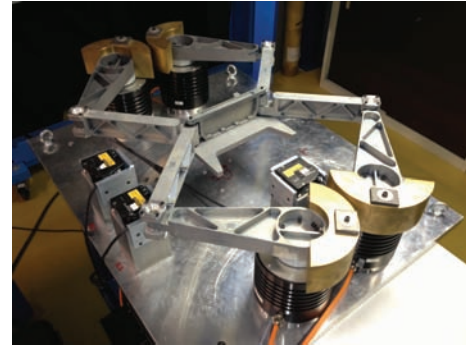
(a) The prototype
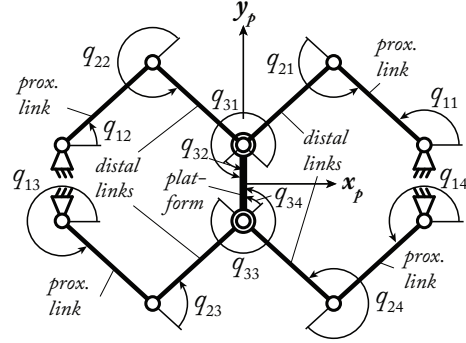
(b) Kinematic description

Fig. 2. The DualV.

The geometric parameters of the virtual open-loop tree structure are described in Table I using the modified Denavit and Hartenberg notation (*MDH*) [10] (in this table, $\gamma_1 = 15.52$deg, $\gamma_2 = 164.48$deg, $\gamma_3 = -164.48$deg and $\gamma_4 = -15.52$deg). The platform and payload are considered as supplementary bodies, the payload being fixed on the platform. They are respectively numbered as bodies 4 and 5.

The *MDH* notation being well known, the parameters of Table I will not be defined here. For more information concerning the *MDH* parameters, the reader should refer to [10].

TABLE I
MDH PARAMETERS FOR THE FRAMES CORRESPONDING TO i-TH ROBOT
LEG ($i = 1, ..., 4$).

j_i	a_{j_i}	μ_{ji}	σ_{ji}	γ_{ji}	d_{ji}	θ_{ji}	r_{ji}
1_i	0	1	0	γ_i	$d_1 = 0.41$m	$q_{1i} - \gamma_i$	0
2_i	1_i	0	0	0	$d_2 = 0.28$m	q_{2i}	0
3_i	2_i	0	0	0	$d_3 = 0.28$m	q_{3i}	0

The DualV is actuated by four ETEL RTMB0140-100 direct drive actuators, which can deliver maximal torques of 127Nm. The robot is able to achieve accelerations of 25g in its workspace. The current amplifier can provide directly the measure of the input torque produced by the actuator. The controller of the DualV was developed within the framework of an industrial PhD thesis which is still confidential for now [20], therefore we are not able to give further explanations
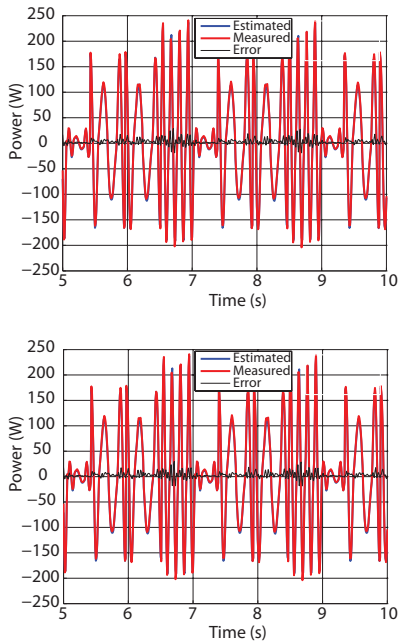
Fig. 3. Robot consumpted power, estimated from input torques using (6) (red lines) and rebuilt using identified parameters (blue lines) with the payload of 5.37 kg.

on this point.

B. Identification Results

In this part, experimentations are performed and the dynamic parameter identification is carried out on the DualV using the modeling approach presented in Section II and the identification procedure proposed in Section III. To estimate the quality of the identification procedure, a payload mass of 5.37kg which has been accurately weighed is mounted of the platform and will be identified in the same time as the robot parameters.

Even paying attention to the choice of the exciting trajectories, some small parameters remain poorly identifiable because they have no significant contribution in the joint torques. These parameters have no significant estimations and can be canceled in order to simplify the dynamic model. Thus parameters such that the relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is too high are canceled to keep a set of essential parameters of a simplified dynamic model with a good accuracy [18]. The essential parameters are calculated using an iterative procedure starting from the base parameters estimation. At each step the base parameter which has the largest relative standard deviation is canceled. A new *IDIM-WLS* parameter estimation of the simplified model is carried out with new relative error standard deviations $\%\sigma_{\hat{\chi}_{ri}}$. The procedure ends when $\max\left(\%\sigma_{\hat{\chi}_{ri}}\right)/\min\left(\%\sigma_{\hat{\chi}_{ri}}\right) < r_{\sigma}$, where r_{σ} is a ratio ideally chosen between 10 and 30 depending on the level of perturbation in $\mathbf{Y}$ and $\mathbf{W}$. In the following of the paper, this ratio is fixed to 10.

Table II presents the identification results for two cases of identification:

- **Case 1**: in the *PIM* of the DualV, it is not *a priori*

		Case 1		Case 2	
Param.	*A priori*	Id. Val.	$\%\sigma_{\hat{\chi}_{ri}}$	Id. Val.	$\%\sigma_{\hat{\chi}_{ri}}$
zz_{11R}	$4.9e-2$	$5.2e-2$	4.26	$3.9e-2$	2.28
zz_{12R}	$4.9e-2$	$4.1e-2$	4.49	$3.9e-2$	2.28
zz_{13R}	$4.9e-2$	$2.6e-2$	8.35	$3.9e-2$	2.28
zz_{14R}	$4.9e-2$	$4.0e-2$	5.58	$3.9e-2$	2.28
zz_4	$2.2e-2$	$1.9e-2$	3.30	$2.0e-2$	3.23
m_4	$2.0e+0$	$2.1e+0$	1.22	$2.1e+0$	1.17
zz_5	N/A	$1.8e-2$	5.13	$1.8e-2$	5.11
mx_5	N/A	$-1.9e-1$	23.39	$-2.0e-1$	22.07
m_5	5.37	$5.4e+0$	0.20	$5.4e+0$	0.20

considered that the proximal links are identical for the four robot legs, i.e. that they don't have exactly the same dynamic parameters;
- **Case 2**: it is now considered that the proximal links are identical for the four robot legs, i.e. that they have exactly the same dynamic parameters.

Subscript "R" stands for the parameters that have been regrouped using the procedure presented in Section II. It can be observed that, in Case 2, the identified values are closer from the *a priori* values. Moreover, the payload of 5.37kg has been very accurately identified in both cases.

The robot consumed power is shown in Fig. 3 for a trajectory different from the one used for the identification process (*i.e. the results are cross-validated*). It can be observed that it is well rebuilt in both cases. Finally, the value the measured input torques, the estimated input torques and the value of the overconstraint $\mathbf{c}$ computed using the methodology presented in [5] are shown at Figs. 4 and 5 for Case 2 (the results are very similar for Case 1). It can be seen that the estimated torques are very close to the measured ones and that the average overconstraint in the robot legs is about 10N.

V. CONCLUSION

This paper has presented a method for the identification of the inertial parameters of parallel robots with actuation redundancy. Contrary to serial robots or parallel robots without actuation redundancy for which the dynamic identification methods are based on the use of the *IDIM* which calculates the joint forces/torques that are linear in relation to the dynamic parameters, for actuation redundant parallel robot that are overconstrained, the *IDIM* has infinity of solutions for the force/torque prediction, depending of the value of the desired overconstraint that is *a priori* unknown. As a result, the *IDIM* cannot be used.

This paper proposed to use the *PIM* of actuation redundant robots for identification purpose as it has a unique formulation and contains the same dynamic parameters as the *IDIM*. This is a newly discovered advantage of the *PIM*: for parallel robots with actuation redundancy, for which the usual *IDIM* cannot be computed, the unique formulation of the *PIM* makes it possible to achieve the identification of the dynamic parameters. The identification of the inertial parameters of a planar parallel robot with actuation
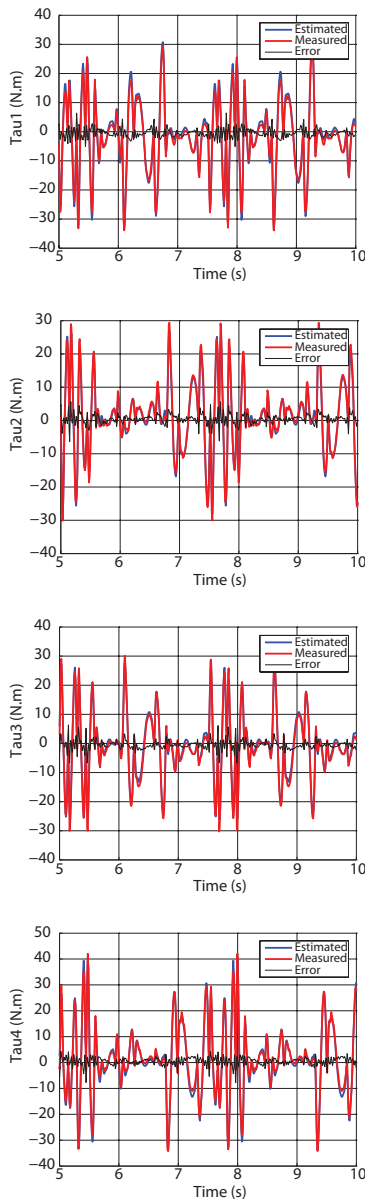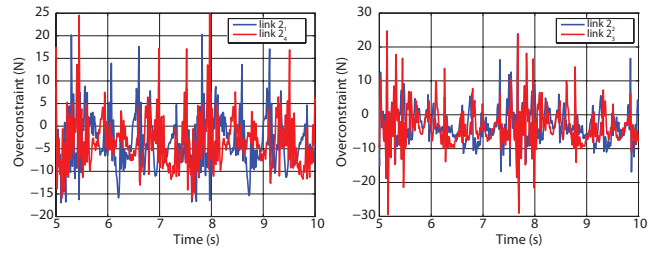
(a) in the distal links 2_1 and 2_4

(b) in the distal links 2_2 and 2_3

Fig. 5. Value of the overconstraint.

Fig. 4. Measured (red lines) and estimated (blue lines) input torques rebuilt using identified parameters with the payload of 5.37 kg in Case 2.

redundancy, the DualV, was then performed using its *PIM*. Experimental results show that the inertial parameters of the robot were correctly identified. Moreover, for validation purpose, a known payload mass has been added on the robot to be sure that the identification process was correct. This mass has been very accurately identified. Finally, it has been shown that the torque prediction with the newly identified parameters was correct.

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