# A MULTIRESOLUTION APPROACH FOR THE CODING OF EDGES OF STILL IMAGES USING ADAPTIVE ARITHMETIC CODING

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**ABSTRACT**

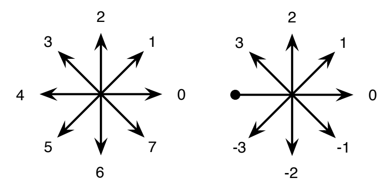
An edge coding scheme based on chain code representation in a multiresolution image coding context is presented. Our method enhances the coding schemes that describe the source structure with Markov models, by using also an *a priori* knowledge from the previous decoded resolution images. Experiments using adaptive arithmetic coding have shown up to a 5% improvement for the bitrate compared to a Markovian scheme.

***Index Terms*—** chain coding, Markov processes, arithmetic coding, adaptive coding, multiresolution image, Freeman codes

## 1. INTRODUCTION

Chain coding is required in many applications such as edge coding or segmentation map coding. Freeman proposed an eight-directional representation with fixed-length codewords for the coding of eight-connected chains of pixels. Each chain is represented by a head: the coordinates of the first pixel, and a path: the directional chain codes numbered from 0 to 7 indicating the direction of the next pixel (see Fig. 1-a). With this coding scheme, the probability distribution of a symbol is quite uniform, therefore the bitrate for the path is around 3 bits per link excluding key symbols such as termination codes.

To improve this bitrate, Freeman therefore proposed a chain difference coding where the link to the next pixel in the chain is represented by a shift in the direction given by the last two freeman codes. These new codes are numbered from $-3$ to 3 (see Fig. 1-a), but provided the regularity of the chains, the probability distribution of these codes is not uniform and therefore a variable-length coding can be applied leading to a bitrate typically around 2 bits/link.




(a) Freeman codes (b) Differential Freeman codes

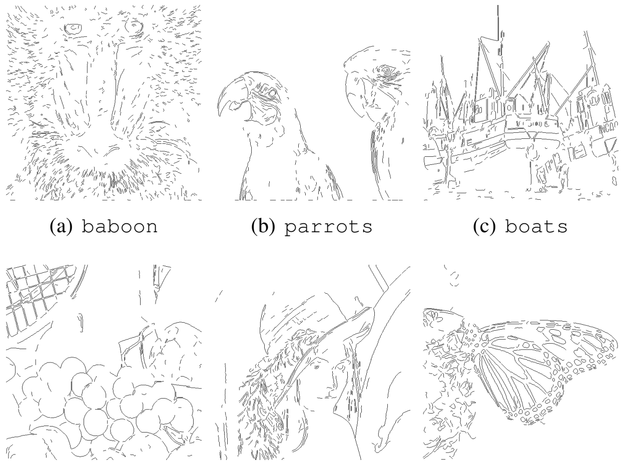
**Fig. 1**. Chain and chain-difference codes.

Lu and Dunham [1] improved chain coding using Markov models for the source structure with empirical statistics computed from a set of contour maps. Based on it, Chan and Siu [2] proposed an implementation using adaptive arithmetic coding where the initialization of the coder uses empirical statistics of the message. These statistics need to be first transmitted in a header. An adaptive arithmetic coder is also proposed by Pateux and Labit [3], but there the coder is initialized using an *a priori* law such as the statistics proposed by Lu and Dunham.

In the context of multiresolutional image coding, some geometric characteristics of an image may be used in order to adapt the transform to the content. For example in [4] we propose a multiresolution approach for still image coding based on oriented wavelet transform where the filtering is driven by the positions of the edges in the image. These edges, represented by chains of pixels positions, need to be encoded for each resolution.

Without any *a priori* knowledge between those extracted edges at the different resolutions, we propose to improve the methods proposed above by using an additional information that is available at the decoder: the decoded version of the image at the lower resolution. So our edge coding method can be used as long as a multiresolution image decomposition is realized.

We first study the improvement (in terms of entropy) provided by the additional information from the decoded image at the lower resolution. A comparison to the methods of the state of the art is presented. We then implement our approach



(a) `baboon` (b) `parrots` (c) `boats`

(d) `fruits` (e) `lena` (f) `monarch`

**Fig. 2**. Some examples of extracted edges.

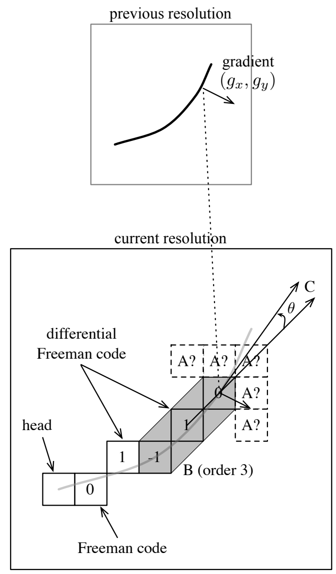



**Fig. 3**. $A$: the next differential Freeman symbol to code/decode, $B$: the $N$ last differential Freeman symbols to code/decode, $C$: the deviation of the orthogonal direction of the gradient at the lower resolution from the direction given by the last coded/decoded pixels' positions.

with arithmetic coding, and analyze the results.

In the following study, for each tested image, three levels of decomposition with a $9/7$ wavelet transform are produced. The process of edges extraction for each decomposition level consists in a thresholding with hysteresis of the local extrema of the Sobel gradient. Edge elements (*edgels*) above a given low threshold are detected, then they are linked. Edges that are not long enough or without any edgel above a given high threshold are not kept. Fig. 2 shows these extracted edges on ten test images for the full resolution.

## 2. MARKOVIAN AND DIFFERENTIAL FREEMAN ARITHMETIC CODING

Both Freeman and differential Freeman code can be modeled as Markov sequences. In theory, a differential Freeman code modeled as a Markov sequence of order $N$ is equivalent to a freeman code modeled with an order $N+1$.

Considering the results in [1], adaptive coding performs better than Huffman coding. In the implementation of arithmetic coding [5], the conditional probabilities are represented by counters. These counters are updated each time a symbol is encoded. These counters can be initialized using a uniform distribution (e.g. counters set to one) or using *a priori* empirical statistics.

The efficiency of arithmetic coding is related to the number of those counters compared to the size of the message to code. For adaptive coding from a uniform law, the higher the Markov order is, the larger the message size has to be in order to get a good estimation of its statistics. Using an *a priori* law for the counters initialization is supposed to accelerate the convergence of the estimation because the statistics of the message are supposed to be closer to this *a priori* distribution than to a simple uniform law. However if the training ratio, i.e. the ratio of the number of realizations used to initialize the counters to the number of counters, is too high, the coder will not manage to adapt efficiently. Therefore the coding will lead to poor results if the statistics of the message do not fit the *a priori* prediction.

Let $A$ be the random variable representing the link to the next pixel of the chain, and $B$ the random variable representing the Markov sequence of the last coded/decoded links. The conditional probability used in the arithmetic coding from the literature is thus $P(A \mid B)$.

## 3. ENTROPIC STUDY USING MULTIRESOLUTION REPRESENTATION

In a multiresolution image coding context, our goal is to take advantage of all the information available at the decoder in order to enhance the prediction of the next link.

Our first idea was to use the decoded edges of the lower resolution for this prediction. But as some details at the current resolution do not appear at the lower resolution, the prediction is sometimes impossible. Moreover even when the information does exist, it is generally quantized too much to be useful.

Therefore we decided to use the previous decoded resolution image as prediction. This information is always available, and its gradient can offer a good hint for the local direction of the edges at the current resolution.

In order to optimize the prediction of the next link, the gradient orientation of the already coded/decoded low-resolution image is compared to the orientation of the previous two decoded edgels. Let $P_i$ be the last coded/decoded pixel position, $\theta_{P_i}$ be the orientation orthogonal to the direction of the gradient at the point $(\frac{P_{i x}}{2}, \frac{P_{i y}}{2})$ of the last decoded resolution. $\theta_{P_i}$ is in $[-\pi, \pi]$. $C$ is defined as the deviation of $\theta_{P_i}$ from the direction of the line $(P_{i-1}P_i)$ (see Fig. 3).

In Fig. 4 we compare the entropy of the conditional probability of the next link, given the Markov sequence ($B$) for order 1 up to 3, the information from the lower resolution ($C$), or both. The presented entropy is the average of the entropies computed respectively for the three levels of decomposition.

As already observed in the literature, the higher the order of the Markov sequence, the lower the entropy. Here with an order below three, the entropy of $P(A \mid B \cap C)$ outperforms the entropy of $P(A \mid C)$ which also outperforms the entropy of $P(A \mid B)$.

These conditional probabilities are now implemented using an arithmetic coder.

## 4. ARITHMETIC CODING IMPLEMENTATION

The first side effect of implementation that prevents us from reaching the entropy is due to the length of the extracted edges. The shorter these edges are, the less efficient the high orders Markov sequence are. As a consequence, more terminal codes are needed, which are not well predicted.

As described in the section 2, the larger the number of realizations of a random variable (and the number of counters needed in the implementation of the adaptive arithmetic coder), the lower the bitrate. Therefore the order of the Markov sequence can not be very high, and the information from $C$ has to be quantized. This quantization has to be precise enough to be efficient but not too much to limit the number of counters .

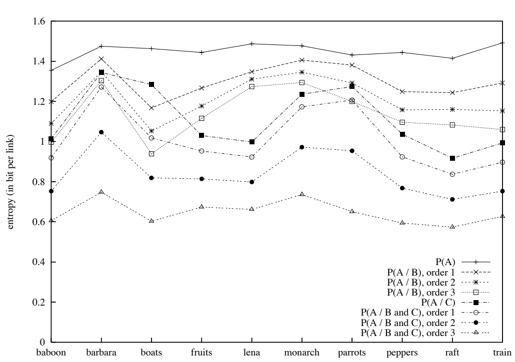



**Fig. 4**. Entropies of the conditional probabilities of $A$ given $B$, $C$ or $B \cap C$ for ten test images.

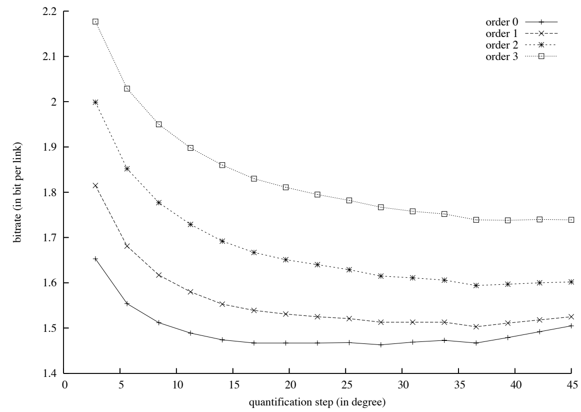



**Fig. 5**. Average bitrate of the paths of the extracted edges for 10 test images, using $P(A \mid B \cap C)$. The adaptive arithmetic coding is with different Markov orders and in function of the quantization step of $C$.

In the following sub-sections, first for different orders of the Markov sequence, we exhibit the best quantization of $C$ that reaches this tradeoff. Then we compare different setups for the adaptive arithmetic coding, and finally we improve the method by initializing the arithmetic coder with the statistics of the coded edges of the lower resolution.

### 4.1. Quantization of C

In Fig. 5, the average bitrate of extracted edges (for ten test images) in function of the quantization step of $C$ is presented. These bitrates are reached with an adaptive arithmetic coder using $C$ and $B$ (with an order from zero to three). For an order equals to zero, namely with only the information from $C$, the best value of the quantization step is about $20°$. The higher the order of the Markov sequence, the larger the quantization step, in order to minimize the bitrate.

However the results show that the lowest bitrate is reached without taking into account the information from $B$ that is the past at the current resolution. This is contrary to our expectation from the entropy presented in section 3. These results are explained by the large number of combinations needed in the implementation as the order grows. But as the order grows, we observe that the quantization has also to grow up in order to produce a competitive bitrate (a step of $37°$ for the order 1). This causes $C$ to become useless with such a quantization.

### 4.2. Comparison of different setups for adaptive arithmetic coding

The results of adaptive arithmetic coding initialized with a uniform law using the conditional probabilities of $A$ given $B$, given $C$, and given $B \cap C$ are presented in Fig. 6 for each test image. The results for $P(A \mid B)$, representing the methods of the state of the art [3, 2], are only presented from a

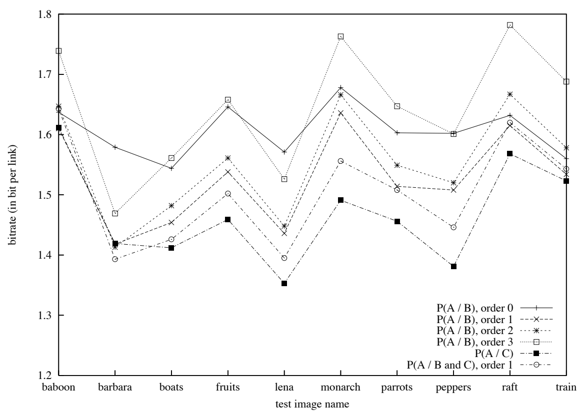



**Fig. 6**. Comparison of adaptive arithmetic coding of the paths of the extracted edges using $P(A \mid B \cap C)$ and $P(B)$ on ten test images.

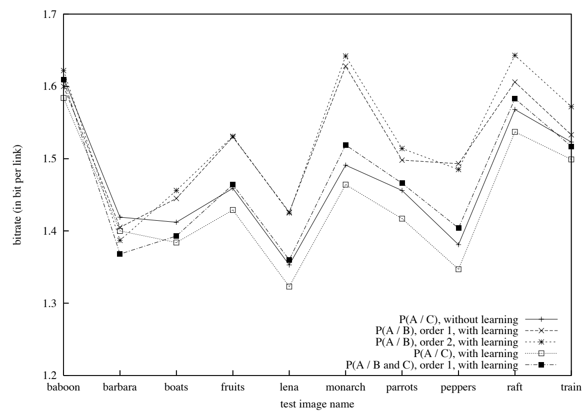



**Fig. 7**. Comparison of adaptive arithmetic coding of the paths of the extracted edges: using $P(A \mid C)$ from a uniform law, or using $P(A \mid B \cap C)$ and $P(B)$ from the statistics of the previous decoded resolution edges.

zero order up to a third order because this higher order does not improve, on average, the bitrate of the coded paths for the previous order. Except for the `baboon` and `barbara` images, $P(A \mid C)$ outperforms the former method. These exceptions with these two images come from the fact that, some extracted edges at the full resolution match with high frequencies that are not present at the lower resolution. In this case the gradient is not useful to predict these edges.

### 4.3. Initialization of the arithmetic coder

Arithmetic coders can be initialized with a uniform law or with an *a priori* law. In [1] for example, empirical statistics computed on different line drawings are employed. In order to make use of the multiresolution process, we propose that the statistics used as an initialization at a current resolution, are built from the adaptivity of the coder at the lower resolutions. This proposed initialization improves all tested methods. The results given on Fig. 7 show that the state of the art methods still remain less effective than our method whatever the initialization.

## 5. CONCLUSION

For edge coding in the context of multiresolution images, we proposed a new method for edge coding using the information of the previous decoded resolution. This is proposed in addition to Markov modeled differential Freeman chain coding from the state of the art.

We show that the deviation of the orthogonal direction to the gradient at the previous resolution, from the direction given by the last two decoded pixels of the chain at the current resolution, is often correlated with a shift in the direction of the current chain (namely the next differential Freeman code).

A preliminary study of the entropy, using this information and the last $N$ decoded differential Freeman codes of the chain, confirms that this deviation highly enhances the results than with only Markov model.

The implementation with arithmetic coding requires to quantize the deviation information. This important consideration leads to decrease the bitrate of the coded edges as the order of the Markov sequence grows. However arithmetic coding with the information of the previous resolution at the order zero, gives better results than only Markov model at any considered order.

## 6. REFERENCES

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