VECTOR QUANTIZATION BY PACKING OF EMBEDDED TRUNCATED LATTICES

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ABSTRACT

The purpose of this paper is to introduce a new vector quantizer (VQ) for the compression of digital image sequences. Our approach unifies both efficient coding methods : a fast lattice encoding and an unbalanced tree-structured codebook design according to a distortion vs. rate tradeoff. This tree-structured lattices VQ (TSLVQ) is based on the hierarchical packing of embedded truncated lattices. So we investigate the design of the hierarchical set of truncated lattice structures which can be optimaly embedded. We present the simple quantization procedure and describe the corresponding tree-structured codebook. Finally two unbalanced tree-structured codebook design algorithms based on the BFOS [1] distortion vs. rate criterion are used.

1. INTRODUCTION

Consider that we code a stationary memoryless vectorial source. According to the asymptotic equipartition property, as the space dimension increases, the vector probability function becomes essentially localized to a compact region of the vector space where the density is almost uniform [10]. The optimal condition for vector quantization occurs when all the codevectors are confined to a compact support region. So for a given bit rate, the average reconstruction error for the input vectors is decreasing. Because of the uniform density, a codebook designed by truncating the highly regular structure of a lattice is well fitted to vector quantization schemes without learning stage [4].

However, in practice, the design of VQ coder using high-density lattices [2] for large dimensions of the vector space produces codevectors with small representativeness and needs efficient entropy-based indexing module. To overcome these drawbacks, we propose a new lattice VQ scheme based on the hierarchical packing of embedded lattices. This approach has simultaneously the efficient properties of a lattice VQ (i.e no time-consuming learning stage for the codebook design and fast coding-decoding operators for each new input source vector) and the opportunity to locally adapt the packing and the hierarchy of lattices taking into account the statistical partition of the image source vectors. This vector source, even if we preprocessed it (by a wavelet transform for instance) is always a nonstationary signal. So a temporal updating of the codebook [6][8] has to be designed in order to fit the input spatiotemporal source distribution. The generalized Lloyd algorithm [5] provides the locally optimal codebook for a given training set and a given bit rate, but the computation complexity of usual classification methods presents a limitation on their applicability for adaptive schemes. A pre-defined hierarchy of embedded lattices seems to be the adequate and adaptive representation for a lot of commonly used image source vector spaces (i.e DCT coefficients, wavelet coefficients, vector of motion-compensated prediction errors or an hybrid source between all of these.., etc).

2. TSLVQ DESIGN

2.1. Embedded Truncated Lattices

With respect to the space dimension, the optimal lattice is selected among $Z^2, D_4, E_8, \Lambda_{16}$. These lattices give the best sphere packings and coverings in their dimension. Then, considering the L^2 Euclidean metric, the regular structure is specifically truncated for the packing such as the confined space, after contraction, recovers maximaly a Voronoï cell of the support lattice. Thus, if ρ and r are respectively the packing radius and the covering radius of the support lattice, the lattice truncation energy E_T is given by [3]:

$$E_T = ((2 \times k + 1) \times \rho)^2 \text{ with } k \in \mathbb{N}^*$$

Namely, the Voronoï cells totally or partially within the multidimensional sphere of radius $R = \sqrt{E_T}$ constitute the confined space. In high dimension, an upper bound for the number of lattice points that lie within this subset is calculated using the Theta series and considering the points into the sphere of radius θ, with :

$$\theta = (2 \times k + 2) \times r$$

The figure 1 illustrates the method using the simple Z^2 lattice, without any loss of generality our approach can be generalized to higher dimensions by employing high dimension lattices.

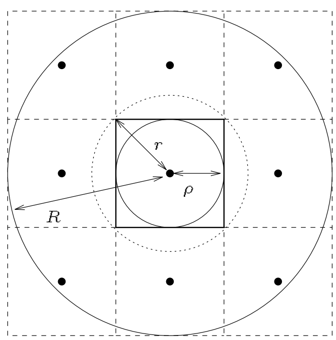


Figure 1: *Truncated Z^2 lattice ($k = 1$). The squares symbolise the Voronoï cells and the dots symbolize the reconstruction points.*

2.2. Hierarchical Set of Truncated Lattices

So, from the previous basic confined space, by shifting its scale, we obtain a hierarchical set of multidimensional regular structures : it is possible, by a simple translation, to embed a lower scale truncated lattice in any Voronoï cell of the next higher scale structure (see figure 2).

2.3. Quantization Procedure

The figure 3 illustrates the quantization procedure using successive scaling and translating operators. We have :

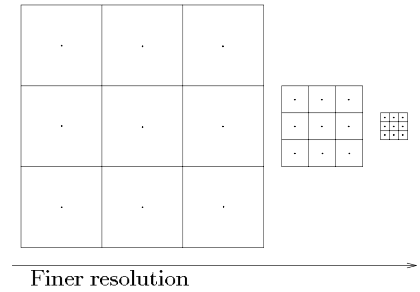


Figure 2: *Hierarchical set corresponding to the previous truncated Z^2 lattice.*

- X : input vectors ;
- Y : truncated lattice reproduction vectors ;
- $E_{\max}$: maximum energy for the source to be encoded ;
- $F_1 = \sqrt{\frac{E_T}{E_{\max}}}$: scaling factor used in order to project X into the basic confined space ;
- $F = \frac{\sqrt{E_T}}{\rho}$: scaling factor used in order to (re)project the vectors into the next finer resolution lattice space ;

Only the parameters $E_{\max}$ and k have to be fixed. At each stage, the quantization is performed with the same truncated lattice structure.

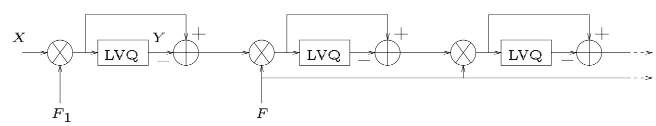


Figure 3: *Quantization scheme.*

2.4. Tree-Structured Codebook

So the codebook has a m-ary tree structure, with m corresponding to the basic confined space points number. A node is a lattice point, its children are the points of the lattice structure embedded into the node Voronoï cell. A tree stage specifies the scale amplitude : the deeper is the tree, the finner is the resolution, the reconstruction errors of the input vectors decrease while the terminal nodes number increases. The figure 4 shows a tree-structured codebooks example.

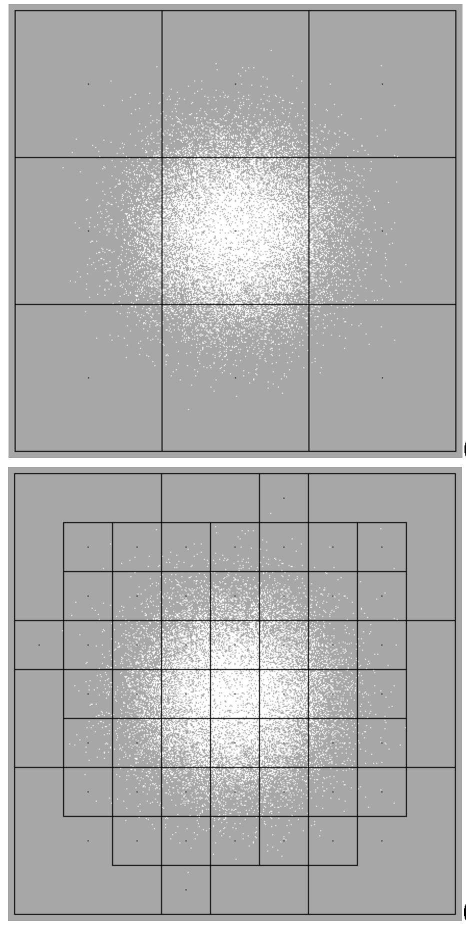
2.5. Unbalanced Tree-Structured Codebook Design

Two classical strategies have been explored in order to obtain a variable rate vector quantizer (i.e an unbalanced tree) : a tree pruning and a greedy tree growing approach [1][9].
In our experiments a training procedure is performed to design the codebook. When encoding the given source, each tree node is characterized by an average distortion and an variable code length. In order to select the branch for pruning (tree pruning approach) or the leaf for splitting (tree growing approach), the BFOS decision criterion is accurate : it performs a locally optimal entropy/distortion trade-off.
An entropy encoding is used for indexing the unbalanced tree terminal leaves. An efficient encoding and indexing is achieved if the basic confined points number is highly restricted, so k is fixed to 1 for the lattice truncation energy calculation E_T.
The average distortion or the average code length associated to the terminal leaves is considered to stop the codebook design procedure.
Now we remark an other aspect in the figure 3 : on the contrary to usual multistage VQ, the quantization stages number is variable for the input vectors.

The overall information that we have to transmit to characterized the TSLVQ codebook is constituted by : $E_{\max}$, the unbalanced tree and the entropy index corresponding to the leaves. Because of the lattice predefined structure, no reproduction vectors have to be transmitted.

3. RESULTS AND CONCLUSION

Comparative experiments applied to i.i.d synthetic sources and real-world images sequence have been performed. A training ratio upper than 100 is used for limiting the reproduction vectors number, so the training sequence size is adapted to the vector dimension.
The figures 6 illustrates the tree pruning approach with Z^2. This figure shows how our method is adapted to differential or hybrid image source coding : for a given rate, the high-density space region (where are located the lowest error magnitudes) is coarsely quantized in order to permit a finer coding of the low-density space region (where lie relevant vectors).



(a)

(b)

Figure 4: *Tree-structured codebooks example using 1 quantization stage (a), and 2 quantization stages (b). The white dots symbolize the input vectors.*

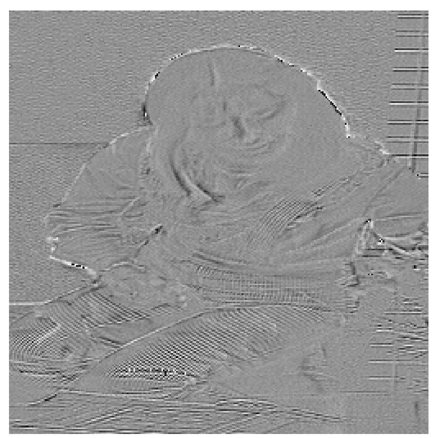
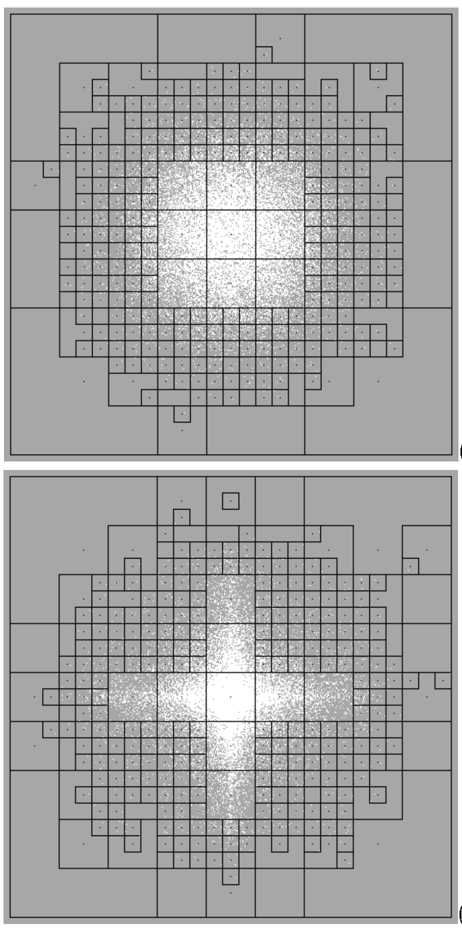


Figure 5: *Image extracted from the differential images sequence source.*

The tree growing approach is illustrated by the figure 7 which shows PSNR (peak signal-to-noise ratio) vs. rate (the codebook entropy) curves, and an image of the differential images sequence vector source is given with the figure 5. For comparison, the curve obtained using the LBG algorithm [7] is shown : the vector dimension is 4 and a full search within the codebook is achieved. The interest of using higher dimensions is demonstrated since TSLVQ with D_4 or Z^4 lattices performs better than the others (a higher PSNR is achieved for a lower rate). Given the vector dimension, TSLVQ with the corresponding best quantizing lattice gives the best result, therefore TSLVQ with D_4 lattice performs better than TSLVQ with Z^4 lattice.



(a)

(b)

Figure 6: *Tree pruning approach illustration considering respectively a Gaussian distribution (a), a Generalized Gaussian distribution (b).*

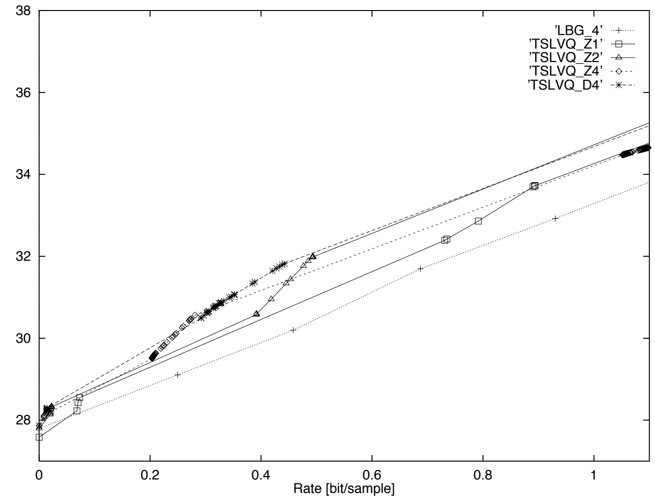


Figure 7: *Tree growing approach : PSNR vs. rate curves obtained by coding the differential source.*

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