Timed automata

Exercise 1 — Coffee machine

Consider a coffee machine specified as follows :
- initially the machine waits for a coin,
- as soon as the coin is provided, the user can choose if she wants a coffee, a tee or getting her coin back,
- after 20 seconds, if she has not chosen yet, she can only get her coin back,
- if she choses a coffee or a tea in less than 20 seconds she gets it.

1. Describe this machine as a timed automaton.

It is possible to add a notion of *urgence* in states of a timed automaton : conditions of the form $x \leq c$ are attached to states, meaning that these states should be exit before $x > c$. We add the following requirements for our coffee machine :
- the coin is automatically returned to the user if she has not chosen her beverage in less than 20 seconds,
- it takes 35 seconds to prepare a tea and 40 seconds to prepare a coffee.

2. Describe this new version of coffee machine as a timed automaton with urgence.

Exercise 2 — Mouse click

1. How would you specify a mouse double click as a timed automaton ?

Exercise 3 — Automata and languages

Fix an alphabet $\Sigma = \{a, b\}$. For each of the following languages over Σ propose a timed automata that recognizes it.

1. $\mathcal{L}_1 = \{(\sigma, \tau) \mid \forall i, t_{i+1} - t_i = 1\}$

2. $\mathcal{L}_2 = \{(\sigma, \tau) \mid \forall i < j, \sigma_i\sigma_{i+1} \dots \sigma_{j-1}\sigma_j = ab^{j-i-1}a \Rightarrow t_j - t_i = 1\}$

3. $\mathcal{L}_3 = \{((abc)^*, \tau) \mid \sigma_i = a \Rightarrow t_{i+3} - t_i \leq 3, \sigma_i = b \Rightarrow t_{i+3} - t_i \geq 2, \sigma_i = c \Rightarrow t_{i+3} - t_i = 4\}$, any observation about this language ?

Exercise 4 — Power of the model

Recall that, for any $\mathcal{L}$ recognized by a timed automata, $Untime(\mathcal{L})$ is rational.

1. Is it true that for any $\mathcal{L}$ such that $Untime(\mathcal{L})$ is rational, there exists a timed automaton recognizing $\mathcal{L}$?

From now, we extend the model, allowing guards of the form $k.x - k'.y = c$ where $k, k', c \in \mathbb{Z}$.

2. Propose a timed automaton recognizing some language $\mathcal{L}$ such that $Untime(\mathcal{L}) = \{a^n b^n \mid n \geq 2\}$.

3. What are your conclusions about this extension of timed automata ?