# Summarising Game Segments

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## Motivations

### Algorithmic

- huge state spaces
- too much non-relevant details
- most algorithms target zero-sum aspect

### Analytic

• conceptual simplification tool

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### Abstraction

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# Summarisation of game segments

#### Procedure summary

- well know method
- control-flow graphs
- avoiding state-space explosion



### Our work

#### Summarization for games

- $\bullet$  game/control flow graph + interactivity between players
- segment/procedure

Segment



Directed graph, nodes  $V, X \subset V$ . In(X): nodes of X having predecessor outside of X. Out(X): nodes of X having successor outside of X.

In(X) and Out(X): interface

## Segment



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### Negociating power of a player.

	Bach	Stravinsky	TV
Bach	Bach	None	τv
Stravinsky	None	Stravinsky	τv

Effectivity of player *i*  

$$\Phi^{i} = \{F \subseteq \Omega | \{\gamma(s^{i}, s^{-i}) | s^{-i} \in S^{-i}\} \subseteq F, \text{ for some } s^{i} \in S^{i}\}$$

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Segment (extensive form)

L. Jezequel, D. Berwanger (ENS Cachan)



- Segment (extensive form)
- Strategies (normal form)



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- Simplify (effectivity)



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## Outcomes in summarized segments



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Outcome: intersection of effectivities (contained).

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## Outcomes in summarized segments



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Effectivity game form

How to play:

- at position v
- choose  $a^i \in A^i(v)$
- next position is taken (non-deterministicaly) in  $\gamma(v, a)$ .

## Remark on effectivity games

• Extensive games can be represented as effectivity games:



• Concurence games also.

## Security level

When payoffs come...

Utility function (worst payoff for some strategy profile)

$$u^i(\mathcal{G},s) = \min\{u^i(\pi)|\pi\in\Omega(\mathcal{G},s)\}$$

Security level of player *i* (what he can guarantee)

$$\mu^i(\mathcal{G}) = \max_{s^i \in S^i} \min_{s^{-i} \in S^{-i}} u^i(\mathcal{G},s)$$

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u-faithful projections

$$h: V \to U \subseteq V$$
 is u-faithful if,  $\forall \pi \in \Omega(\mathcal{G}), u^i(h(\pi)) = u^i(\pi)$ 

Theorem

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#### Theorem

 $\mathcal{G}$  an effectivity game, X a segment if  $h: V \to V \setminus (X \setminus (In(X) \cup Out(X)))$  u-faithful then security level is preserved under summary of X

$$\mu^i(\mathcal{G}) = \mu^i(\mathcal{G}/X)$$

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# Conclusion and further works

#### Main result

Preservation of security level under summary.

#### Further work

- build security strategies from a summarized game
- algorithm for summarizing effectivity games in practice (e.g. composition of effectivities)
- other kind of games (parity...)