Distributed Optimal Planning : an Approach by Weighted Automata Calculus

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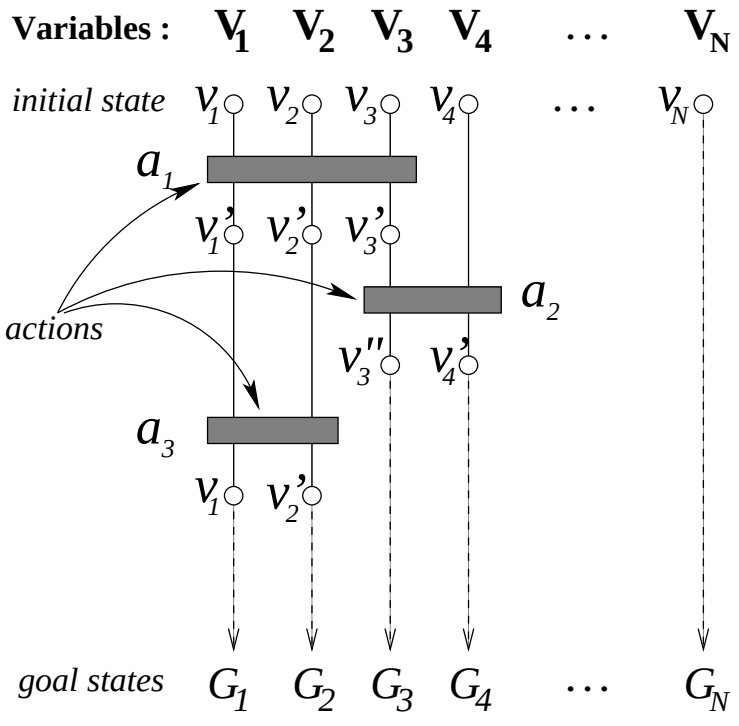
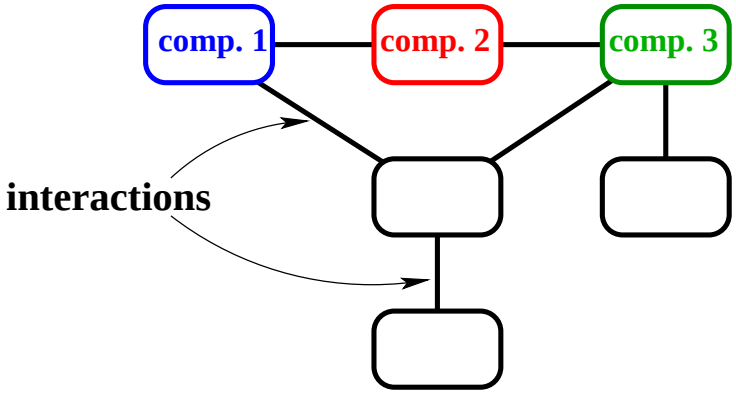
Outline

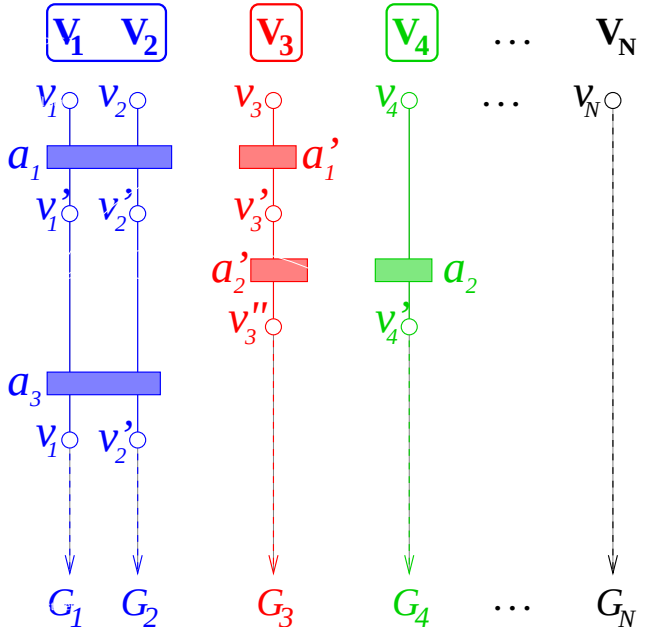
Motivation: optimal planning



Objective: find an action strategy of *minimal cost* to go from system state $(v_n)_{1 \leq n \leq N}$ to one state in the target $G = \times_n G_n$

A *distributed* optimal planning problem





Desired features: a distributed resolution

- each component looks for a local plan
- ensure that local plans are *compatible*
- and that their merging yields an *optimal global plan*
- distributed constraint solving + distributed optimization

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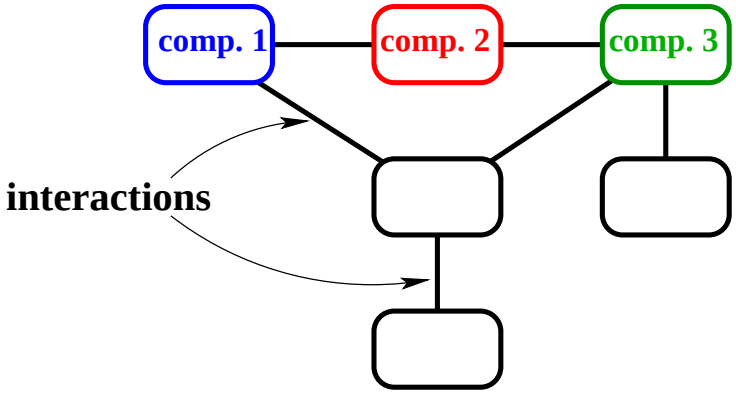

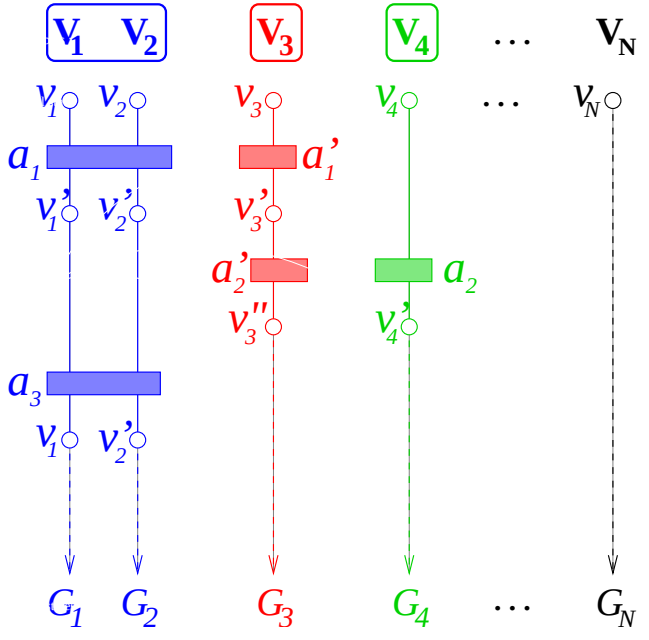

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Components = weighted automata

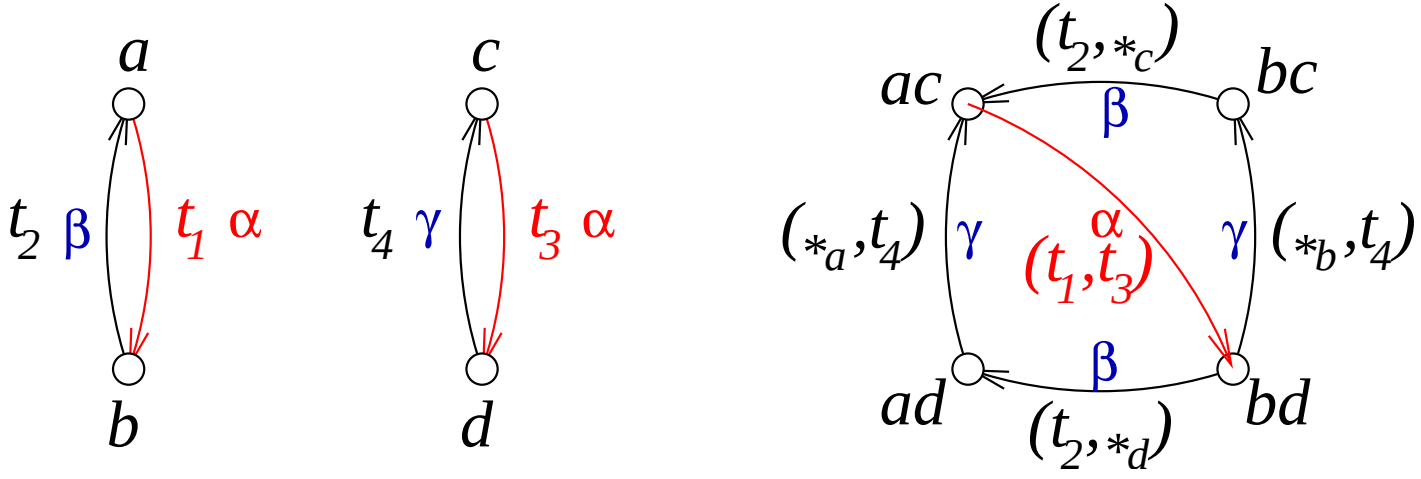
Weighted automaton: $\mathcal{A} = (S, \Sigma, I, F, c, c_i, c_f)$

- S=states, Σ=actions, I=initial states, F=final states
- cost function on transitions: $c : S \times \Sigma \times S \to \mathbb{K}$
  $t = (s, \sigma, s') \Rightarrow c(t)$ is the cost for firing t
- semiring $(\mathbb{K}, \oplus, \otimes, \bar{0}, \bar{1}) = (\mathbb{R}^+ \cup \{+\infty\}, \min, +, +\infty, 0)$
- $c_i : I \to \mathbb{K} \setminus \{\bar{0}\}$ and $c_f : F \to \mathbb{K} \setminus \{\bar{0}\}$ are state costs

Interactions = product of weighted automata

Synchronous product: $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$

- transitions synchronize on common action labels $\Sigma_1 \cap \Sigma_2$
- costs of synchronized actions are added
- transitions carrying a private action label remain private



Problem formulation

Language of a WA (= all weighted plans):

$$\mathcal{L}(\mathcal{A}) = \sum_{u \in \Sigma^*} \mathcal{L}(\mathcal{A}, u) \cdot u$$

$$\mathcal{L}(\mathcal{A}, u) = \min_{\substack{t_1 t_2 \ldots t_n \models \mathcal{A} \\ u = \sigma(t_1 \ldots t_n)}} c_i(^\bullet t_1) + c(t_1) + \ldots + c(t_n) + c_f(t_n^\bullet)$$

Problem:

given the network of weighted automata $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_N$ determine the run(s) u^* of $\mathcal{A}$ with minimal weight:

$$u^* = \arg \min_{u \in \Sigma^*} \mathcal{L}(\mathcal{A}, u)$$

Challenge:

find these runs with a distributed procedure, without computing $\mathcal{A}$ nor $\mathcal{L}(\mathcal{A})$...

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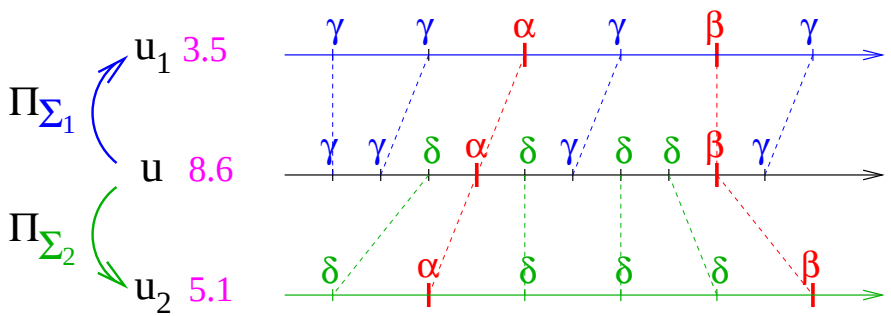
Assembling local plans = product of weighted languages

Product:

- $\mathcal{L}_1, \mathcal{L}_2$ weighted languages on Σ_1 and Σ_2 resp.
- $\mathcal{L}_1 \times_L \mathcal{L}_2$ is a weighted language on $\Sigma_1 \cup \Sigma_2$

$$(\mathcal{L}_1 \times_L \mathcal{L}_2)(u) \quad = \quad \mathcal{L}_1(u_{|\Sigma_1}) + \mathcal{L}_2(u_{|\Sigma_2})$$



Theorem 1

$$\mathcal{L}(\mathcal{A}_1 \times \ldots \times \mathcal{A}_N) = \mathcal{L}(\mathcal{A}_1) \times_L \ldots \times_L \mathcal{L}(\mathcal{A}_N)$$

Consequence: Global plans have a factorized representation.

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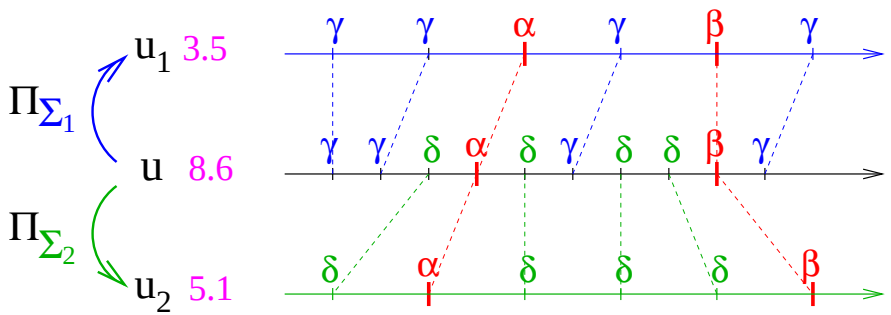

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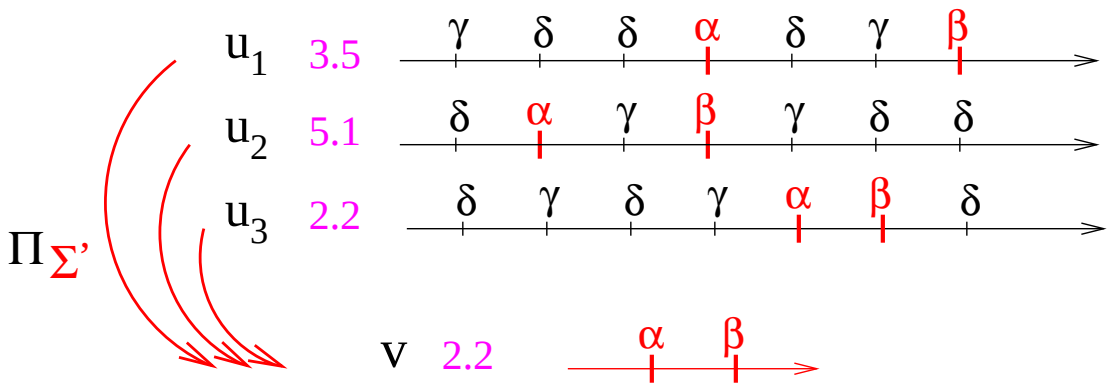
Local view of global plans = projection of a WL

Projection:

- $\mathcal{L}$ weighted language on Σ, and $\Sigma' \subseteq \Sigma$
- $\Pi_{\Sigma'}(\mathcal{L})$ is the weighted language on Σ'

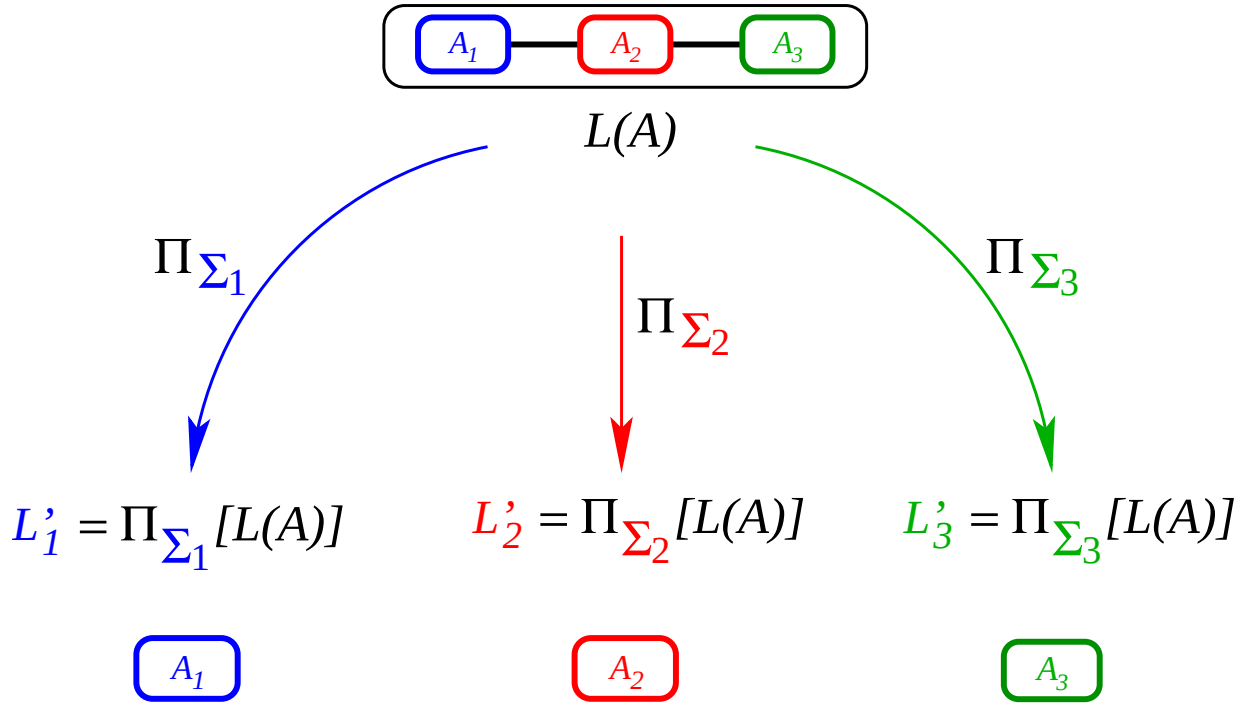
$$[\Pi_{\Sigma'}(\mathcal{L})](v) = \min_{\substack{u \in \Sigma^* \\ u_{|\Sigma'} = v}} \mathcal{L}(u)$$



- $\Rightarrow$ weight minimization over words u with same projection v

Relating optimal planning to projection

Objective: for $\mathcal{A} = \times_{i \in I} \mathcal{A}_i$ compute the $\mathcal{L}'_i = \Pi_{\Sigma_i}[\mathcal{L}(\mathcal{A})]$



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$\Rightarrow$ $u_i^* = u_{|\Sigma_i}$ optimal word in $\mathcal{L}'_i = \Pi_{\Sigma_i}[\mathcal{L}(\mathcal{A})]$, with $\mathcal{L}'_i(u_i^*) = w$

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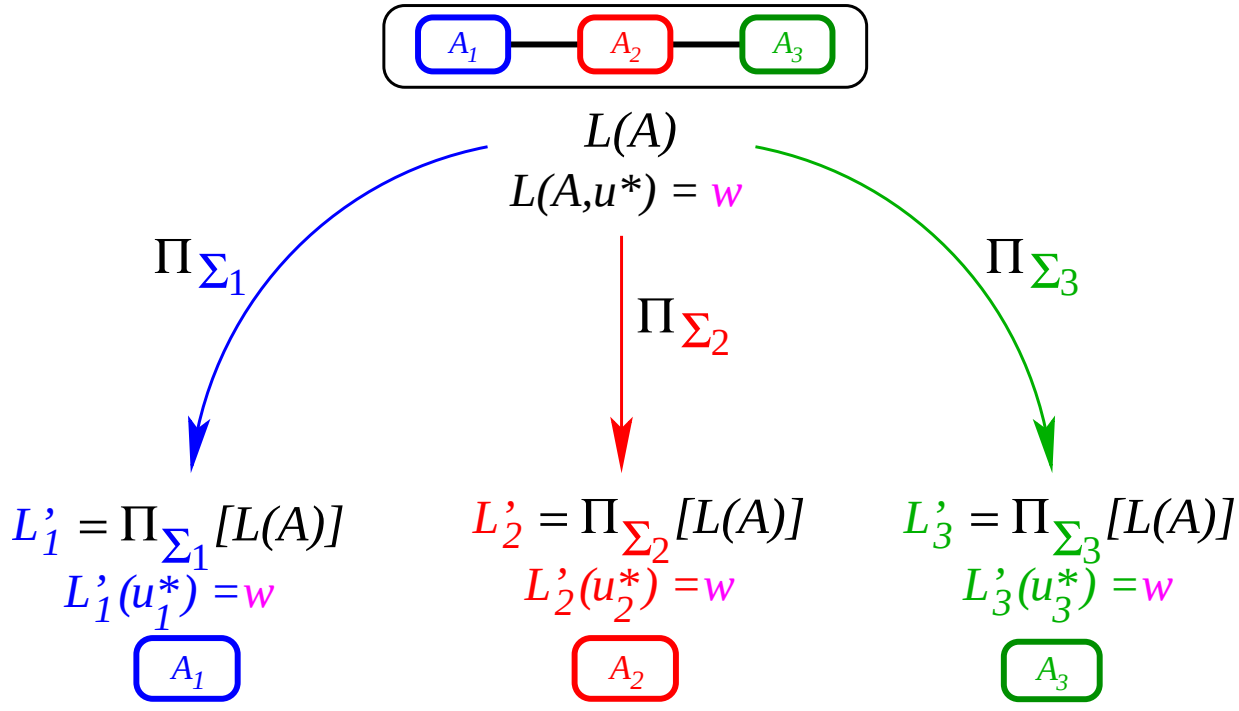

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The key to distributed optimal planning

Theorem 2

Let $\mathcal{L}_1, \mathcal{L}_2$ be weighted languages on Σ_1, Σ_2 resp., and let $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_3 \subseteq \Sigma_1 \cup \Sigma_2$, then

$$\Pi_{\Sigma_3}(\mathcal{L}_1 \times_L \mathcal{L}_2) = \Pi_{\Sigma_3}(\mathcal{L}_1) \times_L \Pi_{\Sigma_3}(\mathcal{L}_2)$$

- allows us to compute the projections $\Pi_{\Sigma_i}(\mathcal{L})$ by local computations
- Example: with $\Sigma_3 = \Sigma_1$, one has

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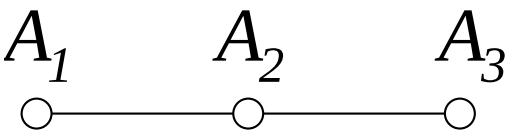
Dynamic programming in space

Interaction graph of components : defined by shared actions
Example: assume $\Sigma_1 \cap \Sigma_3 \subseteq \Sigma_2$



Important : we assume the interaction graph is a tree

Principles of a message passing algorithm: (example)

- By Thm 1 : $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \times_L \mathcal{L}(\mathcal{A}_2) \times_L \mathcal{L}(\mathcal{A}_3)$
- By Thm 2:

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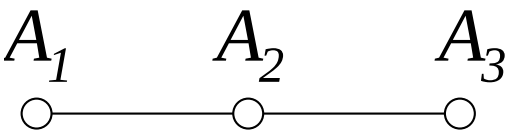

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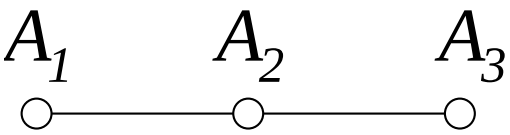

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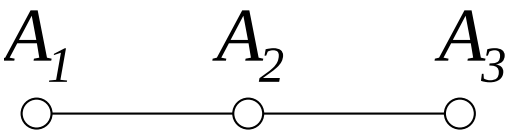

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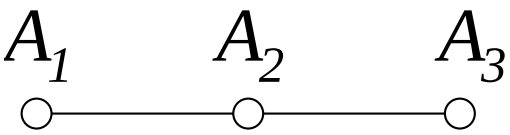

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Implementation is needed and possible

Languages are potentially infinite objects:
languages are not usable in practice.

Solution:

- work directly with automata;
- projection: ϵ-reduction (+ determinization and minimization);
- product: synchronous product of weighted automata.

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Conclusion

Main features:

- unsupervised distributed search of an optimal plan
- all possible/optimal plans are computed
- global plans are computed as tuples of partially synchronized sequences, *i.e.* as *partial orders* of actions

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Planning as games: why?

Improve fairness between agents:

- optimizing the sum may penalize some agent;
- is it possible to be fair?

Reduce communications:

- currently messages are potentially very large (contain all plans);
- in a truly distributed setting they may be numerous;
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Game theory may help in solving these issues. . .

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"Static" games:

- use for fairness? look for equilibria?
- complete or incomplete information?

An example:

- one player per automaton;
- a local plan in automaton A_i is a strategy for player i;
- each player has to maximize her payoff (defined below).

p_i is the strategy/local plan chosen by player i, and p is any global plan corresponding to all players choice (if exists).

Different possible payoffs:

- payoff for any player $= 1/c(p)$ (or 0 if p does not exist);
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"Dynamic" games:

- dynamic construction of local plans;
- all agents have to agree on a synchronization word;
- to deal with message size/quantity?