Summarising Game Segments

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One of the major challenges towards applying game-based methods to the analysis of complex interactive systems is scalability. Besides the state-explosion problem intrinsic to multi-component designs, keeping track of the control flow along game transitions determined by different players adds a layer of mental and computational complexity to system models.

Abstraction is a generic approach to handling state-explosion by down-scaling models in such a way that essential properties are preserved. The approach has been very successful in the static analysis of sequential programs where the basic control-flow problem amounts to a question of reachability. For interactive programs, and in general for games, the control-flow problem corresponds to alternating reachability, an analysis that regards the power of an agent to reach a set of states, independently of the choice of the other agents.

In this contribution, we propose a method for reducing the state space of game models that consists in replacing game segments with so-called called summaries in such a way that the security level of all players, as well as the structure of their security strategies, are preserved.

We consider non-zero-sum games that involve an arbitrary number of players with finitely many payoffs; the state space may be infinite. Our method operates on neighbourhood models [3], a game representation that is intermediary between turn-based and concurrent game structure [1]. The atomic transitions in neighbourhood models are set-valued. They are represented by effectivity functions [2] for the individual players, which link each state to the sets of states into which the player can enforce to outcome of an atomic move.

A segment is a subset of the state space that induces a contiguous fragment of the transition structure and has finitely many input and output states. To summarise a segment, we discard the internal states and view the output states as outcomes of a partial play. Accordingly, each input state is associated with an atomic game obtained by composing the effectivity functions along the transitions in the game segment. To play this game, each player chooses one of the sets of output states available to him; the outcome is a state on which all choices agree.

Clearly, summarisation of a game segment incurs a loss of information with respect to the original model. In particular, the temporal order in which decisions are taken is not represented any more. Nevertheless, we show that, for games games with reachability and safety conditions summarisation of game segments preserves the security level of all players. Moreover, the operation is compositional in the sense that every strategy that guarantees the safety level in the abstracted game can be transformed into one that guarantees the safety level in the original game, by concretising the strategies on the summarised segments independently.

References

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