REGULARIZED DOPPLER RADAR IMAGING FOR TARGET IDENTIFICATION IN ATMOSPHERIC CLUTTER

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ABSTRACT
We develop a method for the formation of Doppler radar images with enhanced features. This problem, when studied as an adaptive spectral estimation problem, is particularly ill-posed because of the small number of data. Our approach is based on a regularized estimation of depth-frequency images which combines a high-resolution Fourier model of the observations with prior information about the nature of the features of interest. We first derive an appropriate model and a regularized criterion for meteorological clutter restoration before addressing the extension to the identification of spot-like targets superimposed to this clutter. We also adapt quasi-Newton algorithms based on Half-Quadratic regularization [1, 2] for the computation of the solution. The practical interest of our approach is validated on simulated and real data.

1. INTRODUCTION
The problem of adaptive spectral estimation has received considerable attention in the signal processing community [3, 4, 5] since it came up in various fields of engineering, especially in radar Doppler imaging. In this context, it consists in searching for a series of spatially depth-wise juxtaposed spectra given the discrete time data, extracted for any depth of interest. The present paper focuses on short-time estimation since only eight time points of complex-valued pulsed Doppler signals are available to estimate each spectrum. Two complementary problems are then studied.

Section 2 focuses on estimating a series of spectra of atmospheric clutter (rain, sea, mixed clutter) with a large variability in terms of spectral content and number of modes. Given a small number of data points, periodogram-based estimates are unreliable. Long AR models have been investigated using quadratic regularization to account for spectral smoothness and depth continuity [6, 3, 5]. Unfortunately, quadratic penalization oversmooths discontinuities between different clutters. To overcome these limitations, we propose a depth-frequency analysis in a nonparametric framework. In [7], we have developed a regularized approach that integrates spectral smoothness for the estimation of a single spectrum. Here, we propose a 2D extension to account for depth continuity as well. The depth-frequency image solution is defined as the global minimizer of a convex criterion, regularized in both directions (depth and frequency) using a nonquadratic penalty term.

Section 3 addresses the problem of restoration of targets superimposed on atmospheric clutter. The proposed approach generalizes the mixed spectrum estimation method proposed in [7]. It amounts to estimating two depth-frequency distributions: one for the targets and one for the clutters. Since target spectra are mostly spiky, a separable penalty term is used to enhance this feature. Finally, the pairs of mixed spectra that result from global optimization of a unique criterion are summed up to provide a map that displays both the targets and the clutter. Efficient numerical solution is achieved through 2D extensions of block-coordinate descent based on HQ regularization [1, 2]. These methods are matched to the complex nature of the estimated quantities. In Section 5, results on synthetic and real Doppler signals are presented.

2. ATMOSPHERIC CLUTTER IMAGING
2.1. Problem statement
In radar Doppler imaging, the data consist of a set of complex-valued signals \( Y = [y_1, \ldots, y_M] \), spatially depth-wise juxtaposed in \( M \) bins, each reflecting a certain depth range. Following [4, 7], each short-time vector of data \( y_m \) is assumed to be a truncated subset of a complex time series \( (y_{nm})_{n \in \mathbb{N}} \). Moreover, it is supposed to be independent from its neighbors \( y_{m \pm 1} \). Fig. 1 gives a Gaussian simulated example over \( M = 96 \) range bins for which \( N = 8 \) data are observed per bin.

Depth-frequency estimation is addressed as the depth-wise extension of spectral analysis. For short-time data sets, this issue can be tackled as a Fourier synthesis problem [4, 7]. Similarly, our goal is to search for the energy distribution of \( (y_{nm})_{n \in \mathbb{N}} \) in the frequency domain. The harmonic frequency model is usually considered for this task. Assuming that the distribution of spectral amplitudes \( X_m(\nu) \) is continuous with respect to frequencies \( \nu \), the inverse discrete-time Fourier transform links the unknown function \( X_m \in \mathbb{L}_C^{2}[0,1] \) to the finite energy series \( (y_{nm})_{n \in \mathbb{N}} \) according to

\[
y_{nm} = \int_0^1 X_m(\nu) e^{2\pi i n \nu} \, d\nu.
\]

Spectral estimation is thus a discrete-time continuous-frequency problem, consisting in recovering \( X_m(\nu) \) given \( y_{nm} \) for \( n \in \mathbb{N} \). Following [4, 7], we resort to a discrete frequency approximation using a large number of sinusoids, say \( P \gg N \), at equally sampled frequencies \( p/P, p \in \mathbb{N}_P \). The approximation of (1) then reads

\[
y_{nm} = \sum_{p=0}^{P-1} X_{pn} w_{0n}^m, \ n \in \mathbb{N}_N,
\]

where \( w_0 = \exp(2j\pi/p) \) and \( X_{pn} \in \mathbb{C} \) are the unknown spectral amplitudes. In vector-matrix form, (2) reads:

\[
y_m = W_{Np} X_m, \quad (3)
\]
is chosen circular, that is
\[
\mathcal{R}(\mathcal{X}) = \mathcal{R}(X_1, \ldots, X_P) = \mathcal{R}(p_1, \ldots, p_P)
\]
with \(p_m = |X_m| = |\rho_{1m}, \ldots, \rho_{Pm}|^{-1}\). For computational simplicity, we focus on convex and continuously differentiable \(C^1\) energies \(\mathcal{R}\) and \(\mathcal{J}\).

2.3. Markovian depth-frequency regularization

In [7], we have proposed the following circular and convex penalty term to enforce spectral smoothness:

\[
R_{\mathcal{R}}(X_m) = \sum_{p=0}^{P-1} (\mu_\mathcal{R} \phi_1(p_{p+1,m} - p_{pm}) + \phi_2(p_{pm})), 
\]
(4)

where \(\mu_\mathcal{R} \geq 0\) tunes the amount of smoothness, \(\phi_2 : \mathbb{R}_+ \to \mathbb{R}\), and \(p_{P+1,m} = p_{0,m}\), because of the 1-periodicity of the discrete Fourier transform. As stated in [7, Corollary 1], function \(R_{\mathcal{R}}\) is circular, i.e., \(R_{\mathcal{R}}(X_m) = R_{\mathcal{R}}(p_m)\) and convex, provided that:

- \(\phi_1\) is even and convex,  
- \(\phi_2\) is convex and nondecreasing,  
- \(\mu_\mathcal{R} \leq \mu_{\text{supp}} = \phi_2'(0^+)/2\phi_1'(\infty)\).  

Inequality (5c) gives an upper bound on the smoothness level that can be chosen while maintaining convexity of \(R_{\mathcal{R}}\). Since \(\mu_{\text{supp}} > 0\) requires \(\phi_2'(0^+) > 0\), \(\phi_2([1,1])\) and \(R_{\mathcal{R}}\) are not \(C^1\) at zero. For \(\phi_2(u) = u\), a \(C^1\) approximation is [7]

\[
R_{\mathcal{R},\varepsilon}(X_m) = \sum_{p=0}^{P-1} (\mu_\mathcal{R} \phi_1(q_{p+1,m} - q_{pm}) + q_{pm}), 
\]
(6)

where \(q_{pm} = \phi_2(p_{pm})\), \(\phi_2(x) = \sqrt{\varepsilon^2 + |x|^2}\), and \(\varepsilon > 0\). Function \(R_{\mathcal{R},\varepsilon}\) is also circular and its convexity is proven in [7, Corollary 2] under conditions (5a)–(5c).

Spectral regularity and depth continuity are simultaneously taken into account in a natural extension of (6) given by

\[
\mathcal{R}(\mathcal{X}) = \sum_{m=1}^{M} R_{\mathcal{R},\varepsilon}(X_m) + \mu_T \sum_{m=1}^{M-1} \sum_{p=0}^{P-1} \phi_3(q_{pm+1} - q_{pm}), 
\]
(7)

where \(\phi_3\) is also convex, and \(\mu_T \geq 0\) tunes the amount of depth continuity. Proceeding as in the previous 1D case, a straightforward extension of (5c) that guarantees convexity of \(\mathcal{R}\) is:

\[
\mu_T \leq \frac{a}{2\phi_3'(\infty)} \quad \text{and} \quad \mu_\mathcal{R} \leq \frac{(1-a)}{2\phi_1'(\infty)} \quad \text{for} \quad a \in [0, 1]. 
\]
(8)

In practice, \(\phi_1\) and \(\phi_3\) are chosen quadratic around zero to avoid ringing artifacts, and linear at infinity, to restore spectral and/or depth discontinuities [8, 9]. Among this set of functions, we retain the hyperbolic potentials: \(\phi_{1,3}(\rho) = \sqrt{\rho^2 + p^2}\). Given these choices, \(\mathcal{R}\) is convex if \(\mu_\mathcal{R}\) and \(\mu_T \leq 1/4\), for a fair compromise between spectral regularity and depth continuity \((a = 1/2)\). Finally, the whole set of hyperparameters is \(\theta = (\lambda, \mu_\mathcal{R}, \tau_1, \mu_T, \tau_3, \varepsilon)\).
3. TARGET IDENTIFICATION IN ATMOSPHERIC CLUTTER

3.1. Depth-frequency mixed model

A mixed spectrum consists of both frequency peaks and smooth spectral components. Mixed spectrum estimation has been addressed in [7]. Here, we propose a depth-wise extension for the restoration of a series of such spectra. Following [7], each vector $X_m$ is split into two sets of unknown variables: the frequency peaks $X_m^f$ and the smoother components $X_m^s$. Then, the observation model reads $y_m = W_{x_f}(X_m^f + X_m^s)$ in range bin $m$. Given the whole set of range bins, adequation of the model to the data is measured by

$$Q(X) = \|Y - W(X^f + X^s)\|^2 = \|Y - W\hat{X}\|^2,$$

where $X^f = [X_1^f, \ldots, X_M^f]$, $X^s = [X_1^s, \ldots, X_M^s]$ and $\hat{X} = [X^f | X^s]$ is a $P \times 2M$ complex matrix. In this form, the problem is still-posed. The construction of the regularization term is now detailed.

3.2. Estimate

Minimizing a penalized criterion defined from (7) does not allow to retrieve spectral peaks embedded in a broadband background [7]. According to [7], mixed spectra restoration is achieved by means of a compound penalized criterion

$$J_p(X) = J_D(X) + \lambda_1 R_1(X^f) + \lambda_2 R_2(X^s), \quad (\lambda_1, \lambda_2) > 0 \quad (9)$$

where $R_1$ is designed to enhance spectral peaks and $R_2$ takes the form of (7). Akin to [4, 7], we introduce a separable circular convex energy $R_1$ to reconstruct a line spectrum $X_m^f$ in all range bins $m$. In the context of Doppler radar imaging, such a penalization becomes

$$R_1(X^f) = \sum_{m=1}^{N} \sum_{p=0}^{P-1} \phi_0(|\rho_p^m|),$$

where $\rho_p^m = |X_p^m|$, and $\phi_0$ is hyperbolic as well as $\phi_1$. Since $\phi_0$ is convex and increasing on $R_1$, $R_2$ is convex [7, Proposition 1]. Given that $R_2$ is also convex and $C^1$, the global criterion $J_p$ inherits the same properties. Its global minimizer is defined by

$$\hat{X} = [\hat{X}_f | \hat{X}_s] = \arg\min_{X} J_p(X).$$

In the Bayesian framework adopted in [4], $(\hat{X}_f, \hat{X}_s)$ corresponds to the joint MAP estimate. Finally, the Doppler image solution is the componentwise squared modulus of the superposition $\hat{X}_f + \hat{X}_s$. Note that eight parameters $\theta = (\lambda_{\pm}, \mu_{\pm}, \lambda_{\pm}, \mu_{\pm}, \epsilon)$ have to be tuned for the estimation of a series of mixed spectra.

4. COMPUTATIONAL ISSUE

We now discuss the minimization stage of $J$ and $J_p$. In [1], we have introduced powerful minimization block-coordinate descent methods for line and smooth spectra restoration. They rely upon two forms of half-quadratic (HQ) regularization: Geman & Reynolds’ construction and Geman & Yang’s one, respectively. The second one allows to derive convex and $C^1$ HQ criteria for Markovian penalty terms such as (6). For this reason, the second algorithm has been generalized for mixed spectra restoration in [2]. Convergence proofs have been stated for convex and $C^1$ criteria such as $J$ and $J_p$. As shown in [10], the 2D depth-frequency extensions for estimation of clutter and target-clutter radar images are straightforward for the following reasons. First, there is no correlation between adjacent range bins in the observation model. Second, the penalization terms involving the frequency and depth directions in (7) do not interact. Note that these techniques are quite similar to the quasi-Newton algorithm developed in [11] where no convergence proof is given.

5. EXPERIMENTAL RESULTS

5.1. Simulated example

The solution spectra have been computed on $P = 64$ frequency points. In practice, taking $P > 64$ does not markedly improve the resolution, while it increases the computational burden.

Fig. 2 shows a comparison between our solution and the spectra series yielded by the long AR regularized technique [5]. In our approach, the hyperparameter values have been empirically selected after several trials, as those that minimize the $L_1$ distance between true and estimated spectra. Consequently, the hyperparameters $\theta = (0.5, 0.4, 400, 6, 10^3)$ have been retained. Note that spectral smoothness is less enforced than depth continuity since $\mu_s < \mu_1$: this allows to account for the presence of the narrow band ground clutter. Moreover, convexity of the corresponding penalization $R$ is not ensured since conditions (8) are not fulfilled. In practice, the restoration of mixed clutters as those in Fig. 2 requires nonconvex energies since $\mu_s = 4, \mu_1 = 6 > 1/4$.

In the AR technique, there are only two hyperparameters, $\beta_1 = \lambda_{\mu_s}$ and $\beta_0 = \lambda_{\mu_1}$ that have been set using the same empirical rule$^1$, namely $\beta_0 = (0.12, 250)$. A qualitative comparison with Fig. 1 leads to four conclusions.

- The effect of regularization is obvious. Spectra estimated with the AR method or our technique are closer to the true ones compared to the periodograms.
- The ground clutter is estimated with a high resolution by both regularized methods.
- The sudden transitions at the beginning and the end of the ground clutter are over-smoothed by the AR method whereas they are preserved by our approach. In addition, the rain clutter and sea echoes are enhanced by our approach.
- The computation of our depth-frequency image requires three times more parameters than using the AR method.

Finally, remark that heterogeneous clutters are better separated with a nonquadratic energy.

A quantitative comparison has been achieved by evaluating $L_1$ distances between true and estimated spectra. The results show an improvement of about 25% from periodograms to the AR method, and 20% from the AR solution to the proposed one.

5.2. Real data of Doppler radar imaging

The proposed technique for estimation of a series of mixed spectra is tested on real data$^2$. The recording ($M = 48$, $N = 8$)
was composed of a meteorological clutter and an isolated target. Fig. 3(a) shows that the periodogram was not able to detect the target. The result of the clutter characterization technique is depicted in Fig. 3(b). The depth-frequency shape of the clutter is accurately restored but the target is lost as well. The clutter estimate has been computed using \( \hat{r} \) and \( \hat{\gamma} \). The target clearly appears in range bin \( m = 29 \), through spots at \( f_1 = -29 \) and \( f_2 = 25 \). This depth-frequency image has been computed with the same set \((\lambda_1, \lambda_2, \kappa)\). Two additional hyperparameters \((\lambda_3, \lambda_4, \epsilon)\) appear in (9). It is \textit{a priori} useful that choose values of \( \lambda_3 \) and \( \lambda_4 \) to have the same order of magnitude, otherwise the over-penalized term would yield a vanishing Doppler image.

### 6. CONCLUSION

This communication addresses depth-frequency spectral estimation in the context of Doppler radar imaging within the regularization framework. Two different problems were tackled. The first was the reconstruction of atmospheric clutters that present rather smooth shapes. The second one was an extension to the identification of spiky objects embedded in such clutters. We proposed solutions to both by the definition of an appropriate regularized criteria and demonstrated the validity of the approach on real data.

### 7. REFERENCES


