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Robust balancing of transfer lines with blocks of uncertain parallel tasks under fixed cycle time and space restrictions

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ABSTRACT

This paper deals with an optimization problem, which arises when a new transfer line has to be designed subject to a limited number of available machines, cycle time constraint, and precedence relations between necessary production tasks. The studied problem consists in assigning a given set of tasks to blocks and then blocks to machines so as to find the most robust line configuration under task processing time uncertainty. The robustness of a given line configuration is measured via its *stability radius*, *i.e.*, as the maximal amplitude of deviations from the nominal value of the processing time of uncertain tasks that do not violate the solution admissibility. In this work, for considering different hypotheses on uncertainty, the stability radius is based upon the Manhattan and Chebyshev norms. For each norm, the problem is proven to be strongly NP-hard and a mixed-integer linear program (MILP) is proposed for addressing it. To accelerate the seeking of optimal solutions, two variants of a heuristic method as well as several reduction rules are devised for the corresponding MILP. Computational results are reported on a collection of instances derived from classic benchmark data used in the literature for the Transfer Line Balancing Problem.

1. Introduction

Transfer lines are traditional manufacturing systems used in the production of parts for the automotive, aerospace, naval and other industries (ElMaraghy, ElMaraghy, Tomiyama, & Monostori 2012, Hitomi 1996). They require a heavy investment due to high installation costs, but remain profitable thanks to a long life and a large production volume. This latter permits to obtain a reduced price per part. Such manufacturing systems consist of a set of machines connected by a conveyor belt. A part, placed on a conveyor, is passed down the line, visiting all the machines in order of their location. Machines execute their tasks by multi-spindle heads (hereafter referred to as blocks), each of which is a combination of tools that are activated in parallel and perform the assigned tasks simultaneously. The order of blocks is fixed for each machine. When the execution of the last block of tasks in a machine is finished on the part, the machine passes this part to the next machine, receives a new part from the previous machine, and repeats its tasks starting from the first block. After leaving the final machine, the part is considered to be complete.

We are interested in a problem of designing the mentioned above transfer lines. The design problem includes several steps. The first two – product design and process selection – determine the information about the work that should be done in the transfer line being designed. In other words, they determine a set of tasks, cycle time and precedence constraints. Thus, for example, the precedence order (i, j) of two tasks i and j means that the task j cannot be allocated to a block which precedes the block with the task i. The next design step is the line balancing, which is a complex combinatorial optimization problem. Subject to all given constraints, it consists in determining a partition of the set of all production tasks into blocks and then these latter into machines such as to optimize a desired production goal (Dolgui, Finel, Vernadat, Guschinsky, & Levin 2005). Among others, this defines a fixed order of activation of blocks into machines.

Besides the precedence constraints mentioned above, there exist some supplementary restrictions which should also be taken into account at the balancing stage. It is assumed that the task processing time does not depend on which machine the corresponding task is performed, or which task precedes or follows it. The time required to perform all the tasks in the block (also called

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block working time) is equal to the longest task processing time of this block. The time necessary to execute all the tasks of the machine (hereafter the machine load) is the sum of the working time of the blocks belonging to this machine. The load of any machine is limited by the cycle time, *i.e.*, the duration that separates the release of two complete parts. By definition, the cycle time is larger than or equal to the load of any machine. The number of machines is considered to be limited, because of cost reasons or space restrictions.

During the life cycle of the transfer line, some product characteristics or materials can be slightly varied in conformity with business needs. Any variation has an effect on the task processing time, which in its turn may affect the productivity of the entire system. We consider two types of tasks: *certain* ones whose processing time does not depend on any event and *uncertain* tasks with a variable processing time. Such uncertainty has to be taken into account on the design stage in order to provide the necessary robustness for the studied production line.

In the next sections, we will present a robust solution approach for the transfer line balancing under task processing time uncertainty. Section 2 gives a review of some similar problems and popular solution methods. A presentation of basic problem notations, definitions and properties as well as the concept of the proposed robust approach are explained in Section 3. Section 4 is dedicated to algorithmic aspects of computing the considered robustness measures for two norms. Some words about the problem complexity as well as the MILP model for each studied norm are presented in Section 5. Pre-processing techniques for the MILP models are described in Section 6. Section 7 presents a series of numerical experiments and discusses the obtained computational results. At the end of the article, we provide a conclusion and present some perspectives for future research.

2. Literature review

In this section, we discuss some approaches, used to handle the task processing time uncertainty for line balancing problems.

The production line is supposed to continuously work without unnecessary interruptions in order to achieve given manufacturing goals. For this purpose, all machines have to stand with the cycle time, which can be challenging with task processing time deviations that may increase the load of machines. Uncertainty is inevitable, but mathematical models can help in identifying solutions that may confront uncertainty by limiting its impact on the line performance as much as possible. In the literature, three major approaches are considered to model the uncertainty: stochastic, fuzzy and robust ones.

Stochastic approach is the most popular one. It consists in a representation of the uncertain task processing time as an independent random variable with a known distribution function. For its application, a special technique for tasks allocation and controlling the cycle time constraint is needed. For example, the chance-constrained one seeks a line balance such that any machine load respects the cycle time with a given probability, also known as confidence level. This technique was applied for straight assembly lines (Chakravarty & Shtub, 1986; Erel, Sabuncuoglu, & Sekerci, 2005; Sarin, Erel, & Dar-El, 1999; Shin, 1990), U-shaped (Agrawal & Tiwari, 2008; Ağpak & Gökçen, 2007; Baykasoğlu & Özbakır, 2007; Guerriero & Miltenburg, 2003; Özcan, Kellegöz, & Toklu, 2011) and two-sided production lines (Chiang, Urban, & Luo, 2016; Özcan, 2010; 2018). This technique usually deals with MILP problem formulations that integrates non-linear stochastic cycle time constraints. As a consequence, it is necessary to apply linearization approaches and introduce new supplementary variables.

As concerns the fuzzy approach, the uncertain task processing time is supposed to belong to a fuzzy set, determined by a given membership function whose image is equal to the interval [0, 1]. In order to interact with this approach, specific arithmetic operations are needed to be introduced. Fuzzy modeling for uncertain task processing times has been applied by Tsujimura, Gen, and Kubota (1995), Hop (2006) and Zacharia and Nearchou (2013) for balancing problems.

The mentioned above techniques require some historic information in order to construct appropriate probability distribution or membership functions. However, such information is not always available at the design stage due to, for example, an originality of the considered line. Since stochastic and fuzzy approaches are hardly applicable, the robust one may be used in that case. This latter approach assumes that only a discrete set of scenarios and/or closed intervals of task processing time realizations are available without any knowledge on their distribution. An optimal solution for the robust approach is an admissible one that has the best performance for its worst case scenario (see Dolgui and Kovalev, 2012; Gurevsky, Hazır, Battaïa, and Dolgui, 2013b; Hazır and Dolgui, 2013).

Some other robust approaches are based on specific indicators. One of the most studied ones called *stability radius*. Introduced in the work of Sotskov, Dolgui, and Portmann (2006) for SALBP-1 problem, it was calculated as the maximal amplitude of deviations from the nominal value of the processing time of uncertain tasks that do not violate the solution optimality. Then, Gurevsky, Battaïa, and Dolgui (2012) showed how to compute the stability radius for the simple assembly line balancing problem maximizing the line efficiency (SALBP-E problem) and for the transfer line balancing problem in Gurevsky, Battaïa, and Dolgui (2013a). In further works, Sotskov, Dolgui, Lai, and Zatsiupa (2015) and Lai, Sotskov, and Dolgui (2019); Lai, Sotskov, Dolgui, and Zatsiupa (2016) proposed some enumeration methods for comparing feasible and optimal line balances by their stability radii and found the condition when the optimal line balance is unstable.

Computing stability radius with respect to the optimality of the studied solution is a difficult NP-hard problem. Thus, in Rossi, Gurevsky, Battaïa, and Dolgui (2016), where the simple assembly line balancing problem under uncertainty was studied, the authors have decided to relax the initial concept of stability radius and use it for controlling the solution admissibility. Moreover, instead of computing this indicator for a given line configuration, they introduced a new optimization problem aiming at seeking a line configuration having the greatest stability radius. To handle this new problem, they proposed a MILP model for each studied norm of stability radius and elaborated some tight combinatorial upper bounds.

The present paper continues to popularize the ideas of Rossi et al. (2016) by applying them to the robust balancing of transfer lines.

3. Basic notations and definitions

It is considered that there exists a non-empty set of *uncertain tasks* $\widetilde{V}$ ($\widetilde{V} \subseteq V$) whose processing time may deviate from its nominal value with regard to time without any additional information. Each other task from $V \setminus \widetilde{V}$ is named *certain*. Detailed basic notations are given in Table 1.

To evaluate the robustness of a feasible solution, we use the concept of *stability radius* whose formal definition requires some supplementary notations. Thus, the stability radius of a feasible solution $s \in F(t)$ can be defined as follows (see Sotskov et al., 2006):

$$\rho(s,t) = \max\{\epsilon \geq 0 \mid \forall \xi \in B(\epsilon) \quad (s \in F(t+\xi))\}, \tag{1}$$

where $B(\epsilon) = \{\xi \in \Xi \mid \|\xi\| \leq \epsilon\}$.

In other words, $\rho(s, t)$ is determined as the value of the radius of the greatest closed ball $B(\cdot)$, called *stability ball*, representing the

Table 1
Basic notations.

V	is the set of all necessary tasks, *i.e.*, $\{1, 2, \ldots, n\}$;
$\widetilde{V}$	is the set of uncertain tasks, where $\widetilde{V} \subseteq V$;
W	is the set of all available machines, *i.e.*, $\{1, 2, \ldots, m\}$;
G	is a directed acyclic graph (V, A) representing the precedence constraints, where A is the set of arcs;
t_j	is a non-negative nominal processing time of the task j;
t	is a vector expressing the nominal processing task times, *i.e.*, $(t_1, t_2, \ldots, t_n)$;
$F(t)$	is the set of all feasible solutions with respect to a given vector t;
Ξ	is the set of vectors, where each of which presents possible non-negative processing time deviations for the uncertain tasks, *i.e.*, $\{\xi \in \mathbb{R}^n_+ \mid \xi_j = 0,\ j \in V \setminus \widetilde{V}\}$;
T	is the cycle time;
$r_{\max}$	is the maximal number of tasks, which can be assigned to one block;
$b_{\max}$	is the maximal number of blocks, which can be allocated into one machine;
U	is the set of all available blocks, *i.e.*, $\{1, 2, \ldots, mb_{\max}\}$;
$U(p)$	is the set of blocks of the machine p, *i.e.*, $\{(p-1) \cdot b_{\max} + 1, \ldots, p \cdot b_{\max}\}$;
V_k	is the set of all tasks assigned to the block k;
$\widetilde{V}_k$	is the set of all uncertain tasks assigned to the block k, *i.e.*, $V_k \cap \widetilde{V}$;
τ_k	is the nominal working time of the block k, *i.e.*, $\max_{j \in V_k} t_j$;
$\widetilde{U}(p)$	is the set of uncertain blocks of the machine p, where each one has at least one uncertain assigned task;
$\widetilde{W}$	is the set of uncertain machines, where each one has at least one uncertain block.

deviations[1] of the nominal processing time of uncertain tasks, for which s remains feasible. Any element ξ of $B(\cdot)$ is evaluated based on a given norm $\|\cdot\|$ defining the distance between vectors t and $t + \xi$ (or the amplitude of deviations from t).

In this paper, two norms ℓ_1 ($\|\cdot\|_1$) and ℓ_∞ ($\|\cdot\|_\infty$) are studied in details, where by definition $\|\xi\|_1 = \sum_{j \in \widetilde{V}} \xi_j$ and $\|\xi\|_\infty = \max_{j \in \widetilde{V}} \xi_j$. As a consequence, the notations ρ_1, $B_1(\cdot)$ and ρ_∞, $B_\infty(\cdot)$ will be used for ℓ_1 and ℓ_∞, respectively.

For a given solution, the stability radius in the ℓ_1-norm is the total amount of processing time increase that this solution can stand while keeping being feasible, regardless of the way this amount of extra processing time is dispatched among uncertain tasks. However, its numerical value is often limited just because it covers situations that may be considered as improbable, as when all the processing time increase is focused on a single uncertain task. This situation can be viewed as unrealistic, in particular when the stability radius value is many times larger than the minimum processing time among uncertain tasks. It can be considered unlikely that the increased processing time of a short uncertain task may reach several times its nominal value. In some contexts, however, such a situation may not be regarded as completely unrealistic. A drilling task, for example, is not expected to last more than a few seconds. However, if the drill bit breaks during the operation, opening the drill chuck, installing a new bit and resuming the task may delay its expected duration by a very large amount of time.

The stability radius in the ℓ_∞-norm corresponds to a different hypothesis on how deviations can affect the processing time of uncertain tasks. It measures the maximum processing time increase that each uncertain task can support (independently of other uncertain tasks) without compromising solution feasibility. It is appropriate when one assumes that all uncertain tasks are subject to the same risk of processing time increase.

[1] Since any decrease of task processing time cannot compromise the solution feasibility, it is sufficient to consider only non-negative task time deviations in this work, *i.e.*, $\xi_j \in \mathbb{R}_+$ for any $j \in \widetilde{V}$.

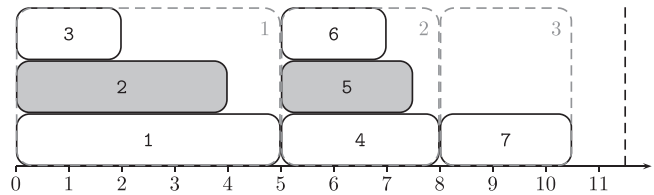

Fig. 1. Example of an assignment of 7 tasks to 3 blocks.

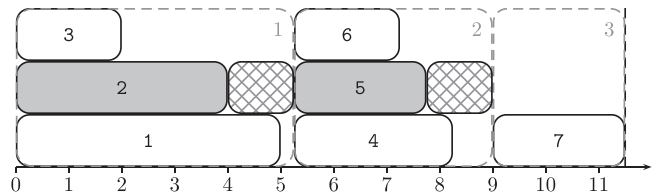

Fig. 2. The processing time of the tasks 2 and 5 is increased by 1.25 simultaneously.

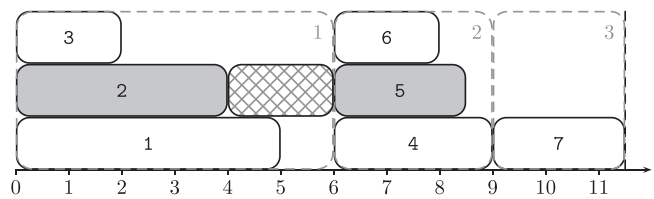

Fig. 3. The processing time of the task 2 is increased by 2.

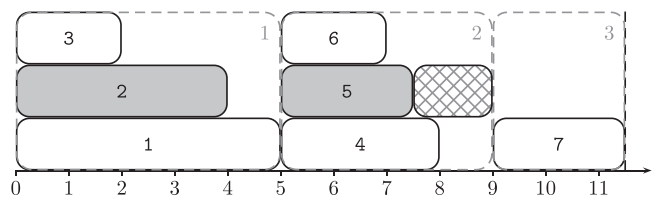

Fig. 4. The processing time of the task 5 is increased by 1.5.

Let us see an illustrative example of the interpretation of the stability radius in the ℓ_∞- and ℓ_1-norms. The following problem instance is considered: $n = 7$, $m = 1$, $\widetilde{V} = \{2, 5\}$, $t = (5, 4, 2, 3, 2.5, 2, 2.5)$, $T = 11.5$, $r_{\max} = 3$. There is no precedence constraint. The feasible assignment is shown in Fig. 1: the total load of the machine is less than the cycle time. Working times of block are $\tau_1 = 5$, $\tau_2 = 3$ and $\tau_3 = 2.5$ by definition. Based on Fig. 2, it is easy to see that the greatest increase (dashed) of the processing time, which can be supported simultaneously by any uncertain task is equal to 1.25. This value defines the stability radius in the ℓ_∞-norm. Regarding Figs. 3 and 4, we can notice that the greatest increase of the processing time, which can be supported individually by any uncertain task can not be greater than 1.5 time units. This latter value determines the stability radius in the ℓ_1-norm.

Before we continue to consider the stability radius let us introduce an important parameter called *save time*. It is determined only for uncertain blocks and calculated as the difference between the block working time and the processing time of its longest uncertain task, *i.e.*, $\tau_k - \max_{j \in \widetilde{V}_k} t_j$, $k \in \widetilde{U}(p)$, $p \in W$. Consequently, the save time can be positive or equal to zero, see Figs. 5 and 6, respectively. Hereafter, the save time of an uncertain block k of the machine p is denoted as $\Delta_k^{(p)}$ and the *minimum save time* of the machine $p \in \widetilde{W}$ as $\Delta_{\min}^{(p)} = \min_{k \in \widetilde{U}(p)} \Delta_k^{(p)}$.

4. Computing stability radii

In this section, we present how to calculate the stability radius for a given feasible solution in two norms, ℓ_1 and ℓ_∞. Aspects of computational complexity are discussed as well. The proofs

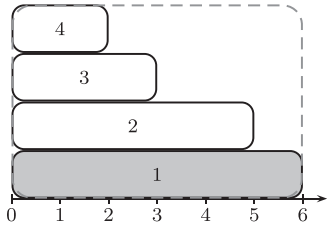

Fig. 5. Block having save time = 0.

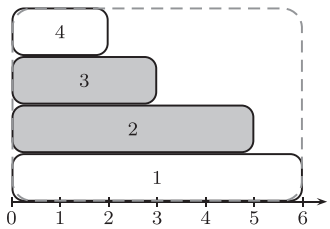

Fig. 6. Block having save time = 1.

of Theorems as well as an illustrative example are provided in Appendix A.

4.1. Stability radius in the ℓ_1-norm

Theorem 1. *The stability radius ρ_1 for a given feasible solution is calculated as follows*

$$\rho_1 = \min_{p\in\widetilde{W}} \left\{ T - \sum_{k\in U(p)} \tau_k + \Delta^{(p)}_{\min} \right\}. \tag{2}$$

It is not difficult to see that computing ρ_1 can be done in $O(n)$ time by a sequential analysis of all the blocks of each machine. In other words, each task is examined only once.

In the case of $r_{\max} = 1$, the configuration of the transfer line is transformed into that of the simple assembly line, since at most one task can be allocated to block. Hence, the save time of all uncertain blocks becomes equal to zero, *i.e.*, $\Delta^{(p)}_k = 0,\ k \in \widetilde{U}(p)$, $p \in \widetilde{W}$. As a consequence, Theorem 1 directly yields

Corollary 1 (Rossi et al. (2016))**.** *The stability radius ρ_1 for a given configuration of simple assembly line is calculated as* $\min_{p\in\widetilde{W}} \left\{T - \sum_{k\in U(p)} \tau_k\right\}$.

4.2. Stability radius in the ℓ_∞-norm

For any uncertain machine p, let us introduce the following function, which is useful for the feather statements:

$$\theta(p,q) = \frac{T - \sum_{k\in U(p)} \tau_k + \sum_{i=1}^{q} \Delta^{(p)}_{\pi_i}}{q},$$

where $\Delta^{(p)}_{\pi_1} \le \Delta^{(p)}_{\pi_2} \le \ldots \le \Delta^{(p)}_{\pi_{|\widetilde{U}(p)|}}$ is the non-decreasing order of the save time for all uncertain blocks of the machine $p \in \widetilde{W}$ and $q \in \{1,\ldots,|\widetilde{U}(p)|\}$.

Theorem 2. *The stability radius ρ_∞ for a given feasible solution is calculated as follows*

$$\rho_\infty = \min_{p\in\widetilde{W}} \min_{q=1,\ldots,|\widetilde{U}(p)|} \theta(p,q). \tag{3}$$

It is not difficult to see that computing ρ_∞ can be done in $O(n + mb_{\max}\log b_{\max})$ time by a sequential analysis of all the blocks and ordering then the save times of each uncertain machine.

Based on the same arguments, developed above for $r_{\max} = 1$ and the ℓ_1-norm case, Theorem 2 implies.

Corollary 2 (Sotskov et al. (2006))**.** *The stability radius ρ_∞ for a given configuration of simple assembly line is calculated as* $\min_{p\in\widetilde{W}} \frac{T-\sum_{k\in U(p)}\tau_k}{|\widetilde{U}(p)|}$.

5. Problem formulations

We note the studied problem as P and distinguish its two versions P_1 and P_∞ respectively for the ℓ_1- and ℓ_∞-norms. The problem P is strongly NP-hard. Indeed, it is sufficient to consider an instance of P, where the maximal number of tasks per block is limited by 1, *i.e.*, $r_{\max} = 1$. Then, P reduces to the same problem as the one investigated in Rossi et al. (2016), which was proven to be NP-hard in the strong sense.

Below, we present two MILP formulations: one for P_1 and another one for P_∞. For both formulations the value of $b_{\max}$ is fixed to $\max\left\{k \mid \sum_{i=1}^{k} t_{\pi_i} \le T\right\}$, where $(\pi_1, \pi_2, \ldots, \pi_n)$ is a permutation of V with respect to the non-decreasing order of their processing times. It is easy to see, that the latter expression can be computed in $O(n\log n)$ time.

5.1. MILP formulation for P_1

P_1 is formulated as a mixed integer linear program on the following decision variables: ρ_1 is the stability radius value to maximize; x_{jk} is a binary variable that is set to one if and only if the task j is allocated to the block k; y_k is equal to 1 if the block k is not empty and 0, otherwise; $\tau_k \ge 0$ determines the working time of the block k; $\Delta^{(p)}_{\min} \ge 0$ represents the minimum value of the save time among all the blocks allocated to the machine p; a_p is a non-negative variable that is positive if the machine p processes at least one assigned uncertain task; z_k is set to 1 if an uncertain task is allocated to the block k and 0, otherwise. The central idea of the MILP formulation for P_1 consists in maximizing ρ_1, the minimum idle time over all the machines that process uncertain tasks (see Theorem 1).

Maximize ρ_1

$$\sum_{k\in U} x_{jk} = 1 \qquad \forall j \in V \tag{4}$$

$$\sum_{j\in V} x_{jk} \le r_{\max} \quad \forall k \in U \tag{5}$$

$$x_{jk} \le y_k \quad \forall k \in U,\ \forall j \in V \tag{6}$$

$$y_k \le \sum_{j\in V} x_{jk} \quad \forall k \in U \tag{7}$$

$$y_{k+1} \le y_k \quad \forall p \in W,\ \forall k \in U(p) \setminus \{pb_{\max}\} \tag{8}$$

$$t_j \cdot x_{jk} \le \tau_k \quad \forall k \in U,\ \forall j \in V \tag{9}$$

$$\Delta_{\min}^{(p)} \le T \cdot (1 - z_k) + \tau_k - t_j \cdot x_{jk} \quad \forall p \in W,\ \forall k \in U(p),\ \forall j \in \widetilde{V} \tag{10}$$

$$x_{jk} \le z_k \quad \forall k \in U,\ \forall j \in \widetilde{V} \tag{11}$$

$$\sum_{k \in U(p)} \tau_k \le T \quad \forall p \in W \tag{12}$$

$$\sum_{q=k}^{|U|} x_{iq} \le \sum_{q=k+1}^{|U|} x_{jq} \quad \forall (i,j) \in A,\ \forall k \in U \tag{13}$$

$$x_{jk} \le a_p \quad \forall p \in W,\ \forall k \in U(p),\ \forall j \in \widetilde{V} \tag{14}$$

$$\rho_1 \le T \cdot (2 - a_p) - \sum_{k \in U(p)} \tau_k + \Delta_{\min}^{(p)} \quad \forall p \in W \tag{15}$$

$$\rho_1 \ge 0$$

$$\Delta_{\min}^{(p)} \ge 0,\ a_p \ge 0 \quad \forall p \in W$$

$$x_{jk} \in \{0,1\} \quad \forall j \in V,\ \forall k \in U$$

$$\tau_k \ge 0,\ z_k \in \{0,1\},\ y_k \in \{0,1\} \quad \forall k \in U$$

Constraints (4) ensure that each task is allocated to exactly one block. Inequalities (5) show that each block contains at most $r_{\max}$ tasks. Any block having at least one assigned task is considered as non-empty, otherwise empty, as enforced by (6) and (7). Constraints (8) ensures that block $k+1$ has to be empty if block k is empty. Indeed, for the sake of breaking symmetry, these constraints enforce that whenever a block is empty, all the subsequent blocks of the considered machine should be closed too. Forbidding empty blocks in between non-empty ones reduces the number of equivalent solutions to the problem, and has a positive impact on solution time. The working time of block is not less than the processing time of any task allocated to it, as provided by (9). Constraints (10) express the definition of $\Delta_{\min}^{(p)}$. Inequalities (11) ensure that the block k is uncertain if there is at least one uncertain task assigned to it, and certain otherwise. As for constraints (12), they state that the load of any machine does not exceed the cycle time. The precedence constraints are expressed by inequalities (13). Constraints (14) – (15) describe the result obtained in Theorem 1. Indeed, it is easy to see that (14) and (15) implies that $a_p \in \{0, 1\}$. As a consequence, if the machine p has no uncertain task, then $a_p = 0$ and (12) together with (15) yields $\rho_1 \le 2T$, which is always valid. Otherwise, when $a_p = 1$, (15) is a corollary of (2).

5.2. MILP formulation for P_∞

P_∞ is formulated as a mixed integer linear program on the following decision variables: ρ_∞ is the stability radius value to maximize; x_{jk} is a binary variable that is set to one if and only if the task j is allocated to the block k; y_k is equal to 1 if the block k is not empty and 0, otherwise; ξ_{jk} is a processing time deviation of the task j on the block k; $\tau_k \ge 0$ determines the working time of the block k. The main idea of the MILP formulation for P_∞ is that the processing time of all uncertain tasks can be increased by ρ_∞ without compromising the solution feasibility.

Maximize ρ_∞

$$\sum_{k \in U} x_{jk} = 1 \quad \forall j \in V \tag{16}$$

$$\sum_{j \in V} x_{jk} \le r_{\max} \quad \forall k \in U \tag{17}$$

$$x_{jk} \le y_k \quad \forall k \in U,\ \forall j \in V \tag{18}$$

$$y_k \le \sum_{j \in V} x_{jk} \quad \forall k \in U \tag{19}$$

$$y_{k+1} \le y_k \quad \forall p \in W,\ \forall k \in U(p) \setminus \{pb_{\max}\} \tag{20}$$

$$\xi_{jk} \le T \cdot x_{jk} \quad \forall k \in U,\ \forall j \in \widetilde{V} \tag{21}$$

$$t_j \cdot x_{jk} + \xi_{jk} \le \tau_k \quad \forall k \in U,\ \forall j \in \widetilde{V} \tag{22}$$

$$t_j \cdot x_{jk} \le \tau_k \quad \forall k \in U,\ \forall j \in V \setminus \widetilde{V} \tag{23}$$

$$\rho_\infty = \sum_{k \in U} \xi_{jk} \quad \forall j \in \widetilde{V} \tag{24}$$

$$\sum_{k \in U(p)} \tau_k \le T \quad \forall p \in W \tag{25}$$

$$\sum_{q=k}^{|U|} x_{iq} \le \sum_{q=k+1}^{|U|} x_{jq} \quad \forall (i,j) \in A,\ \forall k \in U \tag{26}$$

$$\rho_\infty \ge 0$$

$$\xi_{jk} \ge 0,\ x_{jk} \in \{0,1\} \quad \forall j \in V,\ \forall k \in U$$

$$y_k \in \{0,1\} \quad \forall k \in U$$

Constrains (16)–(20), (25) and (26) are same as constraints (4)–(8), (12) and (13) from the MILP formulation of P_1. Inequalities (21) specify that any processing time deviation of any uncertain task cannot be greater than the cycle time. The working time of block is not less than the processing time of any certain task or the deviated processing time of any uncertain task allocated to it, as provided by (22) and (23). Constraints (16) and (21) imply that for a given uncertain task j only one variable from ξ_{jk}, $k \in U$, has a non-zero value. This together with (24) implies that the processing time deviation for any uncertain task is set to ρ_∞.

6. Pre-processing

In this section, we provide some useful techniques for reducing the search space for both MILP formulations, P_1 and P_∞. Thus, Section 6.1 is dedicated to present some methods aiming to reduce the so-called assignment interval for each task and to find unused (or a priori empty) blocks. Then, in Section 6.2, some heuristics are proposed for generating feasible solutions as starting points for MILP formulations. Finally, a global pre-processing approach is given in Section 6.3.

6.1. Assignment intervals and empty blocks

Below, we develop an important algorithm, which returns a lower bound on the number of blocks necessary for assigning a given set of tasks. This can be done not only in a trivial manner as a ratio between the cardinality of this set and $r_{\max}$, but also with respect to the cycle time. The idea is at first to form the blocks of tasks from this set so as the sum of the working time of the obtained blocks is as small as possible and then to find a lower bound on the number of machines necessary to allocate all these blocks. Thus, the seeking number of blocks is equal to the index of the first block of the last needful machine. This algorithm is formally presented below, where D is considered as a given set of tasks.

Algorithm $\alpha(D)$.

1. Let $(\sigma_1, \sigma_2, \ldots, \sigma_{|D|})$ be a permutation of D with respect to the non-increasing order of their processing times.
2. Form $\omega = \left\lceil \frac{|D|}{r_{\max}} \right\rceil$ blocks $B_1, B_2, \ldots, B_\omega$, where each of which has exactly $r_{\max}$ tasks except possibly the last one, such that the task σ_j is in the block $B_{\left\lceil \frac{j}{r_{\max}} \right\rceil}$, $j \in D$. This permits, inter alia, to determine the working time of each obtained block: $\tau_k = \max_{i \in B_k} t_i$, $k = 1, 2, \ldots, \omega$.
3. The lower bound on the number of machines able to allocate all the obtained blocks is equal to $\lambda = \frac{1}{T} \cdot \sum_{k=1}^{\omega} \tau_k$. From this, the returned lower bound on the number of blocks equals $b_{\max} \cdot \lceil \lambda - 1 \rceil + 1$.

Because of Step 1, Algorithm $\alpha(D)$ can be implemented to run in $O(|D| \log |D|)$ time.

While seeking a feasible assignment of all given tasks to blocks and blocks to machines, precedence constraints have to be satisfied. For example, for each task $j \in V$, the set of all its predecessors, denoted by $P(j)$, has to be assigned before j. Similarly, the set of all successors of j, denoted by $S(j)$, has to be assigned after j. Thus, for any task j, we introduce the so-called assignment interval $Q_j = [l_j, u_j]$, where l_j (resp. u_j) is the lowest (resp. uppermost) index of the block able to accommodate the task j. For the efficient solving of problems P_1 and P_∞, these intervals should be as tight as possible. In order to reach this goal, we provide below five different pre-processing rules for reducing Q_j.

Reduction rules.

- Rule 1 is based on the knowledge of the quantity of all predecessors and all successors for a given task in the precedence graph: $l_j^{(1)} = \left\lceil \frac{|P(j)|}{r_{\max}} \right\rceil + 1$ and $u_j^{(1)} = mb_{\max} + 1 - \left(\left\lceil \frac{|S(j)|}{r_{\max}} \right\rceil + 1 \right)$ for any $j \in V$.
- Rule 2 consists in applying Algorithm $\alpha(\cdot)$ on the sets of predecessors and successors of each task: $l_j^{(2)} = \alpha(P(j)) + 1$ and $u_j^{(2)} = mb_{\max} + 1 - (\alpha(S(j)) + 1)$ for any $j \in V$. Here, $\alpha(\cdot)$ is the result of application of Algorithm $\alpha(\cdot)$ on a given set of tasks.
- Rule 3 is similar to Rule 2, but the input set of tasks for Algorithm $\alpha(\cdot)$ includes also the task j itself: $l_j^{(3)} = \alpha(P(j) \cup \{j\})$ and $u_j^{(3)} = mb_{\max} + 1 - \alpha(S(j) \cup \{j\})$.
- Rule 4 analyses the assignment intervals of all direct predecessors and successors for all tasks as follows: $l_j^{(4)} = \max_{(i,j) \in A} l_i + 1$ and $u_j^{(4)} = \min_{(j,i) \in A} u_i - 1$ for any $j \in V$.
- Rule 5 takes also (with respect to Rule 4) into account the cardinality of the sets of direct predecessors $P^*(j)$ and direct successors $S^*(j)$ of j: $l_j^{(5)} = \min_{(i,j) \in A} l_i + \left\lceil \frac{|P^*(j)|}{r_{\max}} \right\rceil$ and $u_j^{(5)} = \max_{(j,i) \in A} u_i - \left\lceil \frac{|S^*(j)|}{r_{\max}} \right\rceil$.

The reduced assignment intervals serve at detecting a set of unused (or a priori empty) blocks, and then to take advantage of this piece of information in order to reduce even further these intervals. Algorithm $\beta(p)$ presents a procedure of seeking unused blocks for a given machine $p \in W$.

Algorithm $\beta(p)$.

1. Compute the set $V(p) = \{j \in V \mid Q_j \cap U(p) \neq \emptyset\}$ of assignable tasks to machine p.
2. Based on $V(p)$, determine the maximal number of blocks $b_{\max}^{(p)}$ for the machine p by solving $\max \left\{ k \;\middle|\; \sum_{i=1}^{k} t_{\pi_i} \leq T \right\}$, where $(\pi_1, \pi_2, \ldots, \pi_{|V(p)|})$ is a permutation of $V(p)$ with respect to the non-decreasing order of their processing times. Thus, the set $\widehat{U}(p) = \{(p-1)b_{\max} + 1 + b_{\max}^{(p)}, \ldots, pb_{\max}\}$ of unused (or a priori empty) blocks is introduced for the machine p.
3. For each task $j \in V(p)$, update its interval Q_j as follows. If $l_j \in \widehat{U}(p)$, then set $l_j = pb_{\max} + 1$. If $u_j \in \widehat{U}(p)$, then set $u_j = (p-1)b_{\max} + b_{\max}^{(p)}$.

Because of Step 2, the running time of Algorithm $\beta(\cdot)$ is not greater than $O(n \log n)$.

The whole process of finding unused blocks and reducing assignment intervals is described in Approach Reduction, where $[1, mb_{\max}]$ is the initial assignment interval for each task.

Approach Reduction.

1. For each task $j \in V$, compute $Q_j^{(1)}$, $Q_j^{(2)}$ and $Q_j^{(3)}$ by applying respectively Rules 1, 2 and 3. Update the intervals Q_j as follows : $l_j = \max\{l_j, l_j^{(1)}, l_j^{(2)}, l_j^{(3)}\}$ and $u_j = \min\{u_j, u_j^{(1)}, u_j^{(2)}, u_j^{(3)}\}$.
2. For each task $j \in V$, compute $Q_j^{(4)}$ and $Q_j^{(5)}$ by applying respectively Rules 4 and 5. Update the intervals Q_j as follows : $l_j = \max\{l_j, l_j^{(4)}, l_j^{(5)}\}$ and $u_j = \min\{u_j, u_j^{(4)}, u_j^{(5)}\}$.
3. For each machine $p \in W$, apply Algorithm $\beta(p)$ and update the intervals Q_j with respect to the constructed sets $\widehat{U}(p)$ of unused blocks.
4. If there exists a task $j \in V$ such that Q_j was reduced on Steps 2 or 3, then go to Step 2. Otherwise stop.

The running time of Approach Reduction does not exceed $O(n^2 mb_{\max} G_{\max})$, where $G_{\max} = \max\{\max_{j \in V} |P^*(j)|, \max_{j \in V} |S^*(j)|\}$.

6.2. Heuristic

Based on the ideas presented in Gurevsky et al. (2013a), in this sub-section, we describe a general heuristic methodology applicable for both problems P_1 and P_∞. Then, its multi-start version is provided as well.

The heuristic aims to construct a feasible solution having the stability radius (ρ_1 for P_1 or ρ_∞ for P_∞) greater than a given value ρ. Its main principle is to assign as many tasks as possible to the current block of the current machine. At the beginning, a partially constructed solution contains only one machine having one empty block. The heuristic assigns tasks to this block until no task can be added because of the existing constraints. Then, a new empty block (or a new machine with one empty block) is opened and becomes current. This continues until all tasks are assigned and a feasible solution is obtained or there is no possibility to construct a feasible solution.

The heuristic uses the so-called candidate list $\mathcal{CL}(k, p)$, which contains all tasks assignable to the current block k of the current machine p. This list is built in the following way: the set of

unassigned tasks is analyzed and task j is added to $\mathcal{CL}(k, p)$ if all following conditions are satisfied:

1. j has no predecessors or all predecessors of j are already assigned,
2. assigning of j does not violate the cycle time constraint,
3. assigning of j does not exceed the maximal number of tasks per block $r_{\max}$,
4. assigning of j assures that the partially constructed solution has the stability radius grater than ρ,

The formal description of the heuristic as well as the usage of the candidate list is presented below.

Heuristic $\mathcal{H}^1(\rho)$.
1. Open the first machine having one empty block, *i.e.*, set $k = 1$ and $p = 1$.
2. If there exists at least one unassigned task, then construct a candidate list $\mathcal{CL}(k, p)$ for the block k of the machine p. Otherwise, go to Step 6.
3. If $\mathcal{CL}(k, p) = \emptyset$ and $k < b_{\max}$, then open a new empty block on the current machine, *i.e.*, set $k = k + 1$, and go to Step 2. If $\mathcal{CL}(k, p) = \emptyset$, $k = b_{\max}$ and $p < m$, then open a new machine having one empty block, *i.e.*, set $p = p + 1$ and $k = 1$, and go to Step 2. If $\mathcal{CL}(k, p) = \emptyset$, $k = b_{\max}$ and $p = m$, then go to Step 5.
4. If $\mathcal{CL}(k, p)$ is not empty, then choose randomly one task from $\mathcal{CL}(k, p)$, assigned it to the current block k and go to Step 2.
5. Stop, a feasible solution is not found.
6. Stop, a feasible solution is found.

In order to diversify the search strategy, a second variant of the proposed above heuristic method, denoted as $\mathcal{H}^2(\rho)$, is also applied. Its unique difference from the first one consists in analyzing the precedence graph (and, as a consequence, composing the list $\mathcal{CL}(k, p)$) in the opposite manner, *i.e.*, $\mathcal{H}^2(\rho)$ constructs feasible solutions starting from the last machine and not from the first one.

A multi-start version for $\mathcal{H}^1(\rho)$ (resp. $\mathcal{H}^2(\rho)$) is naturally appeared. Each attempt of $\mathcal{H}^1(\rho)$ (resp. $\mathcal{H}^2(\rho)$) consists in trying to construct a new feasible solution, whose stability radius is greater than for the best-known one. The number of attempts without improvement is limited by `mslimit`. The formal description of such a version is presented below, where $\mathcal{M}^1(\rho)$ (resp. $\mathcal{M}^2(\rho)$) refers to a multi-start version of heuristic $\mathcal{H}^1(\rho)$ (resp. $\mathcal{H}^2(\rho)$).

Multi-start heuristic $\mathcal{M}^1(\rho)$ (resp. $\mathcal{M}^2(\rho)$).
1. Set $k = 0$.
2. Apply Heuristic $\mathcal{H}^1(\rho)$ (resp. $\mathcal{H}^2(\rho)$). If a new feasible solution with the stability radius $\rho^* > \rho$ is found, then memorize this solution and set $\rho = \rho^*$, $k = 0$ and repeat Step 2. Otherwise, set $k = k + 1$.
3. If $k <$ `mslimit`, then go to Step 2. Otherwise, stop and return the best found feasible solution.

6.3. Global pre-processing approach

Based on the heuristic strategies and reduction rules, this subsection provides a global approach of enhancements for both MILP formulations, P_1 and P_∞. One of the principal ideas of this approach can be found in the following statements.

Statement 1. Let (s^0, ρ^0) be a pair composed respectively of a feasible solution and its stability radius for an instance $\mathcal{I}$ of the problem P_1. Then s^0 is also a feasible solution for the instance $\mathcal{I}'$, which is obtained from $\mathcal{I}$ by increasing the processing time of exactly any one uncertain task by ρ^0. Moreover, all assignment intervals and unused blocks of $\mathcal{I}'$ are also valid for $\mathcal{I}$.

Statement 2. Let (s^0, ρ^0) be a pair composed respectively of a feasible solution and its stability radius for an instance $\mathcal{I}$ of the problem P_∞. Then s^0 is also a feasible solution for the instance $\mathcal{I}'$, which is obtained from $\mathcal{I}$ by increasing the processing time of all uncertain tasks by ρ^0. Moreover, all assignment intervals and unused blocks of $\mathcal{I}'$ are also valid for $\mathcal{I}$.

In other words, Statements 1 and 2 indicate that analyzing $\mathcal{I}'$ may provide the tighter assignment intervals for $\mathcal{I}$. This analysis requires taking into account $|\widetilde{V}|$ instances for P_1, and a single one for P_∞. The difference between two problems is shown below in the following global pre-processing approaches. Here, the multi-start heuristic $\mathcal{M}^1$ starts seeking admissible solutions using any strictly negative value (for example -1 as below) as a lower bound of the stability radius. The best stability radius found by $\mathcal{M}^1$ is then used as a starting point for $\mathcal{M}^2$.

Pre-processing for P_1.
1. Let (s, ρ) (resp. (s^*, ρ^*)) be respectively the best feasible solution and its stability radius value after applying $\mathcal{M}^1(-1)$ (resp. $\mathcal{M}^2(\rho)$).
2. For each uncertain task $j \in \widetilde{V}$, produce the following instructions:
   2.1. Set $t_j = t_j + \rho^*$.
   2.2. Apply Approach Reduction to construct the sets of unused blocks $\widehat{U}(p)$, $p \in W$, and reduce intervals Q_j, $j \in V$.
   2.3. Set $t_j = t_j - \rho^*$.
3. Consider s^* as a starting feasible solution for the corresponding MILP formulation and add the following optimality-based cuts:

$$x_{jk} = 0, \quad \forall j \in V, \ \forall k \notin Q_j, \tag{27}$$

$$y_k = 0, \quad \forall p \in W, \ \forall k \in \widehat{U}(p). \tag{28}$$

Pre-processing for P_∞.
1. Let (s, ρ) (resp. (s^*, ρ^*)) be respectively the best feasible solution and its stability radius value after applying $\mathcal{M}^1(-1)$ (resp. $\mathcal{M}^2(\rho)$).
2. For each uncertain task $j \in \widetilde{V}$, set $t_j = t_j + \rho^*$.
3. Apply Approach Reduction to construct the sets of unused blocks $\widehat{U}(p)$, $p \in W$, and reduce intervals Q_j, $j \in V$.
4. For each uncertain task $j \in \widetilde{V}$, set $t_j = t_j - \rho^*$.
5. Consider s^* as a starting feasible solution for the corresponding MILP formulation and add the following optimality-based cuts:

$$x_{jk} = 0, \quad \forall j \in V, \ \forall k \notin Q_j,$$

$$y_k = 0, \quad \forall p \in W, \ \forall k \in \widehat{U}(p).$$

Here, constraints (27) model the assignment interval for any task $j \in V$ and constraints (28) describe all unused blocks. These

Table 2
Characteristics of the series S1, S2 and S3.

Series	T	n	m	OS
S1	70	25	5	0.50
S2	70	25	5	0.15
S3	70	50	10	0.90

Table 3
Results for MILP models **without** pre-processing for $r_{\max} = 2$.

Series	$\lvert\widetilde{V}\rvert$	P_1			P_∞		
		#OPT	Avg. GAP	Avg. CPU	#OPT	Avg. GAP	Avg. CPU
S1	$\lceil \frac{n}{2} \rceil$	6	0.448	563.53	12	0.933	533.20
	$\lceil \frac{3n}{4} \rceil$	6	0.587	556.95	4	2.069	583.30
	n	8	0.432	547.42	0	3.391	600.00
S2	$\lceil \frac{n}{2} \rceil$	0	0.487	600.00	0	1.405	600.00
	$\lceil \frac{3n}{4} \rceil$	0	0.720	600.00	0	2.647	600.00
	n	0	0.726	600.00	0	4.221	600.00
S3	$\lceil \frac{n}{2} \rceil$	0	**NSF**	600.00	0	**NSF**	600.00
	$\lceil \frac{3n}{4} \rceil$	0	**NSF**	600.00	0	**NSF**	600.00
	n	0	**NSF**	600.00	0	**NSF**	600.00

Table 4
Results for MILP models **without** pre-processing for $r_{\max} = 3$.

Series	$\lvert\widetilde{V}\rvert$	P_1			P_∞		
		#OPT	Avg. GAP	Avg. CPU	#OPT	Avg. GAP	Avg. CPU
S1	$\lceil \frac{n}{2} \rceil$	38	0.051	298.23	48	0.018	224.04
	$\lceil \frac{3n}{4} \rceil$	44	0.016	203.12	42	0.141	325.01
	n	50	0.000	81.93	42	0.212	293.88
S2	$\lceil \frac{n}{2} \rceil$	4	0.161	566.53	13	0.204	514.95
	$\lceil \frac{3n}{4} \rceil$	7	0.203	545.74	5	0.942	564.61
	n	25	0.110	396.34	18	0.696	466.89
S3	$\lceil \frac{n}{2} \rceil$	0	**NSF**	600	0	**NSF**	600
	$\lceil \frac{3n}{4} \rceil$	0	**NSF**	600	0	**NSF**	600
	n	0	**NSF**	600	0	**NSF**	600

constraints do not remove any optimal solution, but exclude some feasible ones.

7. Computational results

Three series S1, S2 and S3 of well-known benchmark instances,[2] have been used to test our MILP models and pre-processing approaches. All the instances from the same series share the same cycle time, number of tasks and machines as well as the same order of strength (OS) for the corresponding precedence graph. The OS of a precedence graph is defined as the number of arcs in its transitive closure divided by $n \cdot (n-1)/2$. Each series contains 50 instances, whose aforementioned characteristics can be found in Table 2. In addition, we assume that the number of uncertain tasks $|\widetilde{V}|$ is in $\{\lceil \frac{n}{2} \rceil, \lceil \frac{3n}{4} \rceil, n\}$, where $\widetilde{V}$ is generated by taking the first $|\widetilde{V}|$ elements of a random permutation of $\{1, \ldots, n\}$, associated with each instance. Finally, the maximum number of tasks per block $r_{\max}$ is varied in $\{2, 3\}$. The tests have been performed on an Apple MacBook Pro, equipped with a 2.7 GHz Intel Core i5 and having 8 GB RAM. GUROBI 9.0 has been used as a MIP solver for addressing the MILP formulations of the problems P_1 and P_∞. The computation time has been limited to 600 seconds per instance and the `mslimit` parameter was fixed to $100n$.

First, we have addressed P_1 and P_∞ with MILP models only (without pre-processing) as they are presented in Section 5. Table 3 shows the number of optimal solutions, average GAP and CPU time for $r_{\max} = 2$. It can be seen that the problem is difficult to solve. The best result is obtained for ℓ_∞-norm with 50% of uncertain tasks in series S1, for which only 12 optimal solutions out of 50 have been found. In series S2, only feasible solutions are obtained, and the average GAP provided by the solver is quite large. With S3, no solution at all is found, which is indicated by "**NSF**" in the GAP column. Table 4 presents the results with $r_{\max} = 3$. The situation remains unchanged with series S3, as no feasible solution is found within the allocated time. With S2, the results are slightly better than with $r_{\max} = 2$, since several instances are solved to optimality. The improvement is more visible with series S1, since a greater number of optimal solutions is found, however only 88% of the instances in S1 are solved to optimality. All these observations lead us to conclude that basic MILP formulations (without pre-processing) are not competitive even for small and medium size instances.

The enhanced MILP models, enriched by a heuristic solution as a warm start and new constraints, provided from computing tighter assignment intervals based on pre-processing techniques, presented in Section 6, have been tested on the same instances. The corresponding computational results are presented in Tables 5 and 6.

The first observation that can be made is that the order of strength (OS) has an important impact on the computational efficiency of our algorithms. For example, when $r_{\max} = 2$ in series S1, which has an OS value of 0.5, meaning that the transitive closure

[2] http://pagesperso.ls2n.fr/~gurevsky-e/data/R-TLBP.zip.

Table 5
Results for MILP models **with** pre-processing for $r_{\max} = 2$.

Series	$\lvert\widetilde{V}\rvert$	P_1			P_∞		
		#OPT	Avg. GAP	Avg. CPU	#OPT	Avg. GAP	Avg. CPU
S1	$\lceil \frac{n}{2} \rceil$	16	0.227	436.41	35	0.196	238.91
	$\lceil \frac{3n}{4} \rceil$	11	0.452	488.83	28	0.252	337.82
	n	14	0.274	457.64	41	0.111	176.46
S2	$\lceil \frac{n}{2} \rceil$	0	0.405	600.00	2	1.201	585.38
	$\lceil \frac{3n}{4} \rceil$	0	0.700	600.00	1	2.008	591.29
	n	1	0.680	594.45	3	1.975	568.70
S3	$\lceil \frac{n}{2} \rceil$	40	0.177	239.55	47	0.022	97.02
	$\lceil \frac{3n}{4} \rceil$	42	0.152	226.66	49	0.020	43.77
	n	44	0.105	209.35	50	0.000	12.85

Table 6
Results for MILP models **with** pre-processing for $r_{\max} = 3$.

Series	$\lvert\widetilde{V}\rvert$	P_1			P_∞		
		#OPT	Avg. GAP	Avg. CPU	#OPT	Avg. GAP	Avg. CPU
S1	$\lceil \frac{n}{2} \rceil$	49	0.006	56.79	50	0.000	10.72
	$\lceil \frac{3n}{4} \rceil$	47	0.008	87.73	50	0.000	11.09
	n	50	0.000	43.78	50	0.000	0.33
S2	$\lceil \frac{n}{2} \rceil$	20	0.095	421.44	28	0.102	324.80
	$\lceil \frac{3n}{4} \rceil$	12	0.187	498.49	17	0.689	426.47
	n	24	0.097	361.66	46	0.003	71.28
S3	$\lceil \frac{n}{2} \rceil$	42	0.136	185.21	48	0.064	104.84
	$\lceil \frac{3n}{4} \rceil$	42	0.151	227.29	49	0.020	48.32
	n	45	0.154	181.34	50	0.000	9.92

of the precedence graph includes half of the edges from the complete graph, 41 instances are solved to optimality in ℓ_1-norm (see Table 5), whereas a single instance is solved to optimality in series S2 for which $OS = 0.15$. Moreover, our approach achieves good results for series S3 (for which $OS = 0.9$), despite the fact that the instance size is twice larger.

The results are similar with the ℓ_∞-norm, even if a greater number of optimal solutions is found in that case. This is visible in particular for series S3 where all the instances for which all the tasks are uncertain are solved to optimality with an average CPU time of 12.85 seconds.

The detailed results for each individual instance with $r_{\max} = 2$ can be found in Appendix B: Tables B.1– B.3. All the tables include information for both considered norms with the instance number and the ratio of uncertain tasks. The first two columns correspond to the best value of the stability radius obtained by the heuristic, with its corresponding CPU time. They are called LB and CPU, respectively. They are followed by four columns related to the results of the commercial solver GUROBI: LB and UB – the best lower and upper bounds obtained after 10 minutes (or less if an optimal solution is found); GAP – the gap to optimality returned by GUROBI at the end of search; CPU – computational time for GUROBI (this value is set to 600.00 if no provable optimal solution was found).

When $r_{\max} = 3$, a greater number of optimal solutions was found, as can be seen in Table 6. All the instances in S1 are solved to optimality in ℓ_∞-norm, and the average CPU time does not exceeds 12 seconds. Significant improvements are achieved for series S2 and S3. The output for all instances is presented in Tables B.4–B.6.

8. Conclusion and perspectives

This paper deals with robust balancing for transfer lines. It consists in finding a line configuration with the greatest stability radius subject to restricted number of machines, fixed cycle time, precedence constraints, and task time variability. The stability radius is evaluated in both ℓ_1- and ℓ_∞-norms. For each norm, the corresponding problem, denoted respectively as P_1 and P_∞, was proven to be strongly NP-hard. A MILP formulation was proposed for each problem. Two variants of a heuristic method, five reduction rules for assignment intervals and pre-processing techniques have been developed as well in order to enhance each MILP model. Numerical results show that the proposed enhancements improve significantly the seeking of optimal solutions.

The proposed MILP models are a first attempt to address the studied problems. The second natural step of our future research is the development of an efficient branch-and-bound procedure with appropriate upper bounds on stability radius for both norms. Another attractive way is to investigate more powerful reduction rules for assignment intervals based on the reformulation of the studied problem as a scheduling one.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2020.08.038.

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Appendix A. Proofs of Theorems for Section 4

Appendix A.1. Results for the ℓ_1-norm

Proof of Theorem 1. Let us denote the right-hand side of (2) as φ_1. To prove the present theorem, it needs to show that $\rho_1 \geq \varphi_1$ and $\rho_1 \leq \varphi_1$.

First start with $\rho_1 \geq \varphi_1$. Let p be an uncertain machine, exposed to stand the processing time deviations $\xi \in B_1(\varphi_1)$. It is not difficult to see that its *perturbed* load can not be greater than the value of the following expression

$$\sum_{k \in U(p)} \tau_k + \left[\sum_{j \in \widetilde{V}} \xi_j - \Delta_{\min}^{(p)} \right]^+, \tag{A.1}$$

where $[x]^+ = \max\{0, x\}$, $x \in \mathbb{R}$. Moreover, taking into account the fact that $\xi \in B_1(\varphi_1)$ and the inequality $\varphi_1 \leq T - \sum_{k \in U(p)} \tau_k + \Delta_{\min}^{(p)}$ that is valid for any $p \in \widetilde{W}$ due to the definition of φ_1, we obtain

$$(\text{A.1}) \leq \sum_{k \in U(p)} \tau_k + \left[\varphi_1 - \Delta_{\min}^{(p)} \right]^+ \leq \sum_{k \in U(p)} \tau_k + \left[T - \sum_{k \in U(p)} \tau_k \right]^+ = T.$$

The latter demonstrates that the load of the machine p does not exceed the cycle time, whatever $\xi \in B_1(\varphi_1)$. This proves $\rho_1 \geq \varphi_1$.

Now, let us show that $\rho_1 \leq \varphi_1$. To do this, it is sufficient to check that for any $\delta > \varphi_1$ there exists a vector of processing time deviations $\xi^* \in B_1(\delta)$, which causes the considered feasible solution to be unfeasible.

As above, based on the definition of φ_1, we deduce that there exists an uncertain machine p^* so as $\varphi_1 = T - \sum_{k \in U(p^*)} \tau_k + \Delta_{\min}^{(p^*)}$. Let k^* be an uncertain block of this machine having the least save time, *i.e.*, $\Delta_{k^*}^{(p^*)} = \Delta_{\min}^{(p^*)}$, and let j^* be a longest uncertain task of this block, *i.e.*, $t_{j^*} = \max_{j \in \widetilde{V}_{k^*}} t_j$. Then, setting $\xi^* \in B_1(\delta)$, where $\xi_j^* = \delta$, if $j = j^*$ and 0 otherwise, we notice that the *perturbed* load of the machine p^* violates the cycle time constraint, since

$$\sum_{k \in U(p^*)} \tau_k + \left[\delta - \Delta_{\min}^{(p^*)} \right]^+ > \sum_{k \in U(p^*)} \tau_k + \left[\varphi_1 - \Delta_{\min}^{(p^*)} \right]^+ = \sum_{k \in U(p^*)} \tau_k + \left[T - \sum_{k \in U(p^*)} \tau_k \right]^+ = T$$

that proves $\rho_1 \leq \varphi_1$. □

Appendix A.2. Results for the ℓ_∞-norm

The following evident equality is valid for any $p \in \widetilde{W}$ and $q \in \{1, \ldots, |\widetilde{U}(p)| - 1\}$:

$$\Delta^{(p)}_{\pi_{q+1}} = (q+1) \cdot \theta(p, q+1) - q \cdot \theta(p, q). \tag{A.2}$$

The next lemma shows that, for a fixed uncertain machine p, the function $\theta(p, q)$ has no more than one local minimum, which corresponds to the global one.

Lemma 1. *Let p be a given uncertain machine having $|\widetilde{U}(p)| \geq 3$ and let there be $q^\star \in \{1, \ldots, |\widetilde{U}(p)| - 1\}$ such that $\theta(p, q^\star + 1) \geq \theta(p, q^\star)$, then*

$$\theta(p, q^\star + \alpha + 1) \geq \theta(p, q^\star + \alpha) \tag{A.3}$$

holds for any $\alpha \in \{0, \ldots, |\widetilde{U}(p)| - q^\star - 1\}$.

Proof of Lemma 1. This lemma is proven by recurrence on α. First, it can immediately be checked that if there exists $q^\star \in \{1, \ldots, |\widetilde{U}(p)| - 1\}$ such that $\theta(p, q^\star + 1) \geq \theta(p, q^\star)$, then (A.3) is satisfied for $\alpha = 0$. Now, assume that (A.3) holds up to some integer $\alpha \leq |\widetilde{U}(p) - q^\star - 2|$, and prove that it also holds up to $\alpha + 1$. To do this, let us develop inequality (A.3). Based on expression (A.2), we obtain

$$\theta(p, q^\star + \alpha + 1) - \theta(p, q^\star + \alpha) = \frac{\Delta^{(p)}_{\pi_{q^\star+\alpha+1}} - \theta(p, q^\star + \alpha)}{q^\star + \alpha + 1}.$$

Since the latter expression is non-negative due to (A.3), then we have $\Delta^{(p)}_{\pi_{q^\star+\alpha+1}} \geq \theta(p, q^\star + \alpha)$. Multiplying it by $(q^\star + \alpha)$ and then adding $\Delta^{(p)}_{\pi_{q^\star+\alpha+1}}$, yields $\Delta^{(p)}_{\pi_{q^\star+\alpha+1}} \geq \theta(p, q^\star + \alpha + 1)$. By definition of the permutation π, we have $\Delta^{(p)}_{\pi_{q^\star+\alpha+2}} \geq \Delta^{(p)}_{\pi_{q^\star+\alpha+1}}$, and, as a consequence, we deduce that

$$\Delta^{(p)}_{\pi_{q^\star+\alpha+2}} \geq \theta(p, q^\star + \alpha + 1). \tag{A.4}$$

Now, let us show that $\theta(p, q^\star + \alpha + 2) \geq \theta(p, q^\star + \alpha + 1)$. By analogy with the previous development, we have

$$\theta(p, q^\star + \alpha + 2) - \theta(p, q^\star + \alpha + 1) = \frac{\Delta^{(p)}_{\pi_{q^\star+\alpha+2}} - \theta(p, q^\star + \alpha + 1)}{q^\star + \alpha + 2},$$

which is non-negative due to (A.4). This proves Lemma 1. □

The next two auxiliary lemmas are useful for a better explanation of the formula for computing the stability radius in the ℓ_∞-norm.

Lemma 2. *Let $q_{\min}^{(p)}$ denote an index on which the function $\theta(p,q)$ reaches its minimum for a given uncertain machine p, then $\Delta^{(p)}_{\pi^{(p)}_{q_{\min}}} \leq \theta(p, q_{\min}^{(p)})$ and additionally $\theta(p, q_{\min}^{(p)}) \leq \Delta^{(p)}_{\pi^{(p)}_{q_{\min}+1}}$, if $q_{\min}^{(p)} < |\widetilde{U}(p)|$.*

Proof of Lemma 2. Prove $\Delta^{(p)}_{\pi^{(p)}_{q_{\min}}} \leq \theta(p, q_{\min}^{(p)})$, which is evident if $q_{\min}^{(p)} = 1$. Let $q_{\min}^{(p)} \geq 2$. From the definition of $q_{\min}^{(p)}$, we deduce that $\theta(p, q_{\min}^{(p)}) \leq \theta(p, q_{\min}^{(p)} - 1)$. Then, using the latter inequality and representation (A.2), we obtain

$$\Delta^{(p)}_{\pi^{(p)}_{q_{\min}}} = q_{\min}^{(p)} \cdot \theta(p, q_{\min}^{(p)}) - (q_{\min}^{(p)} - 1) \cdot \theta(p, q_{\min}^{(p)} - 1) \leq$$

$$q_{\min}^{(p)} \cdot \theta(p, q_{\min}^{(p)}) - (q_{\min}^{(p)} - 1) \cdot \theta(p, q_{\min}^{(p)}) = \theta(p, q_{\min}^{(p)}).$$

Now, prove $\theta(p, q_{\min}^{(p)}) \leq \Delta^{(p)}_{\pi^{(p)}_{q_{\min}+1}}$, if $q_{\min}^{(p)} < |\widetilde{U}(p)|$. As above, the definition of $q_{\min}^{(p)}$ implies that $\theta(p, q_{\min}^{(p)}) \leq \theta(p, q_{\min}^{(p)} + 1)$, which yields the following

$$\Delta^{(p)}_{\pi^{(p)}_{q_{\min}+1}} = (q_{\min}^{(p)} + 1) \cdot \theta(p, q_{\min}^{(p)} + 1) - q_{\min}^{(p)} \cdot \theta(p, q_{\min}^{(p)}) \geq$$

$$(q_{\min}^{(p)} + 1) \cdot \theta(p, q_{\min}^{(p)}) - q_{\min}^{(p)} \cdot \theta(p, q_{\min}^{(p)}) = \theta(p, q_{\min}^{(p)}).$$

□

Lemma 3. *Any uncertain machine p supports each processing time deviation from $B_\infty(\theta(p, q_{\min}^{(p)}))$.*

Proof of Lemma 3. Let p be an uncertain machine of some feasible solution, exposed to stand the processing time deviations from $B_\infty(\theta(p, q_{\min}^{(p)}))$. It is not difficult to see that its *perturbed* load can not be greater than the value of the following expression

$$\sum_{k \in U(p)} \tau_k + \sum_{l \in \widetilde{U}(p)} \left[\theta(p, q_{\min}^{(p)}) - \Delta_l^{(p)}\right]^+, \tag{A.5}$$

where $[x]^+ = \max\{0, x\}$, $x \in \mathbb{R}$. Then, based on Lemma 2 and the definition of the function $\theta(p,q)$, we obtain

$$(\text{A.5}) = \sum_{k \in U(p)} \tau_k + \sum_{j=1}^{q_{\min}^{(p)}} \left(\theta(p, q_{\min}^{(p)}) - \Delta^{(p)}_{\pi_j}\right) = \sum_{k \in U(p)} \tau_k + q_{\min}^{(p)} \cdot \theta(p, q_{\min}^{(p)}) - \sum_{j=1}^{q_{\min}^{(p)}} \Delta^{(p)}_{\pi_j} = T$$

that proves the present lemma. □

Proof of Theorem 2. Let us denote the right-hand side of (3) as φ_∞. To prove the present theorem, it needs to show that $\rho_\infty \geq \varphi_\infty$ and $\rho_\infty \leq \varphi_\infty$. The inequality $\rho_\infty \geq \varphi_\infty$ is a direct consequence of Lemma 3.

Now, let us show that $\rho_\infty \leq \varphi_\infty$. To do this, it is sufficient to check that for any $\delta > \varphi_\infty$ there exists a vector of processing time deviations $\xi^* \in B_\infty(\delta)$, which causes the considered feasible solution to be unfeasible.

As above, based on the definition of φ_∞, we deduce that there exists an uncertain machine p^* and an index q^* such that $q^* \cdot \varphi_\infty = T - \sum_{k \in U(p^*)} \tau_k + \sum_{i=1}^{q^*} \Delta_{\pi_i}^{(p^*)}$. Then, setting $\xi^* \in B_\infty(\delta)$, where $\xi_j^* = \delta$, for any $j \in \widetilde{V}_k$, $k \in \widetilde{U}(p^*)$, we notice that the *perturbed* load of the machine p^* violates the cycle time constraint, since

$$\sum_{k \in U(p^*)} \tau_k + \sum_{i=1}^{|\widetilde{U}(p^*)|} \left[\delta - \Delta_{\pi_i}^{(p^*)}\right]^+ \geq \sum_{k \in U(p^*)} \tau_k + \sum_{i=1}^{q^*} \left[\delta - \Delta_{\pi_i}^{(p^*)}\right]^+ \geq \sum_{k \in U(p^*)} \tau_k + \left[\sum_{i=1}^{q^*} \left(\delta - \Delta_{\pi_i}^{(p^*)}\right)\right]^+ >$$

$$\sum_{k \in U(p^*)} \tau_k + \left[\sum_{i=1}^{q^*} \left(\varphi_\infty - \Delta_{\pi_i}^{(p^*)}\right)\right]^+ = \sum_{k \in U(p^*)} \tau_k + \left[q^* \cdot \varphi_\infty - \sum_{i=1}^{q^*} \Delta_{\pi_i}^{(p^*)}\right]^+ =$$

$$\sum_{k \in U(p^*)} \tau_k + \left[T - \sum_{k \in U(p^*)} \tau_k\right]^+ = T$$

that proves $\rho_\infty \leq \varphi_\infty$. □

Let us present a small illustrative example of one machine of the effective load 19 with five uncertain blocks and $T = 21$ (see, Figure A.2). The ordered five save times and the corresponding values of $\theta(\cdot, q)$ are given in Table A.1. Figure A.1 provides the graph of $\theta(\cdot, q)$ having one local minimum $\theta(\cdot, 3) = 1.67$, which is confirmed by Lemma 1. Following Theorem 2, the stability radius ρ_∞ of the studied machine equals $\theta(\cdot, 3)$, which is also approved by Lemma 2, since $\Delta_3^{(\cdot)} \leq \rho_\infty \leq \Delta_4^{(\cdot)}$ are valid (see, Figure A.3).

q	1	2	3	4	5
$\Delta_q^{(\cdot)}$	0.5	1.0	1.5	2.0	2.0
$\theta(\cdot, q)$	2.5	1.75	1.67	1.75	1.8

Table A.1: Values of $\Delta_q^{(\cdot)}$ and $\theta(\cdot, q)$.

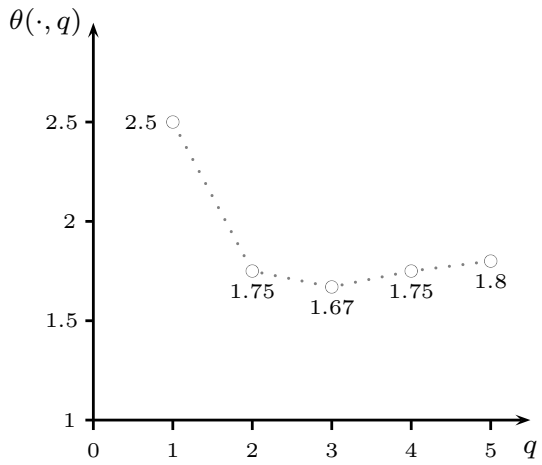

Figure A.1: Graph of $\theta(\cdot, q)$

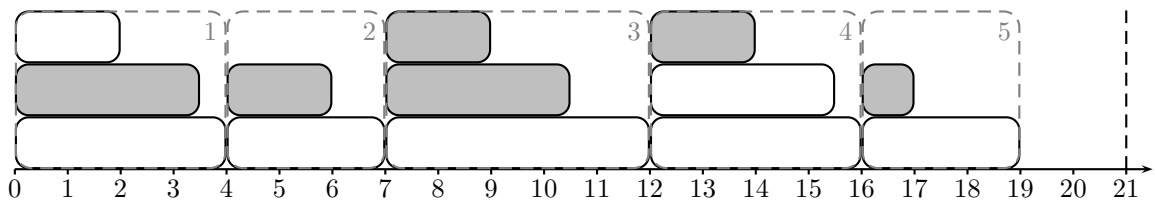

Figure A.2: A machine with five uncertain blocks

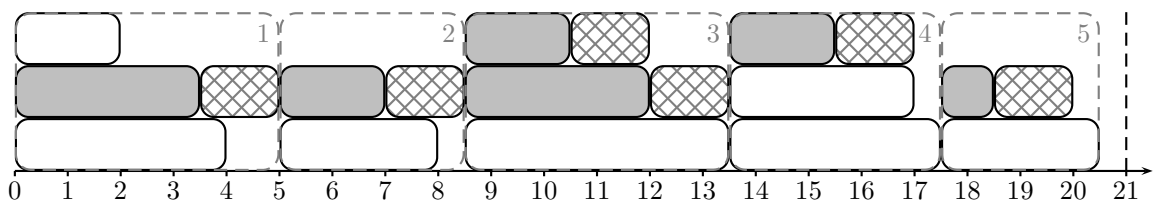

Figure A.3: The machine load perturbed by $\Delta_3^{(\cdot)} = 1.5$ for each uncertain task

Appendix B. Detailed computational results

Table B.1: Detailed results for the series S1 and $r_{\max} = 2$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	28.2	0.11	30.4	30.4	0.000	63.39	18.75	0.22	21.15	21.15	0.000	22.06
	$\lceil\frac{3n}{4}\rceil$	27.6	0.07	30.4	30.4	0.000	65.62	11.867	0.19	13.2	13.2	0.000	14.71
	n	27.5	0.07	30.4	30.4	0.000	67.12	9.467	0.32	10.133	10.133	0.000	7.70
S1.1	$\lceil\frac{n}{2}\rceil$	23.5	0.06	27.3	39.2	0.436	600.00	16	0.29	19.3	19.3	0.000	31.36
	$\lceil\frac{3n}{4}\rceil$	23.8	0.11	23.8	38.8	0.630	600.00	10.35	0.25	11.55	11.55	0.000	221.16
	n	23.4	0.07	23.8	29.1	0.223	600.00	7.8	0.65	7.933	7.933	0.000	96.27
S1.2	$\lceil\frac{n}{2}\rceil$	27.5	0.06	29.1	51.4	0.766	600.00	16.1	0.23	17.55	17.55	0.000	385.53
	$\lceil\frac{3n}{4}\rceil$	26.4	0.07	28.1	36.7	0.306	600.00	12.1	0.38	13	13	0.000	114.05
	n	26.4	0.06	28	28	0.000	250.30	9.167	0.31	9.333	9.333	0.000	12.72
S1.3	$\lceil\frac{n}{2}\rceil$	33.1	0.09	34.5	51.4	0.490	600.00	19.7	0.43	22.05	22.05	0.000	437.29
	$\lceil\frac{3n}{4}\rceil$	28.6	0.09	29.5	51.4	0.742	600.00	12.85	0.43	14.85	25.5	0.717	600.00
	n	28.4	0.08	28.5	51.4	0.804	600.00	9.5	0.36	9.833	19.15	0.947	600.00
S1.4	$\lceil\frac{n}{2}\rceil$	26	0.07	27	40.432	0.497	600.00	16.3	0.20	18.6	18.6	0.000	79.34
	$\lceil\frac{3n}{4}\rceil$	23.7	0.06	26.3	36.4	0.384	600.00	11.15	0.42	12.35	12.35	0.000	94.92
	n	23.9	0.07	26.3	26.4	0.004	600.00	8	0.29	8.767	8.767	0.000	15.47
S1.5	$\lceil\frac{n}{2}\rceil$	30.4	0.10	32.3	37.6	0.164	600.00	18.8	0.25	20.65	20.65	0.000	20.49
	$\lceil\frac{3n}{4}\rceil$	28.4	0.07	29.5	50.9	0.725	600.00	12.8	0.32	14.45	14.6	0.010	600.00
	n	28.4	0.09	29.5	40.9	0.386	600.00	9.633	0.31	9.833	9.833	0.000	71.56
S1.6	$\lceil\frac{n}{2}\rceil$	26.7	0.06	28.5	28.6	0.004	600.00	16	0.25	18.9	20.5	0.085	600.00
	$\lceil\frac{3n}{4}\rceil$	26.5	0.07	27.9	28.6	0.025	600.00	12.5	0.25	13.1	13.1	0.000	282.53
	n	26.5	0.07	27.9	28.6	0.025	600.00	8.833	0.26	9.3	9.3	0.000	10.61
S1.7	$\lceil\frac{n}{2}\rceil$	33.4	0.10	38.5	38.5	0.000	225.93	20	0.32	21.1	43.964	1.084	600.00
	$\lceil\frac{3n}{4}\rceil$	29	0.07	29	50.6	0.745	600.00	13.65	0.33	14.5	22.65	0.562	600.00
	n	29	0.06	29	45.4	0.566	600.00	9.667	0.33	9.867	10.2	0.034	600.00
S1.8	$\lceil\frac{n}{2}\rceil$	32.4	0.07	32.4	35.1	0.083	600.00	17.3	0.31	20.6	20.6	0.000	32.92
	$\lceil\frac{3n}{4}\rceil$	27.5	0.07	28	32.4	0.157	600.00	12.3	0.27	13.7	13.7	0.000	526.83
	n	27.5	0.07	28.9	32.4	0.121	600.00	9.167	0.47	9.633	9.633	0.000	33.39
S1.9	$\lceil\frac{n}{2}\rceil$	23.7	0.04	24.4	24.4	0.000	23.35	12.9	0.12	12.9	12.9	0.000	9.66
	$\lceil\frac{3n}{4}\rceil$	24.4	0.04	24.4	24.4	0.000	36.63	9.467	0.19	9.467	9.467	0.000	4.55
	n	24.4	0.04	24.4	24.4	0.000	28.36	8.133	0.15	8.133	8.133	0.000	0.04
S1.10	$\lceil\frac{n}{2}\rceil$	25.2	0.06	26.7	26.7	0.000	68.51	17.85	0.17	20.2	20.2	0.000	18.42

Table B.1 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	$\lceil\frac{3n}{4}\rceil$	24.9	0.05	25.7	25.7	0.000	69.76	11.367	0.23	11.367	11.367	0.000	19.69
	n	24.9	0.06	25.2	25.2	0.000	41.46	8.3	0.29	8.4	8.4	0.000	3.04
S1.11	$\lceil\frac{n}{2}\rceil$	23	0.05	25.6	25.6	0.000	32.81	12.95	0.13	12.95	12.95	0.000	8.89
	$\lceil\frac{3n}{4}\rceil$	23	0.05	25.4	25.4	0.000	40.33	8.6	0.28	8.633	8.633	0.000	1.68
	n	23	0.04	25.4	25.4	0.000	22.11	7.667	0.20	8.467	8.467	0.000	0.58
S1.12	$\lceil\frac{n}{2}\rceil$	26.5	0.07	27.1	37.8	0.395	600.00	17.8	0.44	18.5	18.5	0.000	138.72
	$\lceil\frac{3n}{4}\rceil$	25.9	0.08	26.6	50.848	0.912	600.00	10.8	0.24	12.05	14.75	0.224	600.00
	n	25.9	0.08	26.6	31.6	0.188	600.00	8.367	0.52	8.867	8.867	0.000	172.18
S1.13	$\lceil\frac{n}{2}\rceil$	25.5	0.05	25.5	25.5	0.000	26.34	15.3	0.18	17.3	17.3	0.000	2.24
	$\lceil\frac{3n}{4}\rceil$	25.5	0.05	25.5	25.5	0.000	21.68	10.5	0.17	11.95	11.95	0.000	8.02
	n	25.5	0.04	25.5	25.5	0.000	26.72	8.5	0.20	8.5	8.5	0.000	0.37
S1.14	$\lceil\frac{n}{2}\rceil$	30.7	0.06	32.1	35.5	0.106	600.00	19	0.20	20.3	20.3	0.000	82.18
	$\lceil\frac{3n}{4}\rceil$	28.7	0.05	30.1	30.3	0.007	600.00	11.45	0.28	12.2	12.2	0.000	18.59
	n	28.7	0.05	30.1	30.1	0.000	147.20	9.567	0.25	10.033	10.033	0.000	7.72
S1.15	$\lceil\frac{n}{2}\rceil$	25.1	0.06	25.1	28.9	0.151	600.00	15.3	0.22	16.05	16.05	0.000	32.97
	$\lceil\frac{3n}{4}\rceil$	24.3	0.10	24.7	38.1	0.543	600.00	10.467	0.27	11.55	11.55	0.000	76.53
	n	24.3	0.09	25.1	28.1	0.120	600.00	8.1	0.29	8.367	8.367	0.000	47.14
S1.16	$\lceil\frac{n}{2}\rceil$	28.4	0.08	29.8	51.2	0.718	600.00	18.05	0.28	21.1	40.2	0.905	600.00
	$\lceil\frac{3n}{4}\rceil$	27.6	0.11	27.6	50.9	0.844	600.00	12.7	0.34	12.85	19.45	0.514	600.00
	n	27.6	0.11	27.6	39.3	0.424	600.00	8.9	0.34	9.2	11.7	0.272	600.00
S1.17	$\lceil\frac{n}{2}\rceil$	32	0.07	32	32	0.000	231.99	18.05	0.25	18.4	18.4	0.000	64.45
	$\lceil\frac{3n}{4}\rceil$	27.3	0.08	28.1	36.2	0.288	600.00	13.1	0.28	14	14	0.000	117.74
	n	27.3	0.08	27.3	36.2	0.326	600.00	8.867	0.25	9.367	9.367	0.000	8.47
S1.18	$\lceil\frac{n}{2}\rceil$	30	0.09	30.7	37	0.205	600.00	18.3	0.27	20.6	20.6	0.000	27.14
	$\lceil\frac{3n}{4}\rceil$	30.3	0.08	30.7	50.6	0.648	600.00	13.2	0.50	13.85	17.4	0.256	600.00
	n	29.8	0.09	30.3	41.2	0.360	600.00	10.1	0.25	10.233	10.233	0.000	317.95
S1.19	$\lceil\frac{n}{2}\rceil$	31	0.08	31.4	31.4	0.000	231.60	18.4	0.25	19.85	19.85	0.000	60.66
	$\lceil\frac{3n}{4}\rceil$	29.1	0.07	29.4	51.8	0.762	600.00	12.4	0.33	13.35	14.6	0.094	600.00
	n	29.1	0.06	29.4	30.6	0.041	600.00	9.7	0.24	9.8	9.8	0.000	9.69
S1.20	$\lceil\frac{n}{2}\rceil$	31.8	0.10	32.8	50.1	0.527	600.00	17.85	0.42	17.85	50.1	1.807	600.00
	$\lceil\frac{3n}{4}\rceil$	25.4	0.07	25.4	50.1	0.972	600.00	10.5	0.33	11.85	45.191	2.814	600.00
	n	25.4	0.07	27.4	50.1	0.828	600.00	8.633	0.52	9.133	11.3	0.237	600.00
S1.21	$\lceil\frac{n}{2}\rceil$	31.1	0.09	34	50.1	0.474	600.00	20.1	0.34	22.05	37.6	0.705	600.00

Table B.1 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	$\lceil\frac{3n}{4}\rceil$	30.3	0.09	30.8	38.8	0.260	600.00	15.15	0.35	15.15	19	0.254	600.00
	n	30.3	0.08	30.8	38	0.234	600.00	10.267	0.74	10.433	10.433	0.000	144.30
S1.22	$\lceil\frac{n}{2}\rceil$	30.6	0.07	33.8	35.4	0.047	600.00	18.35	0.41	19.9	19.9	0.000	108.63
	$\lceil\frac{3n}{4}\rceil$	24.5	0.08	25.3	50.4	0.992	600.00	11.35	0.54	12.05	12.05	0.000	458.85
	n	24.5	0.08	25.4	38.6	0.520	600.00	8.233	0.29	8.467	8.467	0.000	335.67
S1.23	$\lceil\frac{n}{2}\rceil$	32.1	0.07	34	50.8	0.494	600.00	18.6	0.24	19.45	24.5	0.260	600.00
	$\lceil\frac{3n}{4}\rceil$	27.4	0.09	27.4	50.8	0.854	600.00	12.6	0.37	12.7	17.9	0.409	600.00
	n	27.4	0.06	27.4	50.8	0.854	600.00	9.133	0.38	9.233	9.233	0.000	502.78
S1.24	$\lceil\frac{n}{2}\rceil$	33.3	0.09	34	38.2	0.124	600.00	19.2	0.21	20.6	20.6	0.000	226.75
	$\lceil\frac{3n}{4}\rceil$	26.6	0.12	26.8	51.1	0.907	600.00	11.5	0.26	12.7	18.6	0.465	600.00
	n	25.9	0.08	27.6	38.2	0.384	600.00	8.767	0.34	9.233	9.233	0.000	252.94
S1.25	$\lceil\frac{n}{2}\rceil$	30.9	0.06	31.9	52.4	0.643	600.00	20.3	0.30	21.05	21.05	0.000	253.54
	$\lceil\frac{3n}{4}\rceil$	29	0.08	29.6	41.3	0.395	600.00	12.4	0.25	14.15	14.15	0.000	149.11
	n	29	0.08	30	39.8	0.327	600.00	9.667	0.23	10	10	0.000	157.64
S1.26	$\lceil\frac{n}{2}\rceil$	20.8	0.05	21.2	21.2	0.000	17.02	11.3	0.13	13.8	13.8	0.000	2.01
	$\lceil\frac{3n}{4}\rceil$	20.3	0.06	21.2	21.2	0.000	18.99	7.5	0.15	7.5	7.5	0.000	4.72
	n	20.3	0.05	21.2	21.2	0.000	13.34	6.9	0.18	7.067	7.067	0.000	0.63
S1.27	$\lceil\frac{n}{2}\rceil$	30.8	0.06	30.8	40.6	0.318	600.00	19.65	0.38	20.2	30.1	0.490	600.00
	$\lceil\frac{3n}{4}\rceil$	26.1	0.06	29.2	35.7	0.223	600.00	12.8	0.33	12.8	12.8	0.000	483.59
	n	26.1	0.05	29.2	31.6	0.082	600.00	8.933	0.32	9.733	9.733	0.000	16.73
S1.28	$\lceil\frac{n}{2}\rceil$	31.8	0.14	32.9	50.1	0.523	600.00	19.2	0.43	20.05	22.55	0.125	600.00
	$\lceil\frac{3n}{4}\rceil$	25.8	0.11	27.6	50.1	0.815	600.00	12.3	0.38	12.6	24.5	0.944	600.00
	n	25.8	0.11	26.5	50.1	0.891	600.00	8.5	0.37	9	18.1	1.011	600.00
S1.29	$\lceil\frac{n}{2}\rceil$	34.7	0.07	38	38.8	0.021	600.00	19.65	0.37	19.9	31.4	0.578	600.00
	$\lceil\frac{3n}{4}\rceil$	29.3	0.08	29.9	47.875	0.601	600.00	12.55	0.31	14.05	19.35	0.377	600.00
	n	29.3	0.12	31.3	38.8	0.240	600.00	9.533	0.34	10.633	10.633	0.000	48.42
S1.30	$\lceil\frac{n}{2}\rceil$	34.1	0.10	35	35	0.000	121.05	18.6	0.35	19.5	19.5	0.000	84.34
	$\lceil\frac{3n}{4}\rceil$	25.7	0.07	26.3	36.3	0.380	600.00	11.75	0.35	12.15	18.15	0.494	600.00
	n	25.7	0.06	26.5	40.282	0.520	600.00	8.9	0.45	8.9	8.9	0.000	32.54
S1.31	$\lceil\frac{n}{2}\rceil$	30	0.09	32.9	41.3	0.255	600.00	18.9	0.21	20.3	21.7	0.069	600.00
	$\lceil\frac{3n}{4}\rceil$	28.4	0.08	29	51.023	0.759	600.00	11.767	0.29	12.55	12.55	0.000	411.99
	n	28.2	0.05	29	35.7	0.231	600.00	9.367	0.34	9.667	9.667	0.000	15.75
S1.32	$\lceil\frac{n}{2}\rceil$	33.4	0.12	33.4	50.6	0.515	600.00	17	0.24	19.4	43.423	1.238	600.00

Table B.1 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	$\lceil\frac{3n}{4}\rceil$	27.2	0.07	28.5	50.3	0.765	600.00	11.633	0.28	13.1	30.7	1.344	600.00
	n	27.2	0.07	28.5	48.255	0.693	600.00	9.433	0.40	9.8	9.8	0.000	391.23
S1.33	$\lceil\frac{n}{2}\rceil$	30.9	0.07	30.9	30.9	0.000	58.46	17.9	0.18	19.6	19.6	0.000	29.10
	$\lceil\frac{3n}{4}\rceil$	28.1	0.07	28.1	28.1	0.000	398.07	13.05	0.35	13.75	13.75	0.000	21.64
	n	28.1	0.06	28.1	28.1	0.000	67.58	9.3	0.40	9.367	9.367	0.000	4.49
S1.34	$\lceil\frac{n}{2}\rceil$	31.1	0.07	33.7	40.5	0.202	600.00	18.05	0.49	20.25	21.3	0.052	600.00
	$\lceil\frac{3n}{4}\rceil$	25.9	0.13	27.2	50.364	0.852	600.00	11.65	0.40	12.55	21.4	0.705	600.00
	n	25.5	0.08	27.2	43.9	0.614	600.00	8.833	0.31	9.167	15.95	0.740	600.00
S1.35	$\lceil\frac{n}{2}\rceil$	28.5	0.06	30.3	30.3	0.000	57.44	19.35	0.31	19.55	19.55	0.000	33.76
	$\lceil\frac{3n}{4}\rceil$	27.1	0.06	28.1	28.1	0.000	121.60	12.35	0.23	13.45	13.45	0.000	6.82
	n	27.1	0.06	28.1	28.1	0.000	59.93	9.267	0.27	9.367	9.367	0.000	2.37
S1.36	$\lceil\frac{n}{2}\rceil$	31.3	0.05	31.3	31.3	0.000	70.84	15.65	0.23	16.4	16.4	0.000	77.74
	$\lceil\frac{3n}{4}\rceil$	23.7	0.11	25.4	27.9	0.098	600.00	11.3	0.38	11.4	11.4	0.000	42.14
	n	23.6	0.06	25.4	25.5	0.004	600.00	7.867	0.21	8.467	8.467	0.000	11.28
S1.37	$\lceil\frac{n}{2}\rceil$	25	0.07	26.2	26.2	0.000	32.01	16.7	0.16	17.6	17.6	0.000	7.25
	$\lceil\frac{3n}{4}\rceil$	23	0.06	23.4	23.4	0.000	70.79	10.65	0.29	10.85	10.85	0.000	14.05
	n	23	0.05	23.4	23.4	0.000	46.91	7.767	0.24	7.8	7.8	0.000	7.47
S1.38	$\lceil\frac{n}{2}\rceil$	25	0.07	25	25	0.000	57.06	12.75	0.19	13.85	13.85	0.000	20.44
	$\lceil\frac{3n}{4}\rceil$	24.9	0.06	25	25	0.000	37.43	10.7	0.20	12.2	12.2	0.000	9.52
	n	24.9	0.06	25	25	0.000	56.80	8.3	0.25	8.333	8.333	0.000	0.73
S1.39	$\lceil\frac{n}{2}\rceil$	28.1	0.07	28.2	36.4	0.291	600.00	18.2	0.42	18.2	18.2	0.000	74.38
	$\lceil\frac{3n}{4}\rceil$	28.2	0.07	28.2	37.5	0.330	600.00	12.9	0.27	12.9	12.9	0.000	85.21
	n	28.1	0.06	28.1	28.1	0.000	356.30	9.367	0.26	9.367	9.367	0.000	6.59
S1.40	$\lceil\frac{n}{2}\rceil$	27.1	0.11	27.1	36.1	0.332	600.00	16.5	0.20	18.5	18.5	0.000	44.36
	$\lceil\frac{3n}{4}\rceil$	25.1	0.12	25.6	36.1	0.410	600.00	12.1	0.30	12.1	12.1	0.000	121.75
	n	25.1	0.11	26.4	36.1	0.367	600.00	8.433	0.28	8.8	8.8	0.000	16.65
S1.41	$\lceil\frac{n}{2}\rceil$	25.8	0.06	28.8	28.8	0.000	102.11	18.05	0.27	19.65	19.65	0.000	21.18
	$\lceil\frac{3n}{4}\rceil$	25.6	0.05	27.1	27.1	0.000	160.04	11.55	0.34	12.9	12.9	0.000	25.54
	n	25.6	0.05	27.1	27.1	0.000	97.28	8.6	0.24	9.033	9.033	0.000	7.66
S1.42	$\lceil\frac{n}{2}\rceil$	28.7	0.08	31.1	50.1	0.611	600.00	19	0.35	19.65	43	1.188	600.00
	$\lceil\frac{3n}{4}\rceil$	25.7	0.11	25.9	50.1	0.934	600.00	12.15	0.41	12.35	21	0.700	600.00
	n	25.8	0.09	26.8	42	0.567	600.00	8.633	0.43	8.933	18.25	1.043	600.00
S1.43	$\lceil\frac{n}{2}\rceil$	32.8	0.12	34.1	50.4	0.478	600.00	18.05	0.57	20.2	39.8	0.970	600.00

Table B.1 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	$\lceil\frac{3n}{4}\rceil$	27.5	0.13	29.6	50.2	0.696	600.00	12.25	0.50	14.55	20.1	0.381	600.00
	n	26.8	0.09	28.5	50.2	0.761	600.00	8.767	0.57	9.5	19.75	1.079	600.00
S1.44	$\lceil\frac{n}{2}\rceil$	34	0.08	34	50.1	0.474	600.00	19.55	0.38	20.6	20.6	0.000	191.58
	$\lceil\frac{3n}{4}\rceil$	27	0.11	28.5	50.1	0.758	600.00	12	0.39	14.3	19.1	0.336	600.00
	n	27	0.11	28.4	45.623	0.606	600.00	9.3	0.26	9.533	11.3	0.185	600.00
S1.45	$\lceil\frac{n}{2}\rceil$	28.3	0.08	28.3	29.4	0.039	600.00	16.7	0.18	18.2	18.2	0.000	62.10
	$\lceil\frac{3n}{4}\rceil$	22.1	0.05	22.8	40.512	0.777	600.00	10.15	0.35	10.9	11.1	0.018	600.00
	n	22.1	0.05	22.8	29.4	0.289	600.00	7.133	0.19	7.6	7.6	0.000	473.79
S1.46	$\lceil\frac{n}{2}\rceil$	31.1	0.09	31.1	36.5	0.174	600.00	18.25	0.27	18.25	18.25	0.000	34.32
	$\lceil\frac{3n}{4}\rceil$	26.9	0.08	26.9	33	0.227	600.00	12.15	0.35	12.9	12.9	0.000	245.68
	n	26.9	0.08	27.2	33	0.213	600.00	8.967	0.23	9.067	9.067	0.000	10.14
S1.47	$\lceil\frac{n}{2}\rceil$	25.2	0.08	27.2	40.4	0.485	600.00	16.1	0.22	16.25	20.2	0.243	600.00
	$\lceil\frac{3n}{4}\rceil$	22.7	0.06	25	41.777	0.671	600.00	10.05	0.29	11	17.5	0.591	600.00
	n	22.7	0.05	24.7	35.4	0.433	600.00	8	0.27	8.233	8.233	0.000	54.92
S1.48	$\lceil\frac{n}{2}\rceil$	28.7	0.08	29.7	33.1	0.114	600.00	17.3	0.33	19.3	19.3	0.000	142.60
	$\lceil\frac{3n}{4}\rceil$	27.5	0.11	27.7	33.983	0.227	600.00	11.85	0.31	13.7	13.7	0.000	108.92
	n	27.5	0.10	28.4	33.1	0.165	600.00	9.167	0.29	9.467	9.467	0.000	71.84
S1.49	$\lceil\frac{n}{2}\rceil$	31.7	0.10	33.1	39.4	0.190	600.00	16.7	0.23	18.1	18.1	0.000	76.57
	$\lceil\frac{3n}{4}\rceil$	25.3	0.07	25.4	50.6	0.992	600.00	11.65	0.26	12.1	16.55	0.368	600.00
	n	25.3	0.06	25.9	33.217	0.283	600.00	8.633	0.33	8.8	8.8	0.000	41.31

Table B.2: Detailed results for the series S2 and $r_{\max} = 2$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	35.6	0.10	36.7	50.2	0.368	600.00	20.8	0.68	22.8	50.2	1.202	600.00
	$\lceil\frac{3n}{4}\rceil$	29.8	0.11	32.8	50.2	0.530	600.00	14.25	0.56	15.6	50.2	2.218	600.00
	n	29.7	0.08	31.7	50.2	0.584	600.00	10.033	0.60	10.567	25	1.366	600.00
S1.1	$\lceil\frac{n}{2}\rceil$	32.2	0.15	33	50.2	0.521	600.00	17.75	0.90	19.9	50.2	1.523	600.00
	$\lceil\frac{3n}{4}\rceil$	24.9	0.08	26.6	50.2	0.887	600.00	11.2	0.51	13	46.1	2.546	600.00
	n	24.7	0.10	27.1	50.2	0.852	600.00	8.4	0.53	8.867	23.05	1.600	600.00
S1.2	$\lceil\frac{n}{2}\rceil$	33.9	0.08	35.6	44.2	0.242	600.00	20.35	0.69	22.25	51.4	1.310	600.00
	$\lceil\frac{3n}{4}\rceil$	28.3	0.08	29.9	51.4	0.719	600.00	12.6	0.33	14.15	36.7	1.594	600.00
	n	28.6	0.10	29.3	51	0.741	600.00	9.567	0.48	9.967	18.35	0.841	600.00
S1.3	$\lceil\frac{n}{2}\rceil$	34.6	0.13	38	51.4	0.353	600.00	20.65	0.43	22.4	51.4	1.295	600.00
	$\lceil\frac{3n}{4}\rceil$	30.5	0.18	33.6	51.4	0.530	600.00	13.2	0.53	15.2	50.581	2.328	600.00
	n	30	0.13	30.1	51.4	0.708	600.00	9.833	0.85	10.267	44.726	3.356	600.00
S1.4	$\lceil\frac{n}{2}\rceil$	32.6	0.13	34.9	50.5	0.447	600.00	18.6	0.77	20.65	50.5	1.446	600.00
	$\lceil\frac{3n}{4}\rceil$	26.2	0.14	29	50.1	0.728	600.00	11.85	0.78	13.55	50.1	2.697	600.00
	n	25.2	0.09	28.2	50.1	0.777	600.00	8.767	0.56	9.133	44.711	3.895	600.00
S1.5	$\lceil\frac{n}{2}\rceil$	35.4	0.15	37.3	51.8	0.389	600.00	20.1	0.37	22.55	51.8	1.297	600.00
	$\lceil\frac{3n}{4}\rceil$	33.4	0.10	34.1	51.4	0.507	600.00	14.2	0.60	14.9	51.4	2.450	600.00
	n	30.5	0.11	31.6	50.9	0.611	600.00	9.8	0.46	10.467	29.8	1.847	600.00
S1.6	$\lceil\frac{n}{2}\rceil$	26	0.09	28.6	35.2	0.231	600.00	18.85	0.36	21.35	21.35	0.000	244.85
	$\lceil\frac{3n}{4}\rceil$	26	0.12	28.6	35.2	0.231	600.00	12.55	0.50	14.3	14.3	0.000	164.09
	n	26	0.11	28.6	28.6	0.000	322.03	8.7	0.39	9.533	9.533	0.000	13.38
S1.7	$\lceil\frac{n}{2}\rceil$	38.3	0.13	38.5	51.8	0.345	600.00	21.6	0.35	22.7	51.8	1.282	600.00
	$\lceil\frac{3n}{4}\rceil$	30.8	0.08	33.2	50.6	0.524	600.00	14.4	0.48	15.4	50.6	2.286	600.00
	n	30.4	0.11	31.5	50.6	0.606	600.00	9.867	0.51	10.5	37.638	2.585	600.00
S1.8	$\lceil\frac{n}{2}\rceil$	35	0.10	36.8	51.4	0.397	600.00	20.25	0.58	21.05	41.3	0.962	600.00
	$\lceil\frac{3n}{4}\rceil$	28	0.13	29.1	51	0.753	600.00	12.5	0.54	13.5	41.3	2.059	600.00
	n	27.4	0.09	28.8	51	0.771	600.00	9.167	0.47	9.7	20.2	1.082	600.00
S1.9	$\lceil\frac{n}{2}\rceil$	34.2	0.08	35.2	50.7	0.440	600.00	21.8	0.31	22.45	37.58	0.674	600.00
	$\lceil\frac{3n}{4}\rceil$	30.1	0.08	30.3	50.7	0.673	600.00	13.75	0.35	15.8	38.2	1.418	600.00
	n	30.8	0.11	31.2	50.7	0.625	600.00	9.867	0.32	10.4	20.65	0.986	600.00
S1.10	$\lceil\frac{n}{2}\rceil$	34.7	0.10	36.3	51.6	0.421	600.00	20.5	0.47	22.6	51.6	1.283	600.00
	$\lceil\frac{3n}{4}\rceil$	28.6	0.10	29.3	50.2	0.713	600.00	13.75	0.94	14.55	50.2	2.450	600.00

Table B.2 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	28.5	0.08	29.3	50.2	0.713	600.00	9.5	0.76	10.067	45.049	3.475	600.00
S1.11	$\lceil\frac{n}{2}\rceil$	34.8	0.09	36.3	51.7	0.424	600.00	19.8	0.41	22	51.7	1.350	600.00
	$\lceil\frac{3n}{4}\rceil$	29	0.14	29.8	50.9	0.708	600.00	13.85	0.62	14.8	50.9	2.439	600.00
	n	28.4	0.07	29.8	50.9	0.708	600.00	9.8	0.47	10.033	21.5	1.143	600.00
S1.12	$\lceil\frac{n}{2}\rceil$	31.8	0.13	34.1	50.9	0.493	600.00	18.95	0.54	20.5	50.9	1.483	600.00
	$\lceil\frac{3n}{4}\rceil$	26.5	0.11	27.8	50.9	0.831	600.00	11.85	0.58	13.6	50.9	2.743	600.00
	n	25.4	0.09	27.2	50.9	0.871	600.00	8.6	0.51	9.033	43.534	3.819	600.00
S1.13	$\lceil\frac{n}{2}\rceil$	34.2	0.10	34.5	51.1	0.481	600.00	19.2	0.37	21.1	43.2	1.047	600.00
	$\lceil\frac{3n}{4}\rceil$	26.2	0.10	27.2	51.1	0.879	600.00	12.75	0.40	14.3	40.6	1.839	600.00
	n	26	0.07	26.9	51.1	0.900	600.00	8.867	0.46	9.467	20.05	1.118	600.00
S1.14	$\lceil\frac{n}{2}\rceil$	35.8	0.18	36.8	50.1	0.361	600.00	21.35	0.64	21.55	50.1	1.325	600.00
	$\lceil\frac{3n}{4}\rceil$	30.1	0.14	32	50.1	0.566	600.00	13.9	0.66	15.1	50.1	2.318	600.00
	n	30.4	0.08	31.4	50.1	0.596	600.00	10	0.89	10.667	44.711	3.192	600.00
S1.15	$\lceil\frac{n}{2}\rceil$	32	0.14	33.7	50.5	0.499	600.00	18.7	0.34	19.85	50.5	1.544	600.00
	$\lceil\frac{3n}{4}\rceil$	24.8	0.17	26.9	50.1	0.862	600.00	11.7	0.48	12.5	47.433	2.795	600.00
	n	25	0.08	26.6	38.1	0.432	600.00	8.6	0.45	9	19.05	1.117	600.00
S1.16	$\lceil\frac{n}{2}\rceil$	33.6	0.08	35.4	50.9	0.438	600.00	20.5	0.46	21.6	50.9	1.356	600.00
	$\lceil\frac{3n}{4}\rceil$	28	0.10	28.9	50.9	0.761	600.00	12.45	0.35	14.1	41.122	1.916	600.00
	n	27.9	0.09	27.9	50.9	0.824	600.00	9.2	0.58	9.667	22.25	1.302	600.00
S1.17	$\lceil\frac{n}{2}\rceil$	34.9	0.12	36.1	50.4	0.396	600.00	19.55	0.39	20.1	50.4	1.507	600.00
	$\lceil\frac{3n}{4}\rceil$	27.9	0.11	29.9	50.4	0.686	600.00	12.6	0.50	13.95	24.35	0.746	600.00
	n	28.2	0.10	30.3	50.4	0.663	600.00	9.267	0.39	10.233	23.6	1.306	600.00
S1.18	$\lceil\frac{n}{2}\rceil$	35.8	0.12	38.4	50.6	0.318	600.00	20.95	0.48	21.9	50.6	1.311	600.00
	$\lceil\frac{3n}{4}\rceil$	30.4	0.13	30.7	50.6	0.648	600.00	14.3	0.44	15.2	50.6	2.329	600.00
	n	29.7	0.07	30.8	50.6	0.643	600.00	10.033	0.82	10.6	23.35	1.203	600.00
S1.19	$\lceil\frac{n}{2}\rceil$	36.8	0.11	37.7	52	0.379	600.00	21.1	0.53	22.2	52	1.342	600.00
	$\lceil\frac{3n}{4}\rceil$	30.1	0.13	30.7	51.2	0.668	600.00	14.05	0.48	14.7	51.2	2.483	600.00
	n	29.7	0.09	31.5	51.2	0.625	600.00	9.9	0.36	10.567	19.9	0.883	600.00
S1.20	$\lceil\frac{n}{2}\rceil$	32.5	0.09	32.8	50.1	0.527	600.00	19.75	0.53	20.65	50.1	1.426	600.00
	$\lceil\frac{3n}{4}\rceil$	26.5	0.10	28.5	50.1	0.758	600.00	13.35	0.61	13.9	49.654	2.572	600.00
	n	26.4	0.13	28.8	50.1	0.740	600.00	9.033	0.89	9.267	41.41	3.469	600.00
S1.21	$\lceil\frac{n}{2}\rceil$	34.8	0.09	36.2	51.2	0.414	600.00	20.15	0.62	22.5	51.2	1.276	600.00
	$\lceil\frac{3n}{4}\rceil$	29.6	0.10	31.3	51.1	0.633	600.00	13.65	0.51	14.45	38.8	1.685	600.00

Table B.2 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	29.6	0.09	31.3	50.1	0.601	600.00	10.133	0.67	10.533	18.8	0.785	600.00
S1.22	$\lceil\frac{n}{2}\rceil$	32.1	0.10	33.1	50.4	0.523	600.00	18.5	0.87	20.2	50.4	1.495	600.00
	$\lceil\frac{3n}{4}\rceil$	25.7	0.09	27.1	50.4	0.860	600.00	12.45	0.64	13.15	50.4	2.833	600.00
	n	25.5	0.16	26.6	50.4	0.895	600.00	8.433	0.60	8.933	43.157	3.831	600.00
S1.23	$\lceil\frac{n}{2}\rceil$	34.7	0.09	34.7	50.8	0.464	600.00	19.15	0.28	20.8	45.1	1.168	600.00
	$\lceil\frac{3n}{4}\rceil$	27.9	0.10	29.7	50.8	0.710	600.00	13.35	0.48	13.8	23.2	0.681	600.00
	n	27.5	0.09	28.5	50.8	0.782	600.00	9.367	0.49	9.767	22.95	1.350	600.00
S1.24	$\lceil\frac{n}{2}\rceil$	35.5	0.15	36.2	50.8	0.403	600.00	19.5	0.54	21.15	50.8	1.402	600.00
	$\lceil\frac{3n}{4}\rceil$	27	0.16	30.8	50.8	0.649	600.00	12.6	0.59	13.3	50.8	2.820	600.00
	n	26.6	0.09	28.2	50.8	0.801	600.00	9.033	0.79	9.6	43.16	3.496	600.00
S1.25	$\lceil\frac{n}{2}\rceil$	37.6	0.09	39.8	51.1	0.284	600.00	21.7	0.75	22.6	51.1	1.261	600.00
	$\lceil\frac{3n}{4}\rceil$	29.3	0.09	30.8	51.1	0.659	600.00	14.35	0.56	14.75	51.1	2.464	600.00
	n	29.4	0.10	30.8	51.1	0.659	600.00	9.7	0.38	10.2	23.8	1.333	600.00
S1.26	$\lceil\frac{n}{2}\rceil$	30.7	0.11	35.2	50.9	0.446	600.00	17.4	0.38	19.3	23.2	0.202	600.00
	$\lceil\frac{3n}{4}\rceil$	26	0.10	30.2	50.9	0.685	600.00	11.35	0.53	11.85	21.2	0.789	600.00
	n	24.2	0.09	25.2	50.2	0.992	600.00	8.333	0.86	8.667	23.2	1.677	600.00
S1.27	$\lceil\frac{n}{2}\rceil$	32.4	0.09	33.8	45.9	0.358	600.00	19.6	0.31	21.85	22.35	0.023	600.00
	$\lceil\frac{3n}{4}\rceil$	26.9	0.09	29.3	50.5	0.724	600.00	12.15	0.38	13.2	20.15	0.527	600.00
	n	26.8	0.10	29.9	39.1	0.308	600.00	8.933	0.39	9.967	9.967	0.000	178.87
S1.28	$\lceil\frac{n}{2}\rceil$	35.3	0.11	35.8	50.1	0.399	600.00	18.2	0.46	20.3	50.1	1.468	600.00
	$\lceil\frac{3n}{4}\rceil$	26.7	0.15	28.7	50.1	0.746	600.00	12.75	0.53	14.2	50.1	2.528	600.00
	n	26.7	0.14	28.6	50.1	0.752	600.00	9.133	0.64	9.667	38.898	3.024	600.00
S1.29	$\lceil\frac{n}{2}\rceil$	34.4	0.11	38.8	50.2	0.294	600.00	19.8	0.65	21.45	50.2	1.340	600.00
	$\lceil\frac{3n}{4}\rceil$	29.6	0.12	30.4	50.2	0.651	600.00	14.4	0.64	15.55	50.2	2.228	600.00
	n	29.3	0.16	31.2	50.2	0.609	600.00	10.267	0.58	10.833	40.426	2.732	600.00
S1.30	$\lceil\frac{n}{2}\rceil$	34.2	0.13	36.3	50.3	0.386	600.00	20.05	0.58	21	50.3	1.395	600.00
	$\lceil\frac{3n}{4}\rceil$	27.5	0.10	27.8	50.3	0.809	600.00	12.35	0.54	14.15	50.3	2.555	600.00
	n	26.8	0.13	28.3	50.3	0.777	600.00	9.267	0.87	9.5	46.53	3.898	600.00
S1.31	$\lceil\frac{n}{2}\rceil$	33.8	0.13	36.3	41.566	0.145	600.00	19.05	0.34	22.1	26	0.176	600.00
	$\lceil\frac{3n}{4}\rceil$	27.3	0.13	29.7	35.7	0.202	600.00	12.35	0.46	13.75	17.7	0.287	600.00
	n	26.9	0.08	30.2	35.4	0.172	600.00	9.067	0.62	10.067	10.067	0.000	42.32
S1.32	$\lceil\frac{n}{2}\rceil$	33.9	0.20	35.7	50.6	0.417	600.00	19.4	0.37	20.45	50.6	1.474	600.00
	$\lceil\frac{3n}{4}\rceil$	27.3	0.12	29.8	50.3	0.688	600.00	13.55	0.49	14.9	44.596	1.993	600.00

Table B.2 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	27.6	0.08	28.7	50.3	0.753	600.00	9.233	0.62	9.867	30.594	2.101	600.00
S1.33	$\lceil\frac{n}{2}\rceil$	35.8	0.18	38.1	50.1	0.315	600.00	20.15	0.45	21.7	50.1	1.309	600.00
	$\lceil\frac{3n}{4}\rceil$	29.6	0.10	31.4	50.1	0.596	600.00	14.05	0.46	15.75	50.1	2.181	600.00
	n	30.1	0.13	31.3	50.1	0.601	600.00	9.933	0.48	10.5	41.378	2.941	600.00
S1.34	$\lceil\frac{n}{2}\rceil$	32.6	0.10	36.3	50.9	0.402	600.00	18.85	0.55	20.9	46.7	1.234	600.00
	$\lceil\frac{3n}{4}\rceil$	26.3	0.09	28	50.9	0.818	600.00	12	0.69	13.4	23.6	0.761	600.00
	n	27.4	0.11	28	50.5	0.804	600.00	8.933	0.41	9.333	22.8	1.443	600.00
S1.35	$\lceil\frac{n}{2}\rceil$	35.1	0.13	36.8	42.2	0.147	600.00	21.5	0.45	23.05	23.05	0.000	223.58
	$\lceil\frac{3n}{4}\rceil$	30	0.06	32.3	50.2	0.554	600.00	14.1	0.41	15.15	24.3	0.604	600.00
	n	30	0.06	32.3	42	0.300	600.00	10.1	0.57	10.767	11.267	0.046	600.00
S1.36	$\lceil\frac{n}{2}\rceil$	30.6	0.11	30.6	50.4	0.647	600.00	17.95	0.27	19.9	50.4	1.533	600.00
	$\lceil\frac{3n}{4}\rceil$	27.2	0.11	28.1	50.4	0.794	600.00	12.45	0.43	13.4	31.789	1.372	600.00
	n	26.4	0.09	28.3	42.4	0.498	600.00	8.867	0.47	9.467	11.167	0.180	600.00
S1.37	$\lceil\frac{n}{2}\rceil$	31.8	0.10	33.3	50.8	0.526	600.00	18.7	0.39	20.5	50.8	1.478	600.00
	$\lceil\frac{3n}{4}\rceil$	25.8	0.14	27.3	50.3	0.842	600.00	12.5	0.55	13.55	38.623	1.850	600.00
	n	25.5	0.09	27.3	50.3	0.842	600.00	8.5	0.62	9.467	23.5	1.482	600.00
S1.38	$\lceil\frac{n}{2}\rceil$	35.6	0.13	38	50.5	0.329	600.00	20.15	0.58	21.9	50.5	1.306	600.00
	$\lceil\frac{3n}{4}\rceil$	29.3	0.09	33.5	50.5	0.507	600.00	13.65	0.49	15.15	50.5	2.333	600.00
	n	29	0.14	31.3	50.3	0.607	600.00	9.933	0.65	10.567	43.557	3.122	600.00
S1.39	$\lceil\frac{n}{2}\rceil$	35.2	0.13	37.5	50.1	0.336	600.00	20.9	0.70	22.6	50.1	1.217	600.00
	$\lceil\frac{3n}{4}\rceil$	29.4	0.12	31.1	50.1	0.611	600.00	14.1	0.56	15.05	50.1	2.329	600.00
	n	29.6	0.17	31.1	50.1	0.611	600.00	9.833	1.06	10.5	48.572	3.626	600.00
S1.40	$\lceil\frac{n}{2}\rceil$	31.7	0.12	31.8	50.1	0.575	600.00	18.5	0.41	19.2	50.1	1.609	600.00
	$\lceil\frac{3n}{4}\rceil$	26.2	0.12	27.1	50.1	0.849	600.00	12.75	0.44	13.05	48	2.678	600.00
	n	25.7	0.10	26.1	50.1	0.920	600.00	8.667	0.48	9.2	19.55	1.125	600.00
S1.41	$\lceil\frac{n}{2}\rceil$	34.9	0.11	36.3	40.1	0.105	600.00	19.7	0.23	19.85	21.9	0.103	600.00
	$\lceil\frac{3n}{4}\rceil$	27.4	0.09	27.4	50.2	0.832	600.00	11.95	0.55	13.2	20.05	0.519	600.00
	n	27.2	0.13	28.6	41.2	0.441	600.00	9.133	0.89	9.833	16.55	0.683	600.00
S1.42	$\lceil\frac{n}{2}\rceil$	33.2	0.20	35.7	50.1	0.403	600.00	18.3	0.42	20.1	50.1	1.493	600.00
	$\lceil\frac{3n}{4}\rceil$	26.3	0.11	28	50.1	0.789	600.00	11.7	0.68	13.6	49.16	2.615	600.00
	n	25.6	0.11	27.1	50.1	0.849	600.00	8.5	0.47	9.233	36.893	2.996	600.00
S1.43	$\lceil\frac{n}{2}\rceil$	33.3	0.11	35.8	50.9	0.422	600.00	19.3	0.44	21.2	50.9	1.401	600.00
	$\lceil\frac{3n}{4}\rceil$	27.4	0.13	29.4	50.9	0.731	600.00	12.6	0.58	13.65	50.9	2.729	600.00

Table B.2 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	27.4	0.12	29.1	50.2	0.725	600.00	9.133	0.67	9.7	39.699	3.093	600.00
S1.44	$\lceil\frac{n}{2}\rceil$	32.9	0.11	34.7	50.1	0.444	600.00	20.4	0.50	20.4	50.1	1.456	600.00
	$\lceil\frac{3n}{4}\rceil$	27.7	0.11	29.3	50.1	0.710	600.00	12.35	0.45	13.3	34.63	1.604	600.00
	n	28.1	0.13	28.7	50.1	0.746	600.00	9.133	0.38	9.633	23.5	1.439	600.00
S1.45	$\lceil\frac{n}{2}\rceil$	28.4	0.06	29.6	50.8	0.716	600.00	16.7	0.24	19.1	41.139	1.154	600.00
	$\lceil\frac{3n}{4}\rceil$	23.4	0.07	23.9	50.3	1.105	600.00	10.8	0.37	12.45	41.7	2.349	600.00
	n	23.4	0.06	24.3	50.3	1.070	600.00	7.733	0.49	8.333	18.2	1.184	600.00
S1.46	$\lceil\frac{n}{2}\rceil$	30.9	0.09	34.3	50.1	0.461	600.00	19.65	0.34	20.9	50.1	1.397	600.00
	$\lceil\frac{3n}{4}\rceil$	28.5	0.13	29.6	50.1	0.693	600.00	13.35	0.55	14.8	50.1	2.385	600.00
	n	28.8	0.10	30.2	50.1	0.659	600.00	9.3	0.69	9.867	44.332	3.493	600.00
S1.47	$\lceil\frac{n}{2}\rceil$	32	0.09	33.1	50.8	0.535	600.00	16.75	0.45	19.3	50.8	1.632	600.00
	$\lceil\frac{3n}{4}\rceil$	24	0.08	25.6	50.5	0.973	600.00	11.85	0.79	12.05	49.229	3.085	600.00
	n	24.1	0.10	25.6	42.329	0.653	600.00	8	0.38	8.333	21.45	1.574	600.00
S1.48	$\lceil\frac{n}{2}\rceil$	34.3	0.09	35.8	51.8	0.447	600.00	20	0.62	20.65	51.8	1.508	600.00
	$\lceil\frac{3n}{4}\rceil$	27.9	0.14	30.3	50.5	0.667	600.00	13.2	0.54	14.15	50.5	2.569	600.00
	n	27.9	0.14	29	50.5	0.741	600.00	9.1	0.37	10.033	32.635	2.253	600.00
S1.49	$\lceil\frac{n}{2}\rceil$	32.5	0.16	34.9	50.5	0.447	600.00	18.1	0.50	19.75	50.5	1.557	600.00
	$\lceil\frac{3n}{4}\rceil$	25.7	0.09	29	50.5	0.741	600.00	12.8	1.16	13.2	50.5	2.826	600.00
	n	25.8	0.14	27.1	50.5	0.863	600.00	8.533	0.53	9.033	47.471	4.255	600.00

Table B.3: Detailed results for the series S3 and $r_{\max} = 2$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	15.7	0.20	15.7	15.7	0.000	283.37	7.2	0.42	7.2	7.2	0.000	7.86
	$\lceil\frac{3n}{4}\rceil$	15.5	0.20	15.5	15.5	0.000	200.63	4.05	0.51	4.05	4.05	0.000	52.54
	n	15.5	0.20	15.5	15.5	0.000	116.62	3.925	0.53	3.925	3.925	0.000	5.39
S1.1	$\lceil\frac{n}{2}\rceil$	16.4	0.21	17.6	17.6	0.000	143.33	7.2	0.46	7.2	7.2	0.000	52.91
	$\lceil\frac{3n}{4}\rceil$	16.4	0.21	17.6	17.6	0.000	156.71	4.6	0.49	4.6	4.6	0.000	10.18
	n	16.4	0.21	17.6	17.6	0.000	104.54	4.1	0.53	4.4	4.4	0.000	1.11
S1.2	$\lceil\frac{n}{2}\rceil$	17.8	0.20	17.9	17.9	0.000	195.40	9	0.53	9	9	0.000	45.04
	$\lceil\frac{3n}{4}\rceil$	17.8	0.22	17.9	17.9	0.000	200.35	6.65	0.53	6.65	6.65	0.000	22.57
	n	17.8	0.22	17.9	17.9	0.000	116.29	4.5	0.57	4.5	4.5	0.000	1.50
S1.3	$\lceil\frac{n}{2}\rceil$	11.6	0.21	11.6	41.138	2.546	600.00	5.45	0.42	5.45	5.45	0.000	79.11
	$\lceil\frac{3n}{4}\rceil$	11.6	0.21	11.6	41.039	2.538	600.00	3.067	0.48	3.067	3.067	0.000	25.44
	n	11.6	0.21	11.6	11.6	0.000	160.22	2.9	0.53	2.9	2.9	0.000	16.83
S1.4	$\lceil\frac{n}{2}\rceil$	14	0.21	14	14	0.000	23.88	4.95	0.40	4.95	4.95	0.000	20.50
	$\lceil\frac{3n}{4}\rceil$	14	0.21	14	14	0.000	13.61	4.15	0.48	4.15	4.15	0.000	1.65
	n	14	0.20	14	14	0.000	13.74	3.575	0.54	3.575	3.575	0.000	0.43
S1.5	$\lceil\frac{n}{2}\rceil$	14.1	0.21	14.1	36.1	1.560	600.00	7.5	0.47	7.5	7.5	0.000	71.28
	$\lceil\frac{3n}{4}\rceil$	14.1	0.21	14.1	36.1	1.560	600.00	4.7	0.57	4.7	4.7	0.000	56.48
	n	14.1	0.21	14.1	30.801	1.184	600.00	3.525	0.57	3.525	3.525	0.000	31.68
S1.6	$\lceil\frac{n}{2}\rceil$	19.4	0.21	20.7	20.7	0.000	367.06	11.2	0.47	11.2	11.2	0.000	145.17
	$\lceil\frac{3n}{4}\rceil$	18.3	0.22	20.7	20.7	0.000	425.02	7.05	0.56	7.4	7.4	0.000	175.02
	n	18.3	0.21	20.7	20.7	0.000	425.03	6.1	0.61	6.9	6.9	0.000	1.57
S1.7	$\lceil\frac{n}{2}\rceil$	20	0.21	20.7	20.7	0.000	214.01	9.133	0.49	9.7	9.7	0.000	70.37
	$\lceil\frac{3n}{4}\rceil$	20	0.21	20.7	20.7	0.000	413.57	7.05	0.59	7.05	7.05	0.000	144.51
	n	20	0.21	20.7	20.7	0.000	275.60	5.975	0.58	5.975	5.975	0.000	3.26
S1.8	$\lceil\frac{n}{2}\rceil$	23.3	0.29	24.5	50.4	1.057	600.00	12.45	0.86	12.45	22.25	0.787	600.00
	$\lceil\frac{3n}{4}\rceil$	23.2	0.26	24.2	48.2	0.992	600.00	8.667	0.77	8.667	17.35	1.002	600.00
	n	23.2	0.27	24.2	42.307	0.748	600.00	7.833	1.15	8.167	8.167	0.000	15.02
S1.9	$\lceil\frac{n}{2}\rceil$	4.4	0.19	4.4	4.4	0.000	6.43	2.2	0.30	2.55	2.55	0.000	7.62
	$\lceil\frac{3n}{4}\rceil$	4.4	0.19	4.4	4.4	0.000	5.31	1.1	0.36	1.2	1.2	0.000	21.78
	n	4.4	0.19	4.4	4.4	0.000	8.97	1.1	0.39	1.1	1.1	0.000	13.29
S1.10	$\lceil\frac{n}{2}\rceil$	11	0.21	11	11	0.000	143.84	4.4	0.39	4.4	4.4	0.000	78.68
	$\lceil\frac{3n}{4}\rceil$	11	0.21	11	11	0.000	170.66	3.667	0.46	3.667	3.667	0.000	14.46

Table B.3 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	11	0.20	11	30.814	1.801	600.00	2.75	0.51	2.75	2.75	0.000	55.27
S1.11	$\lceil \frac{n}{2} \rceil$	13	0.23	13	13	0.000	172.07	6.6	0.47	6.6	6.6	0.000	135.07
	$\lceil \frac{3n}{4} \rceil$	13	0.23	13	13	0.000	222.41	4.625	0.60	4.95	4.95	0.000	29.05
	n	13	0.23	13	13	0.000	229.60	3.25	0.55	3.25	3.25	0.000	31.30
S1.12	$\lceil \frac{n}{2} \rceil$	7.3	0.20	9.8	9.8	0.000	23.23	4.333	0.36	4.7	4.7	0.000	7.17
	$\lceil \frac{3n}{4} \rceil$	7.3	0.20	9.8	9.8	0.000	38.51	3.1	0.42	3.25	3.25	0.000	0.34
	n	7.3	0.20	9.8	9.8	0.000	10.85	1.825	0.43	2.45	2.45	0.000	4.59
S1.13	$\lceil \frac{n}{2} \rceil$	11.2	0.20	11.6	11.6	0.000	224.02	4.5	0.39	4.5	4.5	0.000	46.47
	$\lceil \frac{3n}{4} \rceil$	11.2	0.20	11.6	11.6	0.000	95.06	3.733	0.46	3.85	3.85	0.000	10.55
	n	11.2	0.20	11.6	11.6	0.000	156.15	2.8	0.47	2.9	2.9	0.000	11.00
S1.14	$\lceil \frac{n}{2} \rceil$	18.4	0.23	18.4	21.9	0.190	600.00	9.2	0.53	9.2	9.2	0.000	190.33
	$\lceil \frac{3n}{4} \rceil$	18.4	0.22	18.4	18.4	0.000	582.76	7.7	0.63	7.7	7.7	0.000	25.98
	n	18.4	0.22	18.4	18.4	0.000	340.20	6.133	0.63	6.133	6.133	0.000	2.66
S1.15	$\lceil \frac{n}{2} \rceil$	12.9	0.21	12.9	20.3	0.574	600.00	8.867	0.46	10.5	10.5	0.000	50.24
	$\lceil \frac{3n}{4} \rceil$	12.9	0.21	12.9	20.3	0.574	600.00	5.2	0.53	5.2	5.2	0.000	19.54
	n	12.9	0.21	12.9	12.9	0.000	228.76	3.325	0.56	3.325	3.325	0.000	7.34
S1.16	$\lceil \frac{n}{2} \rceil$	14.1	0.21	14.1	14.1	0.000	191.68	7.05	0.46	7.05	7.05	0.000	146.57
	$\lceil \frac{3n}{4} \rceil$	12.2	0.22	12.2	12.2	0.000	294.60	6.1	0.52	6.1	6.1	0.000	4.94
	n	12.2	0.21	12.2	12.2	0.000	255.88	3.05	0.51	3.05	3.05	0.000	24.13
S1.17	$\lceil \frac{n}{2} \rceil$	12.2	0.24	13.7	13.7	0.000	61.08	6.75	0.57	6.75	6.75	0.000	51.03
	$\lceil \frac{3n}{4} \rceil$	9.8	0.22	13.7	13.7	0.000	54.55	3.267	0.49	4.733	4.733	0.000	67.04
	n	9.8	0.22	13.7	13.7	0.000	50.27	2.45	0.51	3.55	3.55	0.000	9.01
S1.18	$\lceil \frac{n}{2} \rceil$	19.9	0.20	20.9	20.9	0.000	558.69	8.933	0.49	10.5	10.5	0.000	105.09
	$\lceil \frac{3n}{4} \rceil$	19.6	0.22	20.9	20.9	0.000	374.86	6.633	0.56	6.633	6.633	0.000	5.20
	n	19.6	0.20	20.9	20.9	0.000	437.28	5.6	0.55	6.175	6.175	0.000	3.47
S1.19	$\lceil \frac{n}{2} \rceil$	13.7	0.21	13.7	13.7	0.000	230.31	7.867	0.49	7.867	7.867	0.000	88.04
	$\lceil \frac{3n}{4} \rceil$	13.7	0.21	13.7	13.7	0.000	278.80	4.567	0.53	4.567	4.567	0.000	20.80
	n	13.7	0.21	13.7	13.7	0.000	156.60	3.425	0.53	3.425	3.425	0.000	23.32
S1.20	$\lceil \frac{n}{2} \rceil$	6.6	0.20	6.6	6.6	0.000	10.16	3.25	0.36	3.25	3.25	0.000	5.15
	$\lceil \frac{3n}{4} \rceil$	6.6	0.20	6.6	6.6	0.000	35.56	2.933	0.43	2.933	2.933	0.000	6.29
	n	6.6	0.20	6.6	6.6	0.000	22.50	1.65	0.43	1.65	1.65	0.000	9.53
S1.21	$\lceil \frac{n}{2} \rceil$	15.6	0.20	15.6	31.083	0.993	600.00	6.6	0.42	6.85	6.85	0.000	64.83
	$\lceil \frac{3n}{4} \rceil$	13.3	0.20	13.3	13.3	0.000	64.22	4.15	0.45	4.15	4.15	0.000	5.11

Table B.3 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	13.3	0.20	13.3	13.3	0.000	73.11	3.9	0.49	3.9	3.9	0.000	0.45
S1.22	$\lceil\frac{n}{2}\rceil$	12.6	0.20	12.6	12.6	0.000	125.57	5.133	0.43	5.133	5.133	0.000	51.23
	$\lceil\frac{3n}{4}\rceil$	12.6	0.21	12.6	12.6	0.000	256.70	4.2	0.47	4.2	4.2	0.000	43.24
	n	12.6	0.20	12.6	12.6	0.000	148.70	3.15	0.49	3.15	3.15	0.000	1.19
S1.23	$\lceil\frac{n}{2}\rceil$	10.2	0.19	10.2	10.2	0.000	54.66	4.35	0.41	4.35	4.35	0.000	54.79
	$\lceil\frac{3n}{4}\rceil$	10.2	0.20	10.2	10.2	0.000	94.82	3.05	0.44	3.05	3.05	0.000	15.98
	n	10.2	0.20	10.2	10.2	0.000	59.18	2.55	0.46	2.55	2.55	0.000	12.18
S1.24	$\lceil\frac{n}{2}\rceil$	10.9	0.20	12.3	12.3	0.000	75.85	4.8	0.40	4.8	4.8	0.000	15.21
	$\lceil\frac{3n}{4}\rceil$	10.9	0.20	12.3	12.3	0.000	80.70	3.15	0.47	3.15	3.15	0.000	35.60
	n	10.9	0.21	12.3	12.3	0.000	79.59	2.725	0.50	3.075	3.075	0.000	36.97
S1.25	$\lceil\frac{n}{2}\rceil$	17.6	0.33	17.6	21	0.193	600.00	7.667	0.50	9	9	0.000	66.94
	$\lceil\frac{3n}{4}\rceil$	16.9	0.24	17.6	17.6	0.000	248.21	6.533	0.57	7.367	7.367	0.000	23.30
	n	16.9	0.24	17.6	17.6	0.000	467.60	4.225	0.64	5.175	5.175	0.000	7.88
S1.26	$\lceil\frac{n}{2}\rceil$	14	0.22	14	14	0.000	156.54	6.55	0.47	6.55	6.55	0.000	109.78
	$\lceil\frac{3n}{4}\rceil$	14	0.23	14	14	0.000	197.93	4.667	0.54	4.667	4.667	0.000	31.51
	n	14	0.22	14	14	0.000	126.02	4.667	0.56	4.667	4.667	0.000	1.00
S1.27	$\lceil\frac{n}{2}\rceil$	11.7	0.21	11.7	11.7	0.000	218.68	9.3	0.43	9.3	9.3	0.000	13.97
	$\lceil\frac{3n}{4}\rceil$	11.7	0.21	11.7	11.7	0.000	96.64	4.05	0.48	4.05	4.05	0.000	33.49
	n	11.7	0.21	11.7	11.7	0.000	139.63	2.925	0.53	2.925	2.925	0.000	27.70
S1.28	$\lceil\frac{n}{2}\rceil$	7.1	0.19	7.1	7.1	0.000	13.15	3.35	0.34	3.35	3.35	0.000	4.98
	$\lceil\frac{3n}{4}\rceil$	7.1	0.19	7.1	7.1	0.000	4.91	1.775	0.39	1.775	1.775	0.000	6.78
	n	7.1	0.19	7.1	7.1	0.000	4.48	1.775	0.44	1.775	1.775	0.000	0.48
S1.29	$\lceil\frac{n}{2}\rceil$	11.2	0.20	11.2	11.2	0.000	251.10	5.6	0.43	5.6	5.6	0.000	45.58
	$\lceil\frac{3n}{4}\rceil$	11.2	0.20	11.2	11.2	0.000	141.61	3.4	0.47	3.4	3.4	0.000	38.11
	n	11.2	0.20	11.2	11.2	0.000	379.18	2.35	0.48	2.35	2.35	0.000	25.89
S1.30	$\lceil\frac{n}{2}\rceil$	18.4	0.22	20.9	36.5	0.746	600.00	8.933	0.49	10.1	10.1	0.000	258.71
	$\lceil\frac{3n}{4}\rceil$	18.4	0.22	20.9	28.989	0.387	600.00	8	0.58	8.7	8.7	0.000	38.39
	n	18.4	0.22	20.9	21	0.005	600.00	6.133	0.60	6.967	6.967	0.000	7.54
S1.31	$\lceil\frac{n}{2}\rceil$	18.4	0.34	18.4	18.4	0.000	268.25	7.667	0.46	8.1	8.1	0.000	202.77
	$\lceil\frac{3n}{4}\rceil$	15.7	0.21	16.1	16.1	0.000	301.79	5.267	0.52	6.4	6.4	0.000	13.89
	n	15.7	0.21	16.1	23.1	0.435	600.00	5.133	0.54	5.133	5.133	0.000	1.53
S1.32	$\lceil\frac{n}{2}\rceil$	14	0.20	14	14	0.000	154.35	6.9	0.45	6.9	6.9	0.000	15.80
	$\lceil\frac{3n}{4}\rceil$	14	0.25	14	14	0.000	161.69	4.667	0.62	4.667	4.667	0.000	9.74

Table B.3 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	14	0.24	14	14	0.000	90.25	3.5	0.75	3.5	3.5	0.000	1.84
S1.33	$\lceil\frac{n}{2}\rceil$	13.8	0.20	13.8	13.8	0.000	108.72	6.367	0.42	6.367	6.367	0.000	52.76
	$\lceil\frac{3n}{4}\rceil$	12.4	0.21	12.7	12.7	0.000	286.79	3.767	0.72	3.767	3.767	0.000	35.71
	n	12.4	0.21	12.7	12.7	0.000	257.96	3.175	0.66	3.175	3.175	0.000	36.17
S1.34	$\lceil\frac{n}{2}\rceil$	5.7	0.19	6.2	6.2	0.000	2.76	2.467	0.33	2.467	2.467	0.000	2.22
	$\lceil\frac{3n}{4}\rceil$	5.7	0.20	6.2	6.2	0.000	3.24	2	0.38	2	2	0.000	0.73
	n	5.7	0.20	6.2	6.2	0.000	2.00	1.425	0.43	1.55	1.55	0.000	1.22
S1.35	$\lceil\frac{n}{2}\rceil$	9.5	0.20	9.5	9.5	0.000	133.79	6.033	0.41	6.067	6.067	0.000	11.70
	$\lceil\frac{3n}{4}\rceil$	9.5	0.21	9.5	9.5	0.000	124.86	4	0.47	4	4	0.000	5.72
	n	9.5	0.20	9.5	9.5	0.000	95.38	2.375	0.47	2.375	2.375	0.000	25.54
S1.36	$\lceil\frac{n}{2}\rceil$	8.3	0.19	8.3	8.3	0.000	28.09	4.15	0.37	4.15	4.15	0.000	27.71
	$\lceil\frac{3n}{4}\rceil$	8.3	0.19	8.3	8.3	0.000	28.01	3.833	0.42	3.833	3.833	0.000	9.73
	n	8.3	0.20	8.3	8.3	0.000	15.68	2.075	0.45	2.075	2.075	0.000	3.47
S1.37	$\lceil\frac{n}{2}\rceil$	18.3	0.24	18.3	35.178	0.922	600.00	11.1	0.70	11.1	12.2	0.099	600.00
	$\lceil\frac{3n}{4}\rceil$	18.3	0.24	18.3	44.739	1.445	600.00	7.667	0.86	7.767	7.767	0.000	87.97
	n	18.3	0.23	18.3	38.076	1.081	600.00	6.1	0.65	6.1	6.1	0.000	34.64
S1.38	$\lceil\frac{n}{2}\rceil$	17.4	0.22	17.4	17.4	0.000	329.71	8.7	0.49	8.7	8.7	0.000	240.52
	$\lceil\frac{3n}{4}\rceil$	17.4	0.22	17.4	17.4	0.000	349.21	7.1	0.58	7.833	7.833	0.000	55.25
	n	17.4	0.21	17.4	17.4	0.000	216.49	4.95	0.57	4.95	4.95	0.000	1.73
S1.39	$\lceil\frac{n}{2}\rceil$	18	0.21	20.3	22	0.084	600.00	8.8	0.51	8.8	10.65	0.210	600.00
	$\lceil\frac{3n}{4}\rceil$	18	0.21	18	19.3	0.072	600.00	6.833	0.59	6.833	6.833	0.000	54.34
	n	18	0.21	18	18	0.000	497.22	6	0.61	6	6	0.000	5.60
S1.40	$\lceil\frac{n}{2}\rceil$	15.7	0.22	15.7	15.7	0.000	357.22	7.85	0.46	7.85	7.85	0.000	67.58
	$\lceil\frac{3n}{4}\rceil$	15.7	0.22	15.7	15.7	0.000	104.01	4.675	0.53	4.675	4.675	0.000	43.32
	n	15.7	0.21	15.7	15.7	0.000	107.97	4.675	0.55	4.675	4.675	0.000	0.93
S1.41	$\lceil\frac{n}{2}\rceil$	7.6	0.21	7.6	7.6	0.000	124.57	3	0.40	3	3	0.000	57.29
	$\lceil\frac{3n}{4}\rceil$	7.6	0.21	7.6	7.6	0.000	70.50	2.8	0.47	2.9	2.9	0.000	69.74
	n	7.6	0.21	7.6	7.6	0.000	134.05	1.9	0.50	1.9	1.9	0.000	16.66
S1.42	$\lceil\frac{n}{2}\rceil$	13.9	0.22	13.9	13.9	0.000	52.61	8.1	0.51	8.1	8.1	0.000	30.38
	$\lceil\frac{3n}{4}\rceil$	13.9	0.21	13.9	13.9	0.000	72.56	5.567	0.63	5.8	5.8	0.000	2.92
	n	13.9	0.21	13.9	13.9	0.000	33.79	4.633	0.55	4.633	4.633	0.000	0.70
S1.43	$\lceil\frac{n}{2}\rceil$	8	0.20	8.7	8.7	0.000	27.83	3.8	0.37	4	4	0.000	72.71
	$\lceil\frac{3n}{4}\rceil$	8	0.20	8.7	8.7	0.000	51.66	2.433	0.42	2.433	2.433	0.000	22.80

Table B.3 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	8	0.21	8.7	8.7	0.000	25.18	2	0.48	2.175	2.175	0.000	23.47
S1.44	$\lceil \frac{n}{2} \rceil$	8.6	0.20	9.6	9.6	0.000	46.45	3.533	0.35	3.533	3.533	0.000	39.69
	$\lceil \frac{3n}{4} \rceil$	8.6	0.20	9.6	9.6	0.000	88.64	2.65	0.43	2.65	2.65	0.000	67.45
	n	8.6	0.20	9.6	9.6	0.000	64.33	2.15	0.44	2.4	2.4	0.000	16.76
S1.45	$\lceil \frac{n}{2} \rceil$	10.6	0.22	11	11	0.000	199.26	8.3	0.44	8.7	8.7	0.000	9.67
	$\lceil \frac{3n}{4} \rceil$	10.6	0.22	11	11	0.000	62.64	3.533	0.50	3.533	3.533	0.000	21.97
	n	10.6	0.21	11	11	0.000	484.67	2.65	0.53	2.75	2.75	0.000	52.31
S1.46	$\lceil \frac{n}{2} \rceil$	8.6	0.21	8.9	8.9	0.000	64.43	3.85	0.40	4.133	4.133	0.000	7.68
	$\lceil \frac{3n}{4} \rceil$	8.6	0.21	8.9	8.9	0.000	31.06	2.967	0.47	2.967	2.967	0.000	5.48
	n	8.6	0.21	8.9	8.9	0.000	61.83	2.15	0.48	2.225	2.225	0.000	6.90
S1.47	$\lceil \frac{n}{2} \rceil$	8.5	0.19	8.5	8.5	0.000	22.75	2.833	0.36	2.833	2.833	0.000	45.99
	$\lceil \frac{3n}{4} \rceil$	8.5	0.19	8.5	8.5	0.000	21.88	2.175	0.40	2.175	2.175	0.000	17.04
	n	8.5	0.19	8.5	8.5	0.000	24.21	2.125	0.42	2.125	2.125	0.000	5.55
S1.48	$\lceil \frac{n}{2} \rceil$	15.8	0.21	15.8	15.8	0.000	297.10	6.9	0.48	8.6	8.6	0.000	73.17
	$\lceil \frac{3n}{4} \rceil$	15.8	0.20	15.8	16.5	0.044	600.00	4.45	0.54	4.45	4.45	0.000	72.79
	n	15.8	0.20	15.8	15.8	0.000	158.66	3.95	0.55	3.95	3.95	0.000	1.13
S1.49	$\lceil \frac{n}{2} \rceil$	7.3	0.19	7.3	7.3	0.000	15.17	4.55	0.34	4.8	4.8	0.000	3.88
	$\lceil \frac{3n}{4} \rceil$	6.2	0.20	7.1	7.1	0.000	25.80	2.85	0.42	2.85	2.85	0.000	6.19
	n	6.2	0.20	7.1	7.1	0.000	11.18	1.55	0.42	1.775	1.775	0.000	13.28

Table B.4: Detailed results for the series S1 and $r_{\max} = 3$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	34.7	0.05	34.7	34.7	0.000	26.99	22.3	0.16	22.3	22.3	0.000	10.54
	$\lceil\frac{3n}{4}\rceil$	34.7	0.05	34.7	34.7	0.000	38.41	19.8	0.18	19.8	19.8	0.000	1.83
	n	34.7	0.05	34.7	34.7	0.000	54.72	17.35	0.17	17.35	17.35	0.000	0.02
S1.1	$\lceil\frac{n}{2}\rceil$	36	0.06	38.6	38.6	0.000	19.86	21.35	0.27	22.25	22.25	0.000	6.33
	$\lceil\frac{3n}{4}\rceil$	35.8	0.07	37	37	0.000	51.38	18.45	0.31	19.4	19.4	0.000	4.38
	n	35.7	0.07	37	37	0.000	39.79	17.9	0.41	18.5	18.5	0.000	0.27
S1.2	$\lceil\frac{n}{2}\rceil$	36.5	0.06	38	38	0.000	24.86	29.3	0.18	31.5	31.5	0.000	12.32
	$\lceil\frac{3n}{4}\rceil$	36.5	0.06	36.7	36.7	0.000	23.91	20.55	0.25	21.1	21.1	0.000	2.81
	n	36.5	0.05	36.7	36.7	0.000	40.13	18.25	0.23	18.35	18.35	0.000	0.04
S1.3	$\lceil\frac{n}{2}\rceil$	41.2	0.07	41.2	41.2	0.000	56.84	36	0.21	36	36	0.000	25.04
	$\lceil\frac{3n}{4}\rceil$	40.9	0.13	41.1	41.1	0.000	66.05	22.4	0.29	22.4	22.4	0.000	29.61
	n	40.4	0.09	41.1	41.1	0.000	30.11	19.9	0.37	20.55	20.55	0.000	0.98
S1.4	$\lceil\frac{n}{2}\rceil$	36	0.05	37.2	37.2	0.000	35.15	32.9	0.24	33.1	33.1	0.000	2.11
	$\lceil\frac{3n}{4}\rceil$	36	0.05	36	36	0.000	25.08	18.75	0.20	19.45	19.45	0.000	2.05
	n	34.8	0.05	35.3	35.3	0.000	31.72	17.4	0.20	17.65	17.65	0.000	0.07
S1.5	$\lceil\frac{n}{2}\rceil$	39.7	0.06	40.2	40.2	0.000	54.01	34	0.26	38.9	38.9	0.000	2.48
	$\lceil\frac{3n}{4}\rceil$	39.7	0.07	40.2	40.2	0.000	59.61	20.45	0.21	20.45	20.45	0.000	2.13
	n	39.7	0.05	40.2	40.2	0.000	53.85	19.85	0.22	20.1	20.1	0.000	0.33
S1.6	$\lceil\frac{n}{2}\rceil$	37.8	0.05	37.8	37.8	0.000	23.93	28.6	0.17	30.3	30.3	0.000	11.97
	$\lceil\frac{3n}{4}\rceil$	36.1	0.05	36.1	36.1	0.000	49.21	18.05	0.18	18.05	18.05	0.000	2.07
	n	36.1	0.04	36.1	36.1	0.000	26.25	18.05	0.18	18.05	18.05	0.000	0.02
S1.7	$\lceil\frac{n}{2}\rceil$	40.8	0.09	41	41	0.000	25.83	36.3	0.22	38.3	38.3	0.000	13.86
	$\lceil\frac{3n}{4}\rceil$	40.2	0.11	40.2	40.2	0.000	71.12	20.65	0.25	20.7	20.7	0.000	25.39
	n	40.2	0.10	40.2	40.2	0.000	36.70	19.75	0.36	20.1	20.1	0.000	0.43
S1.8	$\lceil\frac{n}{2}\rceil$	35.5	0.09	39.1	39.1	0.000	30.25	22.7	0.25	28	28	0.000	19.70
	$\lceil\frac{3n}{4}\rceil$	35.1	0.07	35.1	35.1	0.000	52.68	21.05	0.38	21.05	21.05	0.000	1.94
	n	35.1	0.06	35.1	35.1	0.000	54.53	17.55	0.21	17.55	17.55	0.000	0.05
S1.9	$\lceil\frac{n}{2}\rceil$	23.7	0.04	24.4	24.4	0.000	45.04	12.9	0.12	12.9	12.9	0.000	11.15
	$\lceil\frac{3n}{4}\rceil$	23.7	0.04	24.4	24.4	0.000	54.62	10.467	0.14	10.467	10.467	0.000	10.67
	n	23.7	0.04	24.4	24.4	0.000	40.70	7.9	0.15	8.133	8.133	0.000	0.08
S1.10	$\lceil\frac{n}{2}\rceil$	30.3	0.05	30.3	30.3	0.000	70.51	25.7	0.16	28.8	28.8	0.000	3.16
	$\lceil\frac{3n}{4}\rceil$	30.3	0.05	30.3	30.3	0.000	42.61	17.8	0.15	17.8	17.8	0.000	3.97

Table B.4 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	30.3	0.05	30.3	30.3	0.000	36.69	10.1	0.17	10.1	10.1	0.000	0.16
S1.11	$\lceil \frac{n}{2} \rceil$	26.6	0.04	26.8	26.8	0.000	37.35	13.3	0.13	13.3	13.3	0.000	2.57
	$\lceil \frac{3n}{4} \rceil$	26.6	0.04	26.8	26.8	0.000	42.02	10.833	0.14	10.833	10.833	0.000	4.80
	n	26.6	0.04	26.8	26.8	0.000	9.73	8.867	0.15	8.933	8.933	0.000	0.10
S1.12	$\lceil \frac{n}{2} \rceil$	36.9	0.06	37.3	37.3	0.000	23.36	22.55	0.19	22.55	22.55	0.000	5.49
	$\lceil \frac{3n}{4} \rceil$	36.2	0.06	36.5	36.5	0.000	39.93	19.2	0.31	19.2	19.2	0.000	4.07
	n	36.2	0.06	36.2	36.2	0.000	28.12	18.1	0.27	18.1	18.1	0.000	0.40
S1.13	$\lceil \frac{n}{2} \rceil$	24.3	0.04	25.7	25.7	0.000	21.30	17.3	0.13	17.3	17.3	0.000	1.88
	$\lceil \frac{3n}{4} \rceil$	24.3	0.04	25.7	25.7	0.000	29.17	10.2	0.15	12.15	12.15	0.000	9.62
	n	24.3	0.04	25.7	25.7	0.000	39.12	8.1	0.15	8.567	8.567	0.000	0.18
S1.14	$\lceil \frac{n}{2} \rceil$	40	0.08	40.4	40.4	0.000	37.26	23.5	0.25	24.05	24.05	0.000	2.76
	$\lceil \frac{3n}{4} \rceil$	36.4	0.08	39.3	39.3	0.000	70.42	20.25	0.22	20.5	20.5	0.000	1.06
	n	35.9	0.05	39.3	39.3	0.000	83.20	17.95	0.23	19.65	19.65	0.000	0.04
S1.15	$\lceil \frac{n}{2} \rceil$	36.5	0.06	36.9	36.9	0.000	21.16	19.85	0.18	19.85	19.85	0.000	4.54
	$\lceil \frac{3n}{4} \rceil$	36.5	0.06	36.5	36.5	0.000	24.80	19.85	0.27	19.85	19.85	0.000	0.74
	n	36.5	0.05	36.5	36.5	0.000	32.84	18.25	0.24	18.25	18.25	0.000	0.29
S1.16	$\lceil \frac{n}{2} \rceil$	38.9	0.06	38.9	38.9	0.000	49.17	37	0.21	37	37	0.000	7.06
	$\lceil \frac{3n}{4} \rceil$	38.9	0.10	38.9	38.9	0.000	55.15	19.65	0.26	19.65	19.65	0.000	8.55
	n	38.6	0.10	38.9	38.9	0.000	37.81	19.45	0.26	19.45	19.45	0.000	0.38
S1.17	$\lceil \frac{n}{2} \rceil$	36.2	0.05	36.2	36.2	0.000	27.34	36.1	0.19	36.1	36.1	0.000	3.42
	$\lceil \frac{3n}{4} \rceil$	36.2	0.05	36.2	36.2	0.000	41.57	18.1	0.19	18.1	18.1	0.000	2.10
	n	36.2	0.05	36.2	36.2	0.000	32.42	18.1	0.20	18.1	18.1	0.000	0.04
S1.18	$\lceil \frac{n}{2} \rceil$	39.2	0.07	39.8	39.8	0.000	49.94	21.85	0.18	21.85	21.85	0.000	2.61
	$\lceil \frac{3n}{4} \rceil$	39.2	0.10	39.8	39.8	0.000	37.25	20.55	0.33	20.75	20.75	0.000	3.57
	n	39.2	0.07	39.8	39.8	0.000	26.19	19.6	0.25	19.9	19.9	0.000	0.17
S1.19	$\lceil \frac{n}{2} \rceil$	40.4	0.06	41.3	41.3	0.000	30.65	21.6	0.17	21.6	21.6	0.000	4.81
	$\lceil \frac{3n}{4} \rceil$	40.2	0.07	40.6	40.6	0.000	51.83	20.7	0.21	20.7	20.7	0.000	8.71
	n	40.2	0.07	40.6	40.6	0.000	60.02	20.1	0.23	20.3	20.3	0.000	0.13
S1.20	$\lceil \frac{n}{2} \rceil$	37.1	0.10	38.4	50.1	0.305	600.00	35.8	0.27	36	36	0.000	53.70
	$\lceil \frac{3n}{4} \rceil$	36.8	0.08	37.3	44.4	0.190	600.00	19.55	0.40	20.2	20.2	0.000	139.96
	n	36.8	0.08	37.3	37.3	0.000	81.01	18.55	0.33	18.65	18.65	0.000	1.43
S1.21	$\lceil \frac{n}{2} \rceil$	39.1	0.07	40.2	40.2	0.000	35.90	37.6	0.36	38	38	0.000	55.15
	$\lceil \frac{3n}{4} \rceil$	38.8	0.06	38.8	38.8	0.000	41.56	19.65	0.28	20.1	20.1	0.000	11.79

Table B.4 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	38.8	0.06	38.8	38.8	0.000	71.58	19.4	0.26	19.4	19.4	0.000	0.05
S1.22	$\lceil\frac{n}{2}\rceil$	38.6	0.06	38.6	38.6	0.000	22.46	34.3	0.30	34.3	34.3	0.000	5.95
	$\lceil\frac{3n}{4}\rceil$	38.1	0.06	38.1	38.1	0.000	69.61	19.3	0.23	19.3	19.3	0.000	2.29
	n	37	0.07	37.8	37.8	0.000	20.01	18.9	0.30	18.9	18.9	0.000	0.27
S1.23	$\lceil\frac{n}{2}\rceil$	38.4	0.08	39.1	39.1	0.000	97.44	32.4	0.17	32.4	32.4	0.000	1.51
	$\lceil\frac{3n}{4}\rceil$	38.3	0.08	38.4	47.011	0.224	600.00	21.2	0.22	21.35	21.35	0.000	1.17
	n	38	0.07	38.3	38.3	0.000	86.98	19	0.33	19.15	19.15	0.000	1.18
S1.24	$\lceil\frac{n}{2}\rceil$	39.5	0.06	39.6	39.6	0.000	51.61	34.7	0.21	36.7	36.7	0.000	6.76
	$\lceil\frac{3n}{4}\rceil$	39.1	0.09	39.6	39.6	0.000	33.07	20	0.24	20.4	20.4	0.000	1.63
	n	38.5	0.06	38.5	38.5	0.000	37.34	19.25	0.25	19.25	19.25	0.000	0.19
S1.25	$\lceil\frac{n}{2}\rceil$	40.5	0.07	40.5	40.5	0.000	44.34	36.7	0.25	38.3	38.3	0.000	3.53
	$\lceil\frac{3n}{4}\rceil$	40.1	0.06	40.5	40.5	0.000	64.33	21.35	0.23	21.35	21.35	0.000	3.24
	n	39.8	0.05	40.3	40.3	0.000	31.58	20	0.29	20.15	20.15	0.000	0.22
S1.26	$\lceil\frac{n}{2}\rceil$	21.2	0.04	21.3	21.3	0.000	19.66	13.7	0.14	15.85	15.85	0.000	3.05
	$\lceil\frac{3n}{4}\rceil$	21.2	0.04	21.3	21.3	0.000	40.54	11.35	0.14	13.25	13.25	0.000	1.62
	n	21.2	0.04	21.3	21.3	0.000	22.87	7.067	0.16	7.1	7.1	0.000	0.71
S1.27	$\lceil\frac{n}{2}\rceil$	39.1	0.06	40.3	40.3	0.000	73.87	36.2	0.28	36.2	36.2	0.000	18.63
	$\lceil\frac{3n}{4}\rceil$	37.5	0.06	38.5	38.5	0.000	46.30	21.3	0.24	21.4	21.4	0.000	8.15
	n	37.5	0.07	38.1	38.1	0.000	44.65	18.6	0.23	19.05	19.05	0.000	0.15
S1.28	$\lceil\frac{n}{2}\rceil$	40.7	0.09	40.7	40.7	0.000	148.67	35.8	0.37	37.2	37.2	0.000	13.74
	$\lceil\frac{3n}{4}\rceil$	38.1	0.09	38.1	38.1	0.000	52.42	20.35	0.29	20.45	20.45	0.000	19.86
	n	38	0.14	38	38	0.000	31.30	18.85	0.43	19	19	0.000	0.72
S1.29	$\lceil\frac{n}{2}\rceil$	40.3	0.09	40.8	40.8	0.000	75.34	38.1	0.26	38.6	38.6	0.000	6.09
	$\lceil\frac{3n}{4}\rceil$	38.8	0.06	39.3	39.3	0.000	85.05	21.95	0.36	21.95	21.95	0.000	4.26
	n	38.8	0.06	38.8	38.8	0.000	43.60	19.4	0.24	19.4	19.4	0.000	0.05
S1.30	$\lceil\frac{n}{2}\rceil$	39.6	0.05	40.4	40.4	0.000	37.21	33.3	0.19	33.3	33.3	0.000	5.37
	$\lceil\frac{3n}{4}\rceil$	36.3	0.06	36.3	36.3	0.000	67.11	19.1	0.20	20	20	0.000	7.68
	n	36.3	0.05	36.3	36.3	0.000	20.19	18.15	0.22	18.15	18.15	0.000	0.05
S1.31	$\lceil\frac{n}{2}\rceil$	38	0.08	40.4	40.4	0.000	38.43	35.6	0.18	35.7	35.7	0.000	5.80
	$\lceil\frac{3n}{4}\rceil$	37.8	0.09	38.4	38.4	0.000	80.12	19.85	0.23	20.2	20.2	0.000	0.60
	n	36.8	0.06	37.8	37.8	0.000	60.77	18.9	0.41	18.9	18.9	0.000	0.05
S1.32	$\lceil\frac{n}{2}\rceil$	39.8	0.07	40.2	40.2	0.000	186.51	33.3	0.28	34.3	34.3	0.000	25.24
	$\lceil\frac{3n}{4}\rceil$	37.5	0.07	37.5	37.6	0.003	600.00	20.45	0.43	20.45	20.45	0.000	33.52

Table B.4 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	37.5	0.07	37.5	37.5	0.000	71.65	18.75	0.27	18.75	18.75	0.000	1.44
S1.33	$\lceil\frac{n}{2}\rceil$	34.3	0.05	34.3	34.3	0.000	33.38	25.1	0.15	28.1	28.1	0.000	1.87
	$\lceil\frac{3n}{4}\rceil$	34.1	0.05	34.1	34.1	0.000	31.73	20.2	0.26	20.45	20.45	0.000	1.53
	n	34.1	0.05	34.1	34.1	0.000	27.19	17.05	0.19	17.05	17.05	0.000	0.03
S1.34	$\lceil\frac{n}{2}\rceil$	40.5	0.07	40.5	40.5	0.000	24.22	36	0.30	37.1	37.1	0.000	4.45
	$\lceil\frac{3n}{4}\rceil$	38.2	0.07	39	39	0.000	200.68	21.4	0.48	21.4	21.4	0.000	12.57
	n	37.7	0.07	38.3	38.3	0.000	77.92	18.95	0.46	19.15	19.15	0.000	1.30
S1.35	$\lceil\frac{n}{2}\rceil$	35.3	0.05	35.3	35.3	0.000	44.87	19.55	0.12	22.8	22.8	0.000	16.57
	$\lceil\frac{3n}{4}\rceil$	34.9	0.05	35.3	35.3	0.000	33.61	15.75	0.16	15.75	15.75	0.000	0.91
	n	34.9	0.05	35.3	35.3	0.000	29.94	11.9	0.17	11.9	11.9	0.000	0.15
S1.36	$\lceil\frac{n}{2}\rceil$	33.6	0.05	38.9	38.9	0.000	21.01	20.4	0.15	28.5	28.5	0.000	6.54
	$\lceil\frac{3n}{4}\rceil$	33.6	0.05	34.3	34.3	0.000	83.41	19.2	0.18	19.2	19.2	0.000	2.76
	n	33.6	0.05	34.3	34.3	0.000	38.47	16.8	0.18	17.15	17.15	0.000	0.03
S1.37	$\lceil\frac{n}{2}\rceil$	29.4	0.05	31.6	31.6	0.000	47.93	18.9	0.14	20.9	20.9	0.000	6.70
	$\lceil\frac{3n}{4}\rceil$	29.4	0.05	31.6	31.6	0.000	39.12	14.7	0.16	14.7	14.7	0.000	2.53
	n	29.4	0.05	31.6	31.6	0.000	32.84	9.8	0.18	11.333	11.333	0.000	0.16
S1.38	$\lceil\frac{n}{2}\rceil$	30.8	0.05	30.8	30.8	0.000	21.01	12	0.14	15.8	15.8	0.000	22.03
	$\lceil\frac{3n}{4}\rceil$	30.5	0.05	30.8	30.8	0.000	46.18	12	0.16	15.4	15.4	0.000	11.21
	n	30.5	0.05	30.8	30.8	0.000	22.20	10.167	0.15	10.267	10.267	0.000	0.11
S1.39	$\lceil\frac{n}{2}\rceil$	36.8	0.06	36.8	36.8	0.000	30.87	18.75	0.18	18.75	18.75	0.000	1.52
	$\lceil\frac{3n}{4}\rceil$	36.8	0.06	36.8	36.8	0.000	87.87	18.75	0.21	18.75	18.75	0.000	16.37
	n	36.8	0.06	36.8	36.8	0.000	42.91	18.4	0.22	18.4	18.4	0.000	0.10
S1.40	$\lceil\frac{n}{2}\rceil$	36.9	0.07	36.9	36.9	0.000	45.85	19	0.20	33	33	0.000	14.08
	$\lceil\frac{3n}{4}\rceil$	36.1	0.06	36.9	36.9	0.000	43.20	18.05	0.22	18.55	18.55	0.000	17.33
	n	36.1	0.06	36.9	36.9	0.000	50.85	18.05	0.24	18.45	18.45	0.000	0.07
S1.41	$\lceil\frac{n}{2}\rceil$	36.1	0.05	36.1	36.1	0.000	37.83	19.1	0.16	22.35	22.35	0.000	4.44
	$\lceil\frac{3n}{4}\rceil$	35.9	0.05	36.1	36.1	0.000	38.50	18.05	0.19	18.05	18.05	0.000	8.51
	n	35.9	0.05	36.1	36.1	0.000	84.02	17.95	0.20	18.05	18.05	0.000	0.02
S1.42	$\lceil\frac{n}{2}\rceil$	39.1	0.07	39.4	39.4	0.000	101.94	35.8	0.23	36.4	36.4	0.000	54.34
	$\lceil\frac{3n}{4}\rceil$	38.7	0.07	39.4	39.4	0.000	28.69	20.15	0.25	20.35	20.35	0.000	8.33
	n	37.9	0.08	39.3	39.3	0.000	15.42	19.35	0.34	19.65	19.65	0.000	0.34
S1.43	$\lceil\frac{n}{2}\rceil$	40.3	0.09	40.8	40.8	0.000	30.39	37.5	0.26	39.8	39.8	0.000	19.62
	$\lceil\frac{3n}{4}\rceil$	39.2	0.08	39.8	39.8	0.000	50.60	20.65	0.30	20.65	20.65	0.000	58.79

Table B.4 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	39.4	0.13	39.8	39.8	0.000	53.76	19.6	0.47	19.9	19.9	0.000	1.19
S1.44	$\lceil\frac{n}{2}\rceil$	39.3	0.07	40.1	40.1	0.000	87.83	35.5	0.24	35.5	35.5	0.000	7.24
	$\lceil\frac{3n}{4}\rceil$	38.3	0.16	38.4	38.4	0.000	165.27	19.65	0.33	19.95	19.95	0.000	24.53
	n	38.3	0.15	38.4	38.4	0.000	64.96	19.2	0.54	19.2	19.2	0.000	0.67
S1.45	$\lceil\frac{n}{2}\rceil$	34.3	0.05	34.3	34.3	0.000	23.08	34.1	0.16	34.1	34.1	0.000	2.35
	$\lceil\frac{3n}{4}\rceil$	34.3	0.06	34.3	34.3	0.000	34.51	17.25	0.18	17.4	17.4	0.000	3.93
	n	34.3	0.05	34.3	34.3	0.000	40.56	17.15	0.21	17.15	17.15	0.000	0.17
S1.46	$\lceil\frac{n}{2}\rceil$	36.5	0.06	36.5	36.5	0.000	12.40	19.55	0.17	30.9	30.9	0.000	4.06
	$\lceil\frac{3n}{4}\rceil$	34.8	0.05	34.8	34.8	0.000	47.76	17.4	0.19	17.4	17.4	0.000	1.38
	n	34.8	0.05	34.8	34.8	0.000	52.19	17.4	0.19	17.4	17.4	0.000	0.04
S1.47	$\lceil\frac{n}{2}\rceil$	35.8	0.06	36.6	36.6	0.000	34.55	32.6	0.18	32.6	32.6	0.000	4.57
	$\lceil\frac{3n}{4}\rceil$	35.6	0.06	36.2	36.2	0.000	37.06	18.9	0.28	19.95	19.95	0.000	13.88
	n	35.6	0.05	36.2	36.2	0.000	60.23	17.8	0.22	18.1	18.1	0.000	0.84
S1.48	$\lceil\frac{n}{2}\rceil$	37.3	0.07	37.3	37.3	0.000	30.63	34.1	0.17	34.1	34.1	0.000	2.21
	$\lceil\frac{3n}{4}\rceil$	37.3	0.06	37.3	37.3	0.000	32.55	19.75	0.20	20.15	20.15	0.000	2.01
	n	37.3	0.06	37.3	37.3	0.000	31.91	18.65	0.31	18.65	18.65	0.000	0.16
S1.49	$\lceil\frac{n}{2}\rceil$	37.4	0.07	37.4	37.4	0.000	69.32	22.5	0.17	27	27	0.000	4.88
	$\lceil\frac{3n}{4}\rceil$	35.9	0.06	36.7	36.7	0.000	78.60	20.45	0.24	20.45	20.45	0.000	2.09
	n	35.9	0.05	36.2	36.2	0.000	45.48	17.95	0.23	18.1	18.1	0.000	0.48

Table B.5: Detailed results for the series S2 and $r_{\max} = 3$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	43.3	0.08	45.1	50.2	0.113	600.00	40	0.55	41	41	0.000	324.19
	$\lceil\frac{3n}{4}\rceil$	40.7	0.08	41.8	50.2	0.201	600.00	22.8	0.35	23.2	50.2	1.164	600.00
	n	40.2	0.10	41.5	41.5	0.000	272.85	20.1	0.39	20.75	20.75	0.000	5.84
S1.1	$\lceil\frac{n}{2}\rceil$	40.9	0.14	42.5	50.2	0.181	600.00	36.3	0.49	37.3	50.2	0.346	600.00
	$\lceil\frac{3n}{4}\rceil$	37.6	0.14	37.9	50.2	0.325	600.00	21.25	0.41	21.75	43	0.977	600.00
	n	37.6	0.14	38.3	39.2	0.023	600.00	18.8	0.75	19.15	19.15	0.000	58.38
S1.2	$\lceil\frac{n}{2}\rceil$	41.4	0.11	42.2	42.2	0.000	40.89	38	0.26	40.8	40.8	0.000	16.21
	$\lceil\frac{3n}{4}\rceil$	40.1	0.12	40.8	40.8	0.000	185.65	22.15	0.59	22.45	22.45	0.000	131.08
	n	40.1	0.11	40.7	40.7	0.000	40.54	20	0.49	20.35	20.35	0.000	2.69
S1.3	$\lceil\frac{n}{2}\rceil$	43.3	0.08	44.6	51.4	0.152	600.00	39.9	0.36	41.5	51.4	0.239	600.00
	$\lceil\frac{3n}{4}\rceil$	41.5	0.12	42.7	51.4	0.204	600.00	22.15	0.46	23.2	51.4	1.216	600.00
	n	40.9	0.10	41.3	51.4	0.245	600.00	20.6	0.58	20.65	20.65	0.000	21.51
S1.4	$\lceil\frac{n}{2}\rceil$	41.3	0.13	45.8	50.5	0.103	600.00	37.2	0.61	38.6	50.5	0.308	600.00
	$\lceil\frac{3n}{4}\rceil$	38.8	0.10	40.1	50.1	0.249	600.00	21.35	0.77	22.35	50.1	1.242	600.00
	n	37.9	0.10	38.8	49.658	0.280	600.00	19	0.68	19.4	19.6	0.010	600.00
S1.5	$\lceil\frac{n}{2}\rceil$	42.7	0.09	46	46	0.000	520.03	38.4	0.28	41.1	41.1	0.000	345.68
	$\lceil\frac{3n}{4}\rceil$	41.1	0.13	41.3	51.4	0.245	600.00	22.9	0.42	23.25	37.2	0.600	600.00
	n	40.7	0.08	41.1	41.1	0.000	255.53	20.35	0.53	20.55	20.55	0.000	7.24
S1.6	$\lceil\frac{n}{2}\rceil$	35.4	0.06	35.4	35.4	0.000	32.81	35.4	0.23	35.4	35.4	0.000	1.32
	$\lceil\frac{3n}{4}\rceil$	35.4	0.05	35.4	35.4	0.000	40.52	17.7	0.21	17.7	17.7	0.000	5.55
	n	35.4	0.05	35.4	35.4	0.000	31.66	17.7	0.23	17.7	17.7	0.000	0.03
S1.7	$\lceil\frac{n}{2}\rceil$	45.8	0.09	45.8	51.8	0.131	600.00	39.9	0.32	41.5	41.5	0.000	140.85
	$\lceil\frac{3n}{4}\rceil$	41.5	0.14	41.7	50.6	0.213	600.00	22.7	0.41	22.85	50.6	1.214	600.00
	n	41.3	0.09	41.7	50.6	0.213	600.00	20.7	0.76	20.9	20.9	0.000	5.87
S1.8	$\lceil\frac{n}{2}\rceil$	41.5	0.13	41.5	41.5	0.000	64.10	36.3	0.32	40.4	40.4	0.000	17.97
	$\lceil\frac{3n}{4}\rceil$	40.2	0.13	40.8	40.8	0.000	100.72	21.05	0.42	21.5	29.1	0.353	600.00
	n	39.9	0.09	40.8	40.8	0.000	58.53	19.9	0.81	20.4	20.4	0.000	2.41
S1.9	$\lceil\frac{n}{2}\rceil$	43	0.08	43.7	43.7	0.000	104.34	38.3	0.29	40	40	0.000	56.12
	$\lceil\frac{3n}{4}\rceil$	40.8	0.08	41.3	41.3	0.000	390.87	22.45	0.44	22.8	22.8	0.000	132.94
	n	40.8	0.07	41.3	41.3	0.000	68.05	20.4	0.34	20.65	20.65	0.000	2.98
S1.10	$\lceil\frac{n}{2}\rceil$	43	0.18	44.7	51.6	0.154	600.00	40.2	0.42	40.6	51.6	0.271	600.00
	$\lceil\frac{3n}{4}\rceil$	39.9	0.09	40.3	50.2	0.246	600.00	21.05	0.37	21.8	50.2	1.303	600.00

Table B.5 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	39.9	0.10	41.3	50.2	0.215	600.00	19.95	0.57	20.65	20.65	0.000	57.42
S1.11	$\lceil\frac{n}{2}\rceil$	41.8	0.08	44.9	44.9	0.000	60.06	35.7	0.43	38.8	49.557	0.277	600.00
	$\lceil\frac{3n}{4}\rceil$	40.3	0.12	41.1	50.9	0.238	600.00	22.3	0.39	22.8	50.9	1.232	600.00
	n	40	0.08	40.6	40.6	0.000	140.40	19.8	0.38	20.3	20.3	0.000	6.12
S1.12	$\lceil\frac{n}{2}\rceil$	41.1	0.14	43.5	50.9	0.170	600.00	37.3	0.40	39.4	50.9	0.292	600.00
	$\lceil\frac{3n}{4}\rceil$	38.2	0.09	39.4	50.9	0.292	600.00	21.15	0.67	21.6	50.9	1.356	600.00
	n	37.8	0.10	39.4	50.9	0.292	600.00	19.1	0.54	19.7	19.7	0.000	103.08
S1.13	$\lceil\frac{n}{2}\rceil$	39.2	0.10	39.9	51.1	0.281	600.00	34.2	0.32	37.6	37.6	0.000	37.26
	$\lceil\frac{3n}{4}\rceil$	39	0.15	39.2	39.2	0.000	181.50	20.9	0.48	21.35	21.35	0.000	160.03
	n	38.3	0.07	39.2	39.2	0.000	72.22	19.2	0.55	19.6	19.6	0.000	2.28
S1.14	$\lceil\frac{n}{2}\rceil$	43.1	0.13	43.6	50.1	0.149	600.00	39.7	0.48	41.3	41.9	0.015	600.00
	$\lceil\frac{3n}{4}\rceil$	40.3	0.12	41.3	50.1	0.213	600.00	22.65	0.71	23.55	50.1	1.127	600.00
	n	40.3	0.09	41.1	50.1	0.219	600.00	20.25	0.64	20.55	20.55	0.000	18.56
S1.15	$\lceil\frac{n}{2}\rceil$	39.7	0.10	40.3	40.3	0.000	141.76	35.4	0.26	36.6	36.6	0.000	202.93
	$\lceil\frac{3n}{4}\rceil$	37.5	0.10	38.1	38.1	0.000	46.49	20.25	0.38	21.75	35.4	0.628	600.00
	n	36.9	0.11	38.1	38.1	0.000	16.48	18.45	0.45	19.05	19.05	0.000	1.05
S1.16	$\lceil\frac{n}{2}\rceil$	44.5	0.10	44.6	50.9	0.141	600.00	38.6	0.38	39.9	39.9	0.000	485.21
	$\lceil\frac{3n}{4}\rceil$	40.3	0.14	40.3	50.9	0.263	600.00	21.2	0.35	22	22.25	0.011	600.00
	n	39.8	0.09	40.2	44.5	0.107	600.00	19.8	0.56	20.1	20.1	0.000	17.56
S1.17	$\lceil\frac{n}{2}\rceil$	40.4	0.11	40.7	50.4	0.238	600.00	36.3	0.40	38.1	38.1	0.000	71.73
	$\lceil\frac{3n}{4}\rceil$	39.9	0.16	40.3	50.4	0.251	600.00	21.3	0.34	21.65	21.65	0.000	70.96
	n	39.9	0.09	40.3	40.8	0.012	600.00	19.9	0.46	20.15	20.15	0.000	7.51
S1.18	$\lceil\frac{n}{2}\rceil$	40.9	0.11	41.2	50.6	0.228	600.00	40.3	0.75	40.6	40.6	0.000	101.41
	$\lceil\frac{3n}{4}\rceil$	40.4	0.14	40.9	50.6	0.237	600.00	21.95	0.45	22.45	50.6	1.254	600.00
	n	40.4	0.09	41.1	41.1	0.000	73.53	20.2	0.62	20.55	20.55	0.000	4.01
S1.19	$\lceil\frac{n}{2}\rceil$	46.5	0.09	47.6	47.6	0.000	90.99	39.2	0.47	40.4	40.4	0.000	125.45
	$\lceil\frac{3n}{4}\rceil$	40.5	0.08	41.6	44.3	0.065	600.00	22.05	0.34	22.35	31.7	0.418	600.00
	n	40.5	0.07	41.4	41.4	0.000	26.64	20.25	0.36	20.7	20.7	0.000	1.50
S1.20	$\lceil\frac{n}{2}\rceil$	46.7	0.12	46.8	50.1	0.071	600.00	37.3	0.35	39.2	39.2	0.000	154.73
	$\lceil\frac{3n}{4}\rceil$	39	0.13	39.2	50.1	0.278	600.00	21.4	0.49	22.05	23.3	0.057	600.00
	n	38.7	0.11	39.4	50.1	0.272	600.00	19.4	0.65	19.7	19.7	0.000	67.75
S1.21	$\lceil\frac{n}{2}\rceil$	43.6	0.15	43.9	51.2	0.166	600.00	39.3	0.44	39.6	51.2	0.293	600.00
	$\lceil\frac{3n}{4}\rceil$	40.9	0.15	41.3	41.3	0.000	40.36	21.9	0.45	22.45	22.45	0.000	179.18

Table B.5 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	40.1	0.10	41.2	41.2	0.000	44.42	19.8	0.37	20.6	20.6	0.000	0.71
S1.22	$\lceil\frac{n}{2}\rceil$	40.6	0.11	41.6	50.4	0.212	600.00	35.7	0.38	38.5	50.4	0.309	600.00
	$\lceil\frac{3n}{4}\rceil$	37.8	0.13	38.4	50.4	0.313	600.00	20.75	0.58	21.25	50.4	1.372	600.00
	n	37.8	0.08	37.8	50.4	0.333	600.00	18.7	0.61	19.2	19.25	0.003	600.00
S1.23	$\lceil\frac{n}{2}\rceil$	39.3	0.09	40.5	40.8	0.007	600.00	34.5	0.41	34.7	34.7	0.000	64.77
	$\lceil\frac{3n}{4}\rceil$	39	0.07	40.5	40.5	0.000	581.28	20.3	0.43	21.75	21.75	0.000	19.94
	n	38.9	0.06	40.4	40.4	0.000	64.09	19.5	0.36	20.2	20.2	0.000	5.54
S1.24	$\lceil\frac{n}{2}\rceil$	43.7	0.11	44.7	50.8	0.136	600.00	38.6	0.78	39.5	50.8	0.286	600.00
	$\lceil\frac{3n}{4}\rceil$	39.5	0.09	40.2	50.8	0.264	600.00	21.65	0.52	22.6	50.8	1.248	600.00
	n	39.3	0.12	39.8	50.8	0.276	600.00	19.65	0.61	20.1	20.1	0.000	277.69
S1.25	$\lceil\frac{n}{2}\rceil$	44.2	0.08	44.2	51.1	0.156	600.00	40.1	0.36	41.8	41.8	0.000	28.54
	$\lceil\frac{3n}{4}\rceil$	40.7	0.09	42	51.1	0.217	600.00	22.35	0.38	22.45	49.241	1.193	600.00
	n	40.6	0.08	41.5	43.646	0.052	600.00	20.4	0.63	20.85	20.85	0.000	17.08
S1.26	$\lceil\frac{n}{2}\rceil$	40.9	0.07	40.9	40.9	0.000	95.45	24.55	0.21	24.55	24.55	0.000	4.02
	$\lceil\frac{3n}{4}\rceil$	38.3	0.10	38.4	49.1	0.279	600.00	20.75	0.41	21.7	21.7	0.000	10.42
	n	36.9	0.08	38.4	38.4	0.000	322.43	18.85	0.39	19.2	19.2	0.000	12.30
S1.27	$\lceil\frac{n}{2}\rceil$	41.7	0.12	44.1	44.1	0.000	40.91	35.1	0.25	38.5	38.5	0.000	9.10
	$\lceil\frac{3n}{4}\rceil$	38.9	0.11	39.1	39.1	0.000	65.23	21.3	0.47	21.9	21.9	0.000	25.97
	n	38.3	0.18	39.1	39.1	0.000	32.90	18.9	0.49	19.55	19.55	0.000	0.32
S1.28	$\lceil\frac{n}{2}\rceil$	46.1	0.14	46.1	50.1	0.087	600.00	37.2	0.56	39.3	50.1	0.275	600.00
	$\lceil\frac{3n}{4}\rceil$	38.4	0.15	39.4	50.1	0.272	600.00	21.45	0.61	21.9	50.1	1.288	600.00
	n	38.5	0.14	38.7	50.1	0.295	600.00	19	0.49	19.7	19.7	0.000	41.27
S1.29	$\lceil\frac{n}{2}\rceil$	42.5	0.09	42.6	50.2	0.178	600.00	37.2	0.35	39.8	50.2	0.261	600.00
	$\lceil\frac{3n}{4}\rceil$	40	0.11	41.1	50.2	0.221	600.00	21.25	0.34	21.65	50.2	1.319	600.00
	n	39.6	0.12	40.7	40.7	0.000	357.37	19.8	0.41	20.35	20.35	0.000	9.91
S1.30	$\lceil\frac{n}{2}\rceil$	43.4	0.15	43.5	50.3	0.156	600.00	37.2	0.40	39.9	50.3	0.261	600.00
	$\lceil\frac{3n}{4}\rceil$	39.5	0.11	39.9	50.3	0.261	600.00	21.75	0.62	22.75	50.3	1.211	600.00
	n	39.4	0.13	39.8	50.3	0.264	600.00	19.6	0.51	19.9	20.8	0.045	600.00
S1.31	$\lceil\frac{n}{2}\rceil$	39.5	0.09	43	43	0.000	36.62	36.4	0.35	37.8	37.8	0.000	16.45
	$\lceil\frac{3n}{4}\rceil$	36.3	0.09	36.3	36.3	0.000	31.99	18.75	0.27	18.75	18.75	0.000	11.91
	n	36.3	0.08	36.3	36.3	0.000	35.16	18.15	0.33	18.15	18.15	0.000	0.05
S1.32	$\lceil\frac{n}{2}\rceil$	40.8	0.09	42.2	42.2	0.000	103.06	37.7	0.59	40	40.4	0.010	600.00
	$\lceil\frac{3n}{4}\rceil$	39.3	0.09	40.4	50.3	0.245	600.00	20.5	0.38	21	50.3	1.395	600.00

Table B.5 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	39.2	0.09	40.4	43.391	0.074	600.00	19.9	0.75	20.2	20.2	0.000	45.85
S1.33	$\lceil\frac{n}{2}\rceil$	46.5	0.10	48.2	48.2	0.000	303.68	37.7	0.31	40.8	45.2	0.108	600.00
	$\lceil\frac{3n}{4}\rceil$	40.1	0.12	40.8	50.1	0.228	600.00	21.85	0.64	22.6	50.1	1.217	600.00
	n	40.1	0.11	40.8	50.1	0.228	600.00	20.05	0.51	20.4	20.4	0.000	37.63
S1.34	$\lceil\frac{n}{2}\rceil$	41	0.13	42.8	42.8	0.000	185.45	36.3	0.38	38.9	38.9	0.000	45.75
	$\lceil\frac{3n}{4}\rceil$	38.1	0.14	39.3	50.9	0.295	600.00	21.25	0.41	21.8	21.8	0.000	102.68
	n	38.2	0.13	39.5	40.3	0.020	600.00	19.05	0.76	19.75	19.75	0.000	29.85
S1.35	$\lceil\frac{n}{2}\rceil$	42.3	0.10	42.3	42.3	0.000	62.02	24.8	0.35	24.8	24.8	0.000	11.45
	$\lceil\frac{3n}{4}\rceil$	41.4	0.09	42	42	0.000	261.14	22.55	0.32	23.05	23.05	0.000	13.77
	n	40.1	0.10	42	42	0.000	32.70	20.2	0.68	21	21	0.000	1.10
S1.36	$\lceil\frac{n}{2}\rceil$	39.4	0.10	40.9	40.9	0.000	543.13	36	0.33	37.1	37.1	0.000	37.90
	$\lceil\frac{3n}{4}\rceil$	38.4	0.10	38.8	38.8	0.000	198.23	22.25	0.49	22.35	22.35	0.000	22.16
	n	37.6	0.10	38.5	38.5	0.000	28.48	18.65	0.40	19.25	19.25	0.000	0.65
S1.37	$\lceil\frac{n}{2}\rceil$	40.1	0.11	41.3	50.8	0.230	600.00	37.7	0.39	39.4	39.4	0.000	126.73
	$\lceil\frac{3n}{4}\rceil$	38.4	0.17	39.2	50.3	0.283	600.00	20.55	0.39	21.7	21.7	0.000	372.42
	n	38.4	0.08	39.1	40.1	0.026	600.00	19.2	0.46	19.55	19.55	0.000	26.78
S1.38	$\lceil\frac{n}{2}\rceil$	47.5	0.08	48.4	50.5	0.043	600.00	38.4	0.82	40.7	50.5	0.241	600.00
	$\lceil\frac{3n}{4}\rceil$	40.5	0.11	41	50.5	0.232	600.00	22.95	0.45	23.1	50.5	1.186	600.00
	n	40.3	0.09	40.7	50.3	0.236	600.00	20.25	0.68	20.45	20.45	0.000	10.50
S1.39	$\lceil\frac{n}{2}\rceil$	46	0.14	48.4	50.1	0.035	600.00	39.3	0.63	41.1	50.1	0.219	600.00
	$\lceil\frac{3n}{4}\rceil$	40.2	0.11	40.9	50.1	0.225	600.00	22.3	0.54	22.5	50.1	1.227	600.00
	n	40.1	0.10	40.9	50.1	0.225	600.00	20.1	0.57	20.45	20.45	0.000	47.60
S1.40	$\lceil\frac{n}{2}\rceil$	39.1	0.19	39.1	50.1	0.281	600.00	36.9	0.43	38.3	38.3	0.000	270.86
	$\lceil\frac{3n}{4}\rceil$	37.8	0.09	38.5	50.1	0.301	600.00	19.75	0.42	20	37.4	0.870	600.00
	n	37.8	0.07	38.5	38.5	0.000	154.05	19.25	0.58	19.25	19.25	0.000	9.13
S1.41	$\lceil\frac{n}{2}\rceil$	39.4	0.08	39.7	39.7	0.000	120.37	32.8	0.39	33	33	0.000	27.68
	$\lceil\frac{3n}{4}\rceil$	39.1	0.10	39.1	39.4	0.008	600.00	20.05	0.34	20.35	20.35	0.000	17.30
	n	38.8	0.07	39.1	39.1	0.000	84.50	19.55	0.42	19.55	19.55	0.000	2.55
S1.42	$\lceil\frac{n}{2}\rceil$	42.9	0.10	46.6	46.6	0.000	285.87	36.6	0.41	38.7	50.1	0.295	600.00
	$\lceil\frac{3n}{4}\rceil$	38.6	0.18	39.7	50.1	0.262	600.00	21.5	0.44	21.5	50.1	1.330	600.00
	n	38.4	0.17	39.7	39.7	0.000	161.33	18.9	0.50	19.85	19.85	0.000	20.72
S1.43	$\lceil\frac{n}{2}\rceil$	42.2	0.10	43.2	50.9	0.178	600.00	38	0.56	39.9	43.4	0.088	600.00
	$\lceil\frac{3n}{4}\rceil$	39.6	0.10	40.4	50.9	0.260	600.00	21.2	0.51	21.7	50.9	1.346	600.00

Table B.5 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	39.2	0.14	40.2	46.3	0.152	600.00	19.7	0.72	20.2	20.2	0.000	97.90
S1.44	$\lceil \frac{n}{2} \rceil$	43	0.10	43	43	0.000	158.00	37.8	0.56	39.4	39.4	0.000	85.28
	$\lceil \frac{3n}{4} \rceil$	38.7	0.08	39.7	50.1	0.262	600.00	22.25	0.50	22.4	22.4	0.000	64.64
	n	38.6	0.10	39.6	40.1	0.013	600.00	19.25	0.52	19.8	19.8	0.000	5.44
S1.45	$\lceil \frac{n}{2} \rceil$	40.2	0.06	42.1	42.1	0.000	81.81	34.8	0.26	35.2	35.2	0.000	13.29
	$\lceil \frac{3n}{4} \rceil$	35.6	0.07	36.4	48.9	0.343	600.00	19.35	0.41	19.75	23.4	0.185	600.00
	n	35.6	0.07	36.4	36.4	0.000	56.22	17.8	0.32	18.2	18.2	0.000	6.84
S1.46	$\lceil \frac{n}{2} \rceil$	42	0.13	42	50.1	0.193	600.00	38.9	0.50	40.4	50.1	0.240	600.00
	$\lceil \frac{3n}{4} \rceil$	39.7	0.13	40.8	50.1	0.228	600.00	21.15	0.52	21.55	50.1	1.325	600.00
	n	39.5	0.09	40.5	50.1	0.237	600.00	19.8	0.70	20.25	20.25	0.000	21.26
S1.47	$\lceil \frac{n}{2} \rceil$	39.7	0.08	40.8	50.8	0.245	600.00	35.8	0.41	35.8	35.8	0.000	216.45
	$\lceil \frac{3n}{4} \rceil$	36.6	0.12	37	47.79	0.292	600.00	20.65	0.42	20.95	20.95	0.000	181.75
	n	36.2	0.12	36.8	36.8	0.000	52.61	18.4	0.62	18.4	18.4	0.000	2.57
S1.48	$\lceil \frac{n}{2} \rceil$	42.1	0.08	44	51.8	0.177	600.00	38.2	0.59	39.4	45.9	0.165	600.00
	$\lceil \frac{3n}{4} \rceil$	39.6	0.11	40.5	50.5	0.247	600.00	21.75	0.79	22.55	50.5	1.239	600.00
	n	39.2	0.09	40.3	50.5	0.253	600.00	19.7	0.40	20.15	20.15	0.000	38.81
S1.49	$\lceil \frac{n}{2} \rceil$	42.4	0.15	43.8	50.5	0.153	600.00	38.3	0.75	39.4	50.5	0.282	600.00
	$\lceil \frac{3n}{4} \rceil$	39.2	0.24	39.5	50.5	0.278	600.00	20.95	0.77	21.5	50.5	1.349	600.00
	n	38.8	0.11	39.4	50.5	0.282	600.00	19.4	0.70	19.7	21.2	0.076	600.00

Table B.6: Detailed results for the series S3 and $r_{\max} = 3$.

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
S1.0	$\lceil\frac{n}{2}\rceil$	15.7	0.20	15.7	15.7	0.000	119.41	8.433	0.44	8.433	8.433	0.000	15.17
	$\lceil\frac{3n}{4}\rceil$	15.5	0.20	15.5	15.5	0.000	192.71	5.1	0.48	5.1	5.1	0.000	8.11
	n	15.5	0.20	15.5	15.5	0.000	200.33	3.925	0.52	3.925	3.925	0.000	9.81
S1.1	$\lceil\frac{n}{2}\rceil$	16.4	0.20	18.4	18.4	0.000	85.24	7.2	0.43	7.2	7.2	0.000	35.87
	$\lceil\frac{3n}{4}\rceil$	16.4	0.21	18.4	18.4	0.000	134.22	4.6	0.51	4.6	4.6	0.000	21.99
	n	16.4	0.21	18.4	18.4	0.000	160.60	4.6	0.52	4.6	4.6	0.000	1.17
S1.2	$\lceil\frac{n}{2}\rceil$	17.9	0.21	17.9	17.9	0.000	269.45	9	0.45	9	9	0.000	93.00
	$\lceil\frac{3n}{4}\rceil$	17.9	0.21	17.9	17.9	0.000	445.25	6.8	0.52	6.967	6.967	0.000	41.96
	n	17.9	0.21	17.9	17.9	0.000	168.10	5.233	0.56	5.233	5.233	0.000	2.27
S1.3	$\lceil\frac{n}{2}\rceil$	12.2	0.20	12.2	43.02	2.526	600.00	5.7	0.43	5.7	5.7	0.000	82.39
	$\lceil\frac{3n}{4}\rceil$	12	0.21	12	40.61	2.384	600.00	4	0.48	4.075	4.075	0.000	8.54
	n	12	0.21	12	42.241	2.520	600.00	3	0.49	3	3	0.000	81.85
S1.4	$\lceil\frac{n}{2}\rceil$	14	0.20	14	14	0.000	22.56	4.95	0.46	4.95	4.95	0.000	25.86
	$\lceil\frac{3n}{4}\rceil$	14	0.20	14	14	0.000	17.89	4.667	0.47	4.75	4.75	0.000	0.99
	n	14	0.20	14	14	0.000	10.25	3.575	0.51	3.575	3.575	0.000	0.41
S1.5	$\lceil\frac{n}{2}\rceil$	16.7	0.21	17.2	26.825	0.560	600.00	8.65	0.48	8.65	8.65	0.000	76.73
	$\lceil\frac{3n}{4}\rceil$	16.7	0.21	17.2	36.1	1.099	600.00	7.767	0.55	8.6	8.6	0.000	9.00
	n	16.7	0.21	17.2	17.2	0.000	332.04	5.567	0.58	5.733	5.733	0.000	1.68
S1.6	$\lceil\frac{n}{2}\rceil$	19.4	0.21	20.7	20.7	0.000	263.51	11.2	0.48	11.2	11.2	0.000	121.75
	$\lceil\frac{3n}{4}\rceil$	19.4	0.21	20.7	20.7	0.000	443.13	7.05	0.55	8.1	8.1	0.000	41.57
	n	19.4	0.21	20.7	20.7	0.000	330.12	6.467	0.61	6.9	6.9	0.000	1.85
S1.7	$\lceil\frac{n}{2}\rceil$	22.1	0.21	22.1	23.9	0.081	600.00	9.133	0.49	9.9	9.9	0.000	187.41
	$\lceil\frac{3n}{4}\rceil$	22.1	0.21	22.1	22.1	0.000	413.02	7.367	0.57	7.367	7.367	0.000	131.43
	n	22.1	0.21	22.1	22.1	0.000	146.37	7.367	0.61	7.367	7.367	0.000	0.25
S1.8	$\lceil\frac{n}{2}\rceil$	27.2	0.25	27.7	50.4	0.819	600.00	15.7	0.61	18.25	18.25	0.000	145.90
	$\lceil\frac{3n}{4}\rceil$	27.2	0.26	27.7	38.524	0.391	600.00	14.7	0.72	15.65	15.65	0.000	438.33
	n	27.2	0.25	27.8	28.7	0.032	600.00	9.067	0.75	9.267	9.267	0.000	3.56
S1.9	$\lceil\frac{n}{2}\rceil$	4.4	0.19	4.4	4.4	0.000	9.77	2.2	0.30	2.55	2.55	0.000	4.87
	$\lceil\frac{3n}{4}\rceil$	4.4	0.19	4.4	4.4	0.000	8.29	1.1	0.35	1.2	1.2	0.000	16.20
	n	4.4	0.19	4.4	4.4	0.000	12.77	1.1	0.40	1.1	1.1	0.000	10.81
S1.10	$\lceil\frac{n}{2}\rceil$	12.3	0.20	12.3	12.3	0.000	147.43	4.4	0.40	4.8	4.8	0.000	109.53
	$\lceil\frac{3n}{4}\rceil$	12.3	0.20	12.3	12.3	0.000	202.35	3.667	0.46	3.667	3.667	0.000	12.88

Table B.6 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	12.3	0.20	12.3	42.325	2.441	600.00	3.25	0.52	3.25	3.25	0.000	20.11
S1.11	$\lceil\frac{n}{2}\rceil$	14.2	0.20	15.6	15.6	0.000	74.62	6.6	0.39	6.6	6.6	0.000	45.97
	$\lceil\frac{3n}{4}\rceil$	14.2	0.20	15.4	15.4	0.000	188.21	4.733	0.53	4.95	4.95	0.000	48.60
	n	14.2	0.20	15.4	15.4	0.000	207.36	3.9	0.53	3.9	3.9	0.000	5.97
S1.12	$\lceil\frac{n}{2}\rceil$	7.3	0.19	9.8	9.8	0.000	24.80	4.333	0.36	4.7	4.7	0.000	6.54
	$\lceil\frac{3n}{4}\rceil$	7.3	0.20	9.8	9.8	0.000	56.84	3.1	0.40	3.25	3.25	0.000	0.36
	n	7.3	0.20	9.8	9.8	0.000	9.34	1.825	0.44	2.45	2.45	0.000	7.69
S1.13	$\lceil\frac{n}{2}\rceil$	9.2	0.19	12.9	12.9	0.000	101.35	6	0.38	6	6	0.000	55.27
	$\lceil\frac{3n}{4}\rceil$	9.2	0.20	12.9	12.9	0.000	171.55	3.533	0.44	4.025	4.025	0.000	26.13
	n	9.2	0.20	12.9	12.9	0.000	177.96	2.3	0.48	3.225	3.225	0.000	7.78
S1.14	$\lceil\frac{n}{2}\rceil$	21.9	0.21	21.9	21.9	0.000	152.01	10.4	0.53	10.4	38.712	2.722	600.00
	$\lceil\frac{3n}{4}\rceil$	21.2	0.21	21.2	21.2	0.000	294.16	7.767	0.59	7.767	7.767	0.000	152.84
	n	21.2	0.21	21.2	21.2	0.000	176.38	7.067	0.60	7.067	7.067	0.000	0.28
S1.15	$\lceil\frac{n}{2}\rceil$	13.3	0.20	16.9	16.9	0.000	125.78	8.75	0.43	10.7	10.7	0.000	93.25
	$\lceil\frac{3n}{4}\rceil$	12.9	0.21	13.3	22.207	0.670	600.00	5.1	0.48	5.2	5.2	0.000	42.41
	n	12.9	0.21	13.3	13.3	0.000	136.91	3.325	0.56	3.325	3.325	0.000	4.14
S1.16	$\lceil\frac{n}{2}\rceil$	15	0.20	15	15	0.000	169.47	7.05	0.46	7.05	7.05	0.000	86.78
	$\lceil\frac{3n}{4}\rceil$	12.2	0.21	12.2	12.2	0.000	167.37	5.9	0.51	6.1	6.1	0.000	9.98
	n	12.2	0.21	12.2	12.2	0.000	202.76	3.833	0.51	3.833	3.833	0.000	6.45
S1.17	$\lceil\frac{n}{2}\rceil$	12.9	0.21	14.8	14.8	0.000	91.07	7.767	0.38	8.6	8.6	0.000	29.79
	$\lceil\frac{3n}{4}\rceil$	12.1	0.20	13.9	13.9	0.000	64.46	4.033	0.45	5.3	5.3	0.000	2.06
	n	12.1	0.20	13.9	13.9	0.000	54.55	3.025	0.48	3.7	3.7	0.000	18.65
S1.18	$\lceil\frac{n}{2}\rceil$	19.9	0.21	21	21	0.000	169.91	8.933	0.49	10.55	10.55	0.000	68.12
	$\lceil\frac{3n}{4}\rceil$	19.9	0.21	21	21	0.000	429.29	6.633	0.56	6.7	6.7	0.000	8.21
	n	19.9	0.21	21	21	0.000	152.99	5.6	0.57	6.7	6.7	0.000	1.76
S1.19	$\lceil\frac{n}{2}\rceil$	19.1	0.21	19.1	19.1	0.000	127.05	8.467	0.44	8.467	8.467	0.000	67.50
	$\lceil\frac{3n}{4}\rceil$	19.1	0.21	19.1	19.1	0.000	197.96	6.367	0.54	6.367	6.367	0.000	13.93
	n	19.1	0.21	19.1	19.1	0.000	70.22	6.367	0.59	6.367	6.367	0.000	0.12
S1.20	$\lceil\frac{n}{2}\rceil$	6.6	0.19	6.6	6.6	0.000	66.98	3.3	0.37	3.3	3.3	0.000	6.17
	$\lceil\frac{3n}{4}\rceil$	6.6	0.19	6.6	6.6	0.000	41.59	3.25	0.43	3.25	3.25	0.000	4.32
	n	6.6	0.19	6.6	6.6	0.000	10.07	1.65	0.43	1.65	1.65	0.000	7.43
S1.21	$\lceil\frac{n}{2}\rceil$	17.9	0.19	17.9	17.9	0.000	67.21	6.6	0.41	6.85	6.85	0.000	33.64
	$\lceil\frac{3n}{4}\rceil$	14.8	0.20	15.2	15.2	0.000	45.54	4.15	0.43	4.15	4.15	0.000	4.78

Table B.6 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	14.8	0.20	15.2	15.2	0.000	79.31	3.9	0.48	3.9	3.9	0.000	0.41
S1.22	$\lceil\frac{n}{2}\rceil$	13.3	0.21	13.3	13.3	0.000	64.98	6.15	0.40	7.5	7.5	0.000	33.36
	$\lceil\frac{3n}{4}\rceil$	13.3	0.21	13.3	13.3	0.000	210.30	4.433	0.46	4.433	4.433	0.000	19.83
	n	13.3	0.21	13.3	13.3	0.000	112.61	3.85	0.51	3.85	3.85	0.000	0.74
S1.23	$\lceil\frac{n}{2}\rceil$	10.2	0.19	10.2	10.2	0.000	35.11	4.667	0.41	4.8	4.8	0.000	47.65
	$\lceil\frac{3n}{4}\rceil$	10.2	0.19	10.2	10.2	0.000	60.17	3.333	0.44	3.5	3.5	0.000	11.20
	n	10.2	0.20	10.2	10.2	0.000	75.91	2.55	0.46	2.55	2.55	0.000	10.50
S1.24	$\lceil\frac{n}{2}\rceil$	14.9	0.20	14.9	14.9	0.000	107.24	7.45	0.41	7.45	7.45	0.000	31.65
	$\lceil\frac{3n}{4}\rceil$	14.9	0.20	14.9	14.9	0.000	147.68	4.967	0.47	5	5	0.000	10.26
	n	14.9	0.20	14.9	34.878	1.341	600.00	3.725	0.51	3.725	3.725	0.000	0.95
S1.25	$\lceil\frac{n}{2}\rceil$	19.6	0.22	22.1	22.1	0.000	199.39	12.3	0.47	12.3	12.3	0.000	39.05
	$\lceil\frac{3n}{4}\rceil$	19.6	0.22	21.3	21.3	0.000	290.74	8.433	0.56	8.433	8.433	0.000	174.38
	n	19.6	0.22	21.3	21.3	0.000	162.49	6.533	0.60	7.1	7.1	0.000	6.07
S1.26	$\lceil\frac{n}{2}\rceil$	16.8	0.21	20.1	20.1	0.000	64.17	7.4	0.48	8.067	8.067	0.000	81.45
	$\lceil\frac{3n}{4}\rceil$	16.8	0.21	20.1	20.1	0.000	53.73	5.6	0.54	6.7	6.7	0.000	14.17
	n	16.8	0.21	20.1	20.1	0.000	67.54	5.6	0.56	6.7	6.7	0.000	0.92
S1.27	$\lceil\frac{n}{2}\rceil$	16.2	0.20	16.2	16.2	0.000	132.29	8.633	0.43	9.3	9.3	0.000	8.63
	$\lceil\frac{3n}{4}\rceil$	11.7	0.20	11.7	11.7	0.000	143.52	4.05	0.46	4.05	4.05	0.000	48.60
	n	11.7	0.20	11.7	11.7	0.000	160.95	2.925	0.53	2.925	2.925	0.000	27.09
S1.28	$\lceil\frac{n}{2}\rceil$	7.1	0.19	7.1	7.1	0.000	16.32	3.35	0.34	3.35	3.35	0.000	9.20
	$\lceil\frac{3n}{4}\rceil$	7.1	0.19	7.1	7.1	0.000	4.98	1.775	0.37	1.775	1.775	0.000	6.13
	n	7.1	0.19	7.1	7.1	0.000	3.21	1.775	0.40	1.775	1.775	0.000	0.86
S1.29	$\lceil\frac{n}{2}\rceil$	11.2	0.20	11.2	11.2	0.000	186.00	5.6	0.43	5.6	5.6	0.000	86.46
	$\lceil\frac{3n}{4}\rceil$	11.2	0.20	11.2	11.2	0.000	221.87	3.4	0.48	3.4	3.4	0.000	32.23
	n	11.2	0.20	11.2	11.2	0.000	158.04	2.35	0.48	2.35	2.35	0.000	33.07
S1.30	$\lceil\frac{n}{2}\rceil$	20.9	0.22	21	24.3	0.157	600.00	8.75	0.52	10.1	10.1	0.000	570.03
	$\lceil\frac{3n}{4}\rceil$	20.9	0.22	21	46	1.190	600.00	8.133	0.58	9.2	9.2	0.000	56.55
	n	20.9	0.21	21	21	0.000	588.35	6.967	0.61	7	7	0.000	2.10
S1.31	$\lceil\frac{n}{2}\rceil$	16.1	0.21	18.4	18.4	0.000	291.32	7.667	0.47	8.1	8.1	0.000	194.89
	$\lceil\frac{3n}{4}\rceil$	15.7	0.21	17.5	17.5	0.000	460.42	5.267	0.51	7.667	7.667	0.000	16.84
	n	15.7	0.21	17.5	17.5	0.000	282.96	5.133	0.54	5.133	5.133	0.000	7.79
S1.32	$\lceil\frac{n}{2}\rceil$	14	0.20	15.4	15.4	0.000	102.91	6.9	0.45	6.9	6.9	0.000	191.16
	$\lceil\frac{3n}{4}\rceil$	14	0.21	14.6	14.6	0.000	220.49	4.667	0.52	4.667	4.667	0.000	9.51

Table B.6 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	14	0.21	14.6	14.6	0.000	153.12	3.5	0.54	3.65	3.65	0.000	7.47
S1.33	$\lceil\frac{n}{2}\rceil$	13.8	0.20	13.8	13.8	0.000	177.09	4.4	0.41	6.367	6.367	0.000	445.68
	$\lceil\frac{3n}{4}\rceil$	11.1	0.21	12.7	12.7	0.000	188.41	3.767	0.48	3.767	3.767	0.000	30.61
	n	11.1	0.21	12.7	12.7	0.000	162.45	2.775	0.53	3.175	3.175	0.000	54.12
S1.34	$\lceil\frac{n}{2}\rceil$	6.2	0.19	6.2	6.2	0.000	2.05	2.467	0.32	2.467	2.467	0.000	3.87
	$\lceil\frac{3n}{4}\rceil$	6.2	0.19	6.2	6.2	0.000	2.61	2	0.37	2	2	0.000	0.68
	n	6.2	0.19	6.2	6.2	0.000	1.04	1.55	0.42	1.55	1.55	0.000	0.96
S1.35	$\lceil\frac{n}{2}\rceil$	9.5	0.19	9.5	9.5	0.000	47.91	5.9	0.39	6.067	6.067	0.000	8.37
	$\lceil\frac{3n}{4}\rceil$	9.5	0.20	9.5	9.5	0.000	92.21	4	0.47	4	4	0.000	8.11
	n	9.5	0.20	9.5	9.5	0.000	71.90	2.375	0.47	2.375	2.375	0.000	12.22
S1.36	$\lceil\frac{n}{2}\rceil$	8.3	0.19	8.3	8.3	0.000	26.82	4.15	0.38	5.9	5.9	0.000	21.27
	$\lceil\frac{3n}{4}\rceil$	8.3	0.19	8.3	8.3	0.000	50.40	3.9	0.44	4.05	4.05	0.000	9.62
	n	8.3	0.19	8.3	8.3	0.000	14.52	2.075	0.45	2.075	2.075	0.000	3.58
S1.37	$\lceil\frac{n}{2}\rceil$	20.9	0.23	20.9	50.2	1.402	600.00	11.85	0.54	12	17.5	0.458	600.00
	$\lceil\frac{3n}{4}\rceil$	20.9	0.23	23.3	31.238	0.341	600.00	7.767	0.63	8.65	17.5	1.023	600.00
	n	20.9	0.23	20.9	49.454	1.366	600.00	6.967	0.66	7.767	7.767	0.000	21.30
S1.38	$\lceil\frac{n}{2}\rceil$	17.7	0.21	17.7	17.7	0.000	174.67	9.7	0.47	12.15	12.15	0.000	71.92
	$\lceil\frac{3n}{4}\rceil$	17.7	0.22	17.7	17.7	0.000	113.38	8.1	0.56	8.1	8.1	0.000	49.13
	n	17.7	0.21	17.7	17.7	0.000	179.49	5.9	0.60	5.9	5.9	0.000	0.87
S1.39	$\lceil\frac{n}{2}\rceil$	18	0.22	20.3	42	1.069	600.00	8.8	0.49	10.25	10.25	0.000	409.88
	$\lceil\frac{3n}{4}\rceil$	18	0.22	18	44.743	1.486	600.00	6.95	0.55	6.95	6.95	0.000	79.62
	n	18	0.22	18	18	0.000	515.32	6	0.57	6	6	0.000	1.87
S1.40	$\lceil\frac{n}{2}\rceil$	15.7	0.20	15.7	18.9	0.204	600.00	7.85	0.45	7.85	7.85	0.000	77.67
	$\lceil\frac{3n}{4}\rceil$	15.7	0.21	15.7	15.8	0.006	600.00	4.675	0.55	4.675	4.675	0.000	39.96
	n	15.7	0.21	15.7	15.7	0.000	129.68	4.675	0.55	4.675	4.675	0.000	1.21
S1.41	$\lceil\frac{n}{2}\rceil$	7.6	0.20	7.6	7.6	0.000	131.35	3	0.41	3	3	0.000	75.68
	$\lceil\frac{3n}{4}\rceil$	7.6	0.20	7.6	7.6	0.000	153.87	2.325	0.43	2.9	2.9	0.000	51.04
	n	7.6	0.21	7.6	7.6	0.000	87.22	1.9	0.51	1.9	1.9	0.000	12.34
S1.42	$\lceil\frac{n}{2}\rceil$	16.3	0.21	17.4	17.4	0.000	70.83	7.4	0.48	8.75	8.75	0.000	55.28
	$\lceil\frac{3n}{4}\rceil$	16.3	0.21	17.4	17.4	0.000	41.93	5.433	0.54	5.8	5.8	0.000	10.59
	n	16.3	0.21	17.4	17.4	0.000	80.35	4.667	0.55	5.8	5.8	0.000	1.08
S1.43	$\lceil\frac{n}{2}\rceil$	9.3	0.19	9.3	9.3	0.000	111.94	4.2	0.39	4.65	4.65	0.000	16.38
	$\lceil\frac{3n}{4}\rceil$	9.3	0.20	9.3	9.3	0.000	41.74	3.033	0.43	3.15	3.15	0.000	5.12

Table B.6 – continued from previous page

		P_1						P_∞					
		Heuristics		MIP Solver				Heuristics		MIP Solver			
Inst	$\lvert\widetilde{V}\rvert$	LB	CPU	LB	UB	GAP	CPU	LB	CPU	LB	UB	GAP	CPU
	n	9.3	0.20	9.3	9.3	0.000	21.20	2.325	0.46	2.325	2.325	0.000	18.48
S1.44	$\lceil\frac{n}{2}\rceil$	8.6	0.19	9.6	9.6	0.000	72.12	3.533	0.35	3.533	3.533	0.000	18.29
	$\lceil\frac{3n}{4}\rceil$	8.6	0.20	9.6	9.6	0.000	73.51	2.65	0.42	2.65	2.65	0.000	57.77
	n	8.6	0.20	9.6	9.6	0.000	30.39	2.15	0.44	2.4	2.4	0.000	27.22
S1.45	$\lceil\frac{n}{2}\rceil$	14.9	0.20	16.3	16.3	0.000	80.74	7.767	0.42	8.8	8.8	0.000	32.32
	$\lceil\frac{3n}{4}\rceil$	14.9	0.20	16.3	16.3	0.000	92.01	5.667	0.52	5.667	5.667	0.000	2.71
	n	14.9	0.20	16.3	16.3	0.000	63.44	4.1	0.58	5.1	5.1	0.000	0.94
S1.46	$\lceil\frac{n}{2}\rceil$	8.9	0.20	8.9	8.9	0.000	40.52	4.133	0.40	4.133	4.133	0.000	34.09
	$\lceil\frac{3n}{4}\rceil$	8.9	0.20	8.9	8.9	0.000	50.64	2.967	0.46	2.967	2.967	0.000	6.81
	n	8.9	0.21	8.9	8.9	0.000	57.62	2.225	0.47	2.225	2.225	0.000	7.52
S1.47	$\lceil\frac{n}{2}\rceil$	8.5	0.20	8.5	8.5	0.000	55.42	3.6	0.34	3.6	3.6	0.000	48.15
	$\lceil\frac{3n}{4}\rceil$	8.5	0.20	8.5	8.5	0.000	26.19	2.4	0.40	2.5	2.5	0.000	3.78
	n	8.5	0.20	8.5	8.5	0.000	22.63	2.125	0.42	2.125	2.125	0.000	24.60
S1.48	$\lceil\frac{n}{2}\rceil$	15.4	0.20	17.4	17.4	0.000	162.20	6.9	0.46	8.65	8.65	0.000	33.94
	$\lceil\frac{3n}{4}\rceil$	15.4	0.20	17.1	17.1	0.000	274.92	5.8	0.52	6.1	6.1	0.000	9.19
	n	15.4	0.20	17.1	17.1	0.000	214.22	4.45	0.55	4.75	4.75	0.000	1.69
S1.49	$\lceil\frac{n}{2}\rceil$	7.3	0.19	7.3	7.3	0.000	19.41	4.55	0.34	4.8	4.8	0.000	3.98
	$\lceil\frac{3n}{4}\rceil$	6.4	0.20	7.1	7.1	0.000	34.60	2.85	0.42	2.85	2.85	0.000	6.76
	n	6.4	0.20	7.1	7.1	0.000	11.75	1.6	0.42	1.775	1.775	0.000	13.93