On the Dynamic Properties of Rigid-Link Flexible-Joint Parallel Manipulators in the Presence of Type 2 Singularities

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In our previous work (2008, "Optimal Force Generation in Parallel Manipulators for Passing through the Singular Positions," Int. J. Robot. Res., **27***(8), pp. 967–983), the dynamic properties of rigid-link parallel manipulators, in the presence of type 2 singularities, have been studied. It was shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts of the manipulator are orthogonal to the twist along the direction of the uncontrollable motion. This condition was obtained using the symbolic approach based on the inverse dynamics and the study of the Lagrangian of a general rigid-link parallel manipulator. It was validated by experimental tests carried out on the prototype of a four-degrees-of-freedom parallel manipulator. However, it is known that the flexibility of the mechanism may not always be neglected. Indeed, joint flexibility is the main source contributing to the overall manipulator flexibility and it leads to the trajectory distortion. Therefore, in this paper, the condition for passing through a type 2 singularity of parallel manipulators with flexible joints is studied. The suggested technique is illustrated by the example of a 5R parallel manipulator with flexible joints. It is shown that passing through singularity is possible if the 12th-order polynomial trajectory planning is applied. The obtained results are validated by the numerical simulations carried out using the* ADAMS *software.* [DOI: 10.1115/1.4001121]

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1 Introduction

Over the past decades, with the large development of parallel manipulators, more attention have been paid to their kinematic, kinetostatic, and dynamic properties, and in particular, to their singularities. Several papers deal with singularity analysis of parallel manipulators [1–13]. Most of them present the analysis of singular configurations from a kinematic point of view [1–7]. Algebraically, a singularity analysis is based on the degeneracy of Jacobian matrices of the mechanical structure, or of the system of reciprocal screws (wrenches) applied to the platform by the legs. However, it is also known that, when parallel manipulators have type 2 singularities [1], they lose their stiffness and their quality of motion transmission, and thus, their payload capability. Therefore, the singularity zones in the workspace of manipulators may be analyzed not only in terms of kinematic criterions, from the theoretically perfect model of manipulators, but also in terms of kinetostatic approaches [8–13].

Moreover, while it is demonstrated using the kinetostatic approach that, when subjected to type 2 singularities parallel manipulators lock up, it has been shown experimentally that, via optimal dynamic control of manipulators, it is possible to pass through these singular zones. Thus, it is evident that singular configurations should also be examined in terms of the dynamic aspects.

The further study of singularity in parallel manipulators has revealed an interesting problem that concerns the path planning of parallel manipulators under the presence of singular positions, i.e., the motion feasibility in the neighborhood of singularities. In this case the dynamic conditions can be considered in the design process. One of the most evident solutions for the stable motion generation in the neighborhood of singularities is to use redundant sensors and actuators [14–17]. However, it is an expensive solution to the problem because of the additional actuators and the complicated control of the manipulator caused by actuation redundancy. Another approach concerns with motion planning to pass through singularity [18–24], i.e., a parallel manipulator may track a path through singular poses if its velocity and acceleration are properly constrained. This is a promising way for the solution of this problem. However, only a few research papers on this approach have addressed the path planning for obtaining a good tracking performance. But they have not adequately addressed the physical interpretation of the dynamic aspects.

In our recent work [25], optimal force generation in parallel manipulators for passing through the singular positions has been studied. It was shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts of the manipulator are orthogonal to the twist along the direction of the uncontrollable motion. This paper was concerned with the study of rigid-link parallel manipulators without any flexibility. However, it should be noted that several factors may bring a loss of rigidity in parallel manipulators (elasticity of links, clearance in joints, etc.). But their contributions can be considerably reduced for a properly designed and constrained mechanical system. However, even for the most optimum design of manipulator one main source of flexibility remains and it cannot be easily reduced: it is the flexibility in the actuated joints, due to the use of Harmonic Drive® systems.

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Therefore, in the present work the dynamic condition for passing through the singular positions is defined in general for parallel manipulators, taking into account the elasticity in the actuated joints.

This paper is organized as follows: Sec. 2 presents theoretical aspects of the examined problem. As in our previous work, using the Lagrangian formulation, the condition of force distribution is defined, which allows the passing of any parallel manipulator through the type 2 singular positions. In Sec. 3, the suggested solution is illustrated via a 5R planar parallel manipulator. In Sec. 4, the conclusions are given.

2 Optimal Dynamic Conditions for Passing Through Type 2 Singularity

Let us consider a parallel manipulator of m links, n degrees-of-freedom, and driven by n actuators.

The general Lagrangian dynamic formulation for a manipulator with elasticity in actuated joints can be expressed as [26]

$$\mathbf{0} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_d}\right) - \frac{\partial L}{\partial \mathbf{q}_d} \tag{1a}$$

$$\boldsymbol{\tau} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right) - \frac{\partial L}{\partial \mathbf{q}_a} \tag{1b}$$

where L is the Lagrangian of the manipulator ($L=T-V$, where T is the kinetic energy and V is the potential energy due to gravitational forces, friction, and elasticity), $\mathbf{q}_a=[q_1^a, q_2^a, \ldots, q_n^a]^T$ and $\dot{\mathbf{q}}_a=[\dot{q}_1^a, \dot{q}_2^a, \ldots, \dot{q}_n^a]^T$ represent the vectors of position and velocity of the actuators, respectively, $\mathbf{q}_d=[q_1^d, q_2^d, \ldots, q_n^d]^T$ and $\dot{\mathbf{q}}_d=[\dot{q}_1^d, \dot{q}_2^d, \ldots, \dot{q}_n^d]^T$ represent the vectors of position and velocity of the controlled links, respectively, i.e., the position and velocity of the links that are controlled by the displacement of the actuators in which there are elasticity, and $\boldsymbol{\tau}$ is the vector of the actuators efforts.

However, for a parallel mechanism, the position (velocity, resp.) of the end-effector is a nontrivial function of the position (velocity, resp.) of the controlled links; therefore, it is preferable to rewrite Eq. (1) using the Lagrange multipliers, as follows:

$$\mathbf{0} = \mathbf{W_b} + \mathbf{B}^T\boldsymbol{\lambda}, \quad \mathbf{W}_b = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_d}\right) - \frac{\partial L}{\partial \mathbf{q}_d} \tag{2a}$$

$$\boldsymbol{\tau} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right) - \frac{\partial L}{\partial \mathbf{q}_a} \tag{2b}$$

where $\boldsymbol{\lambda}$ is the Lagrange multipliers vector, which is related to the wrench applied on the platform by

$$\lambda = \mathbf{A}^{-T}\mathbf{W_p}, \quad \mathbf{W_p} = \left(\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{v}}\right) - \frac{\partial L}{\partial \mathbf{x}}\right) \tag{3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the two matrices relating the vectors $\mathbf{v}$ and $\dot{\mathbf{q}}$ according to $\mathbf{Av}=\mathbf{B}\dot{\mathbf{q}}_d$ (they can be found by differentiating the closure equations with respect to time), $\mathbf{x}=[x, y, z, \phi, \psi, \theta]^T$ and $\mathbf{v}=[\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\psi}, \dot{\theta}]^T$ are the trajectory parameters and their derivatives, respectively (x, y, z represent the position of the controlled point, and ϕ, ψ, and θ represent the rotation of the platform about three axes $\mathbf{a}_\phi$, $\mathbf{a}_\psi$, and $\mathbf{a}_\theta$), and $\mathbf{W_p}$ is the wrench applied on the platform by the legs and external forces [27] expressed along axes $\mathbf{a}_\phi$, $\mathbf{a}_\psi$, and $\mathbf{a}_\theta$.

Expressing $\mathbf{W_p}$ in the base frame, one can obtain:

$$0 = \mathbf{W_b} + \mathbf{J}^{T\,\mathbf{R}_0}\mathbf{W_p} \tag{4a}$$

$$\boldsymbol{\tau} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right) - \frac{\partial L}{\partial \mathbf{q}_a} \tag{4b}$$

where $\mathbf{J}=({}^{\mathbf{R}_0}\mathbf{A})^{-1}\mathbf{B}$ is the Jacobian matrix between twist $\mathbf{t}$ of the platform (expressed in the base frame) and $\dot{\mathbf{q}}_d$, ${}^{\mathbf{R}_0}\mathbf{A}=\mathbf{AD}$ is the expression of matrix $\mathbf{A}$ in the base frame, where $\mathbf{D}$ is a transformation matrix, of which expression is given in Ref. [28].

For any prescribed trajectory $\mathbf{x}(t)$, the values of vectors $\ddot{\mathbf{q}}_d$, $\dot{\mathbf{q}}_d$, and $\mathbf{q}_d$ can be found using the inverse kinematics. Thus, taking into account that the manipulator is not in a type 1 singularity [1], the terms $\mathbf{W_b}$ and ${}^{\mathbf{R}_0}\mathbf{W_p}$ can be computed. However, for a trajectory passing through a type 2 singularity, the determinant of matrix $\mathbf{J}$ is indefinite. Numerically, the values of the efforts applied by the actuators become infinite. In practice, the manipulator is either locked in such a position of the end-effector or it cannot follow the prescribed trajectory.

It is known that a type 2 singularity appears when the determinant of matrix ${}^{\mathbf{R}_0}\mathbf{A}$ vanishes, in other words, when at least two of its columns are linearly dependant [28]. So, one may obtain such a relationship

$$\sum_{j=1}^{6} \alpha_j \mathbf{A}_j = \mathbf{0} \tag{5}$$

where $\mathbf{A}_j$ represents the jth column of the matrix ${}^{\mathbf{R}_0}\mathbf{A}$ and α_j are the coefficients, which, in general, can be functions of q_p^d ($p=1,\ldots,n$). It should be noted that the vector $\mathbf{t_s}=[\alpha_1, \alpha_2, \ldots, \alpha_6]^T$ represents the direction of the uncontrollable motion of the platform in a type 2 singularity.

By substituting Eq. (5) into Eq. (3), we obtain

$$\mathbf{A}_j^T\boldsymbol{\lambda} = W_j, \quad j=1,\ldots,6 \tag{6}$$

where W_j is the jth row of vector ${}^{\mathbf{R}_0}\mathbf{W_p}$.

Then, from Eqs. (5) and (6), the following conditions are derived:

$$\sum_{j=1}^{6} (\alpha_j \mathbf{A}_j^T \boldsymbol{\lambda}) = \sum_{j=1}^{6} (\alpha_j W_j) = 0 \tag{7}$$

The right term of Eq. (7) corresponds to the scalar product of vectors $\mathbf{t_s}$ and ${}^{\mathbf{R}_0}\mathbf{W_p}$.

Thus, in the presence of a type 2 singularity, it is possible to satisfy condition (7) if the wrench applied on the platform by the legs and external efforts ${}^{\mathbf{R}_0}\mathbf{W_p}$ are orthogonal to the direction of the uncontrollable motion $\mathbf{t_s}$. Otherwise, the dynamic model is not consistent. Obviously, in the presence of a type 2 singularity, the displacement of the end-effector of the manipulator has to be planned to satisfy Eq. (7). Therefore, our task will be to achieve a trajectory, which will allow the manipulator to pass trough the type 2 singularities, i.e., which will allow the manipulator respecting condition (7).

In the dynamic model of the rigid-link flexible-joint manipulator [26], the efforts $\boldsymbol{\tau}$ applied on the actuators may be expressed as follows:

$$\boldsymbol{\tau} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_d}\right) - \frac{\partial L}{\partial \mathbf{q}_d} = \mathbf{M}(\mathbf{q}_d)\ddot{\mathbf{q}}_d + \mathbf{C}(\mathbf{q}_d, \dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d + \mathbf{D}(\mathbf{q}_d) \tag{8}$$

This equation only depends on the acceleration, velocity, and position of the actuators of the manipulator. Therefore, in order to avoid some discontinuity on the efforts $\boldsymbol{\tau}$, the polynomial used for the trajectory planning should be at least of the fifth degree (because the initial and final positions are known, and the velocities and accelerations at the beginning and end of the trajectory should be equal to 0). In our previous work [25], it was shown that the condition for passing through the type 2 singular configurations added three supplementary conditions, and therefore, the polynomial used for the trajectory planning should be at least of the

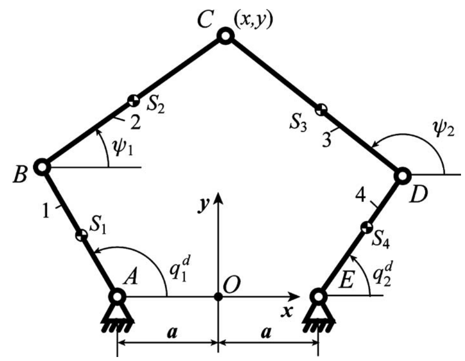

Fig. 1 Kinematic chain of the planar 5R parallel manipulator

eighth degree (the position, velocity, and acceleration when passing through the singularity should be constrained).

In the present study, it will be shown that the degree of the polynomial law should be different when taking into account the flexibility on the actuating system. Indeed, it will be presented further that the efforts $\boldsymbol{\tau}$ applied on the actuators depend not only on the position, velocity, and acceleration of the actuators, but also on the jerk and its first derivative. So, due to the addition of elasticity in the actuated joints, the polynomial used for the trajectory planning should be at least of the 12th degree.

In Sec. 3, an example illustrates the obtained results discussed above. This example presents a planar 5R parallel manipulator, which allows obtaining relatively simple symbolic model for demonstrating the expected results by numerical simulations. The results are validated using the ADAMS software.

3 Illustrative Example

In the planar 5R parallel manipulator, as shown in Fig. 1, the output axis is connected to the base by two legs, each of which consists of three revolute joints and two links. In each of the two legs, the revolute joint connected to the base is actuated. Thus, such a manipulator is able to position its output axis in a plane.

As shown in Fig. 1, the input joints are denoted as A and E with the input parameters q_1^d and q_2^d. The common joint of the two legs is denoted as C, which is also the output point with the controlled parameters x and y. A fixed global reference system xOy is located at the center of AE with the y-axis normal to AE and the x-axis directed along AE. The lengths of the links AB, BC, CD, and DE are denoted as L_1, L_2, L_3, and L_4, respectively. The positions of the centers of masses S_i of links from joint centers A, B, D, and E are denoted by the dimensionless lengths r_1, r_2, r_3, and r_4, respectively, i.e., $AS_1=r_1L_1$, $BS_2=r_2L_2$, $DS_3=r_3L_3$, and $ES_4=r_4L_4$.

Actuators 1 and 2 are connected to links 1 and 4, respectively, via harmonic drive systems, which are presented by a model similar to that given in Ref. [26]. The position of actuator i is denoted as q_i^a. It is assumed that the actuator i is capable to deliver a couple τ_i to the motor shaft, which is elastically coupled to the link j of the robot ($i=1$ or 2, $j=1$ or 4). The flexibility is modeled by a torsion spring with stiffness k_i. The gear ratio is denoted as n_i. In the following parts of this paper, $n_1=n_2=n$ and $k_1=k_2=k$. I_i^a is the axial moment of inertia of the motor i plus the Harmonic drive system.

The singularity analysis of this manipulator [29] shows that the type 2 singularities appear when legs 2 and 3 are parallel (see also Fig. 2 in Ref. [25]).

In both cases, the gained degree-of-freedom is an infinitesimal translation perpendicular to legs 2 and 3. However, if $L_2=L_3$, the gained degree-of-freedom may become a finite rotary motion.

In order to simplify the analytic expressions, we consider that the gravity effects are along the z-axis, and consequently, the input torques are only due to the inertia effects. We also admit that there is no friction in the system. To simplify the computation, it is also preferable to replace the masses of the moving links by the concentrated masses [30,31]. For a link i with mass m_i and its axial moment of inertia I_i, we have

$$\begin{bmatrix} 1 & 1 & 1 \\ r_i & 0 & 1-r_i \\ r_i^2L_i^2 & 0 & (1-r_i)^2L_i^2 \end{bmatrix}\begin{bmatrix} m_{i1} \\ m_{i2} \\ m_{i3} \end{bmatrix} = \begin{bmatrix} m_i \\ 0 \\ I_i \end{bmatrix} \quad (i=1,2,3,4) \tag{9}$$

where m_{ij} $(j=1,2,3)$ are the values of the three point masses placed at the centers of the revolute joints and at the center of masses of the link i.

In this case, the potential energy V can be written as

$$V=\frac{k}{2}(\mathbf{q}_d+\mathbf{q}_a/n)^T(\mathbf{q}_d+\mathbf{q}_a/n) \tag{10}$$

where $\mathbf{q}_d=[q_1^d,q_2^d]^T$ and $\mathbf{q}_a=[q_1^a,q_2^a]^T$, and the kinetic energy T can be written as

$$T=\frac{1}{2}(m_{S1}\mathbf{V}_{S1}^2+m_{S2}\mathbf{V}_{S2}^2+m_{S3}\mathbf{V}_{S3}^2+m_{S4}\mathbf{V}_{S4}^2+m_B\mathbf{V}_B^2+m_C\mathbf{V}_C^2 + m_D\mathbf{V}_D^2)+\frac{1}{2}I_a\dot{\mathbf{q}}_a^T\dot{\mathbf{q}}_a \tag{11}$$

where $m_{S1}=m_{12}$, $m_{S2}=m_{22}$, $m_{S3}=m_{32}$, $m_{S4}=m_{42}$, $m_B=m_{13}+m_{21}$, $m_C=m_{23}+m_{21}$, and $m_D=m_{33}+m_{41}$. The terms m_{ij} $(i=1,2,3,4)$ are deduced from the relation (9), $\mathbf{V}_{Si}$ is the vector of the linear velocities of the center of masses S_i, and $\mathbf{V}_B$, $\mathbf{V}_C$, and $\mathbf{V}_D$ are the vectors of the linear velocities of the corresponding axes.

Thus the dynamic model can be obtained from Eq. (2)

$$\mathbf{0}=\mathbf{W_b}+\mathbf{J}_{5R}^T\mathbf{W_p} \tag{12a}$$

$$\boldsymbol{\tau}=I_a\ddot{\mathbf{q}}_a+\frac{k}{n}(\mathbf{q}_d+\mathbf{q}_a/n) \tag{12b}$$

taking into account that for the examined manipulator

$$\mathbf{W_b}=\mathbf{W_b^*}+k(\mathbf{q}_d+\mathbf{q}_a/n), \quad \mathbf{W_b^*}=\mathbf{J_B^T}\mathbf{F_B}+\mathbf{J_D^T}\mathbf{F_D} \tag{13}$$

where

$$\mathbf{J_B}=\begin{bmatrix} -L_1\sin q_1 & 0 \\ L_1\cos q_1 & 0 \end{bmatrix}, \quad \mathbf{J_D}=\begin{bmatrix} 0 & -L_4\sin q_2 \\ 0 & L_4\cos q_2 \end{bmatrix} \tag{14}$$

$$\mathbf{F_B}=m_{B1}\boldsymbol{\Gamma}_\mathbf{B}+m_{C1}\boldsymbol{\Gamma}_\mathbf{C}, \quad \mathbf{F_D}=m_{D2}\boldsymbol{\Gamma}_\mathbf{D}+m_{C3}\boldsymbol{\Gamma}_\mathbf{C} \tag{15}$$

$$\boldsymbol{\Gamma}_\mathbf{B}=L_1\left(\ddot{q}_1\begin{bmatrix} -\sin q_1 \\ \cos q_1 \end{bmatrix}-\dot{q}_1^2\begin{bmatrix} \cos q_1 \\ \sin q_1 \end{bmatrix}\right), \quad \boldsymbol{\Gamma}_\mathbf{D}=L_4\left(\ddot{q}_2\begin{bmatrix} -\sin q_2 \\ \cos q_2 \end{bmatrix} -\dot{q}_2^2\begin{bmatrix} \cos q_2 \\ \sin q_2 \end{bmatrix}\right), \quad \boldsymbol{\Gamma}_\mathbf{C}=\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \tag{16}$$

$$m_{B1}=m_{S1}r_1^2+m_B+m_{S2}(1-r_2)^2, \quad m_{C1}=m_{S2}r_2(1-r_2) \tag{17}$$

$$m_{C3}=m_{S3}r_3(1-r_3), \quad m_{D2}=m_{S4}r_4^2+m_D+m_{S3}(1-r_3)^2 \tag{18}$$

The term $\mathbf{W_p}$ is given by

$$\mathbf{W_p}=m_{C1}\boldsymbol{\Gamma}_\mathbf{B}+m_{C2}\boldsymbol{\Gamma}_\mathbf{C}+m_{C3}\boldsymbol{\Gamma}_\mathbf{D} \tag{19}$$

$$m_{C2}=m_{S2}r_2^2+m_C+m_{S3}r_3^2 \tag{20}$$

and the Jacobian matrix $\mathbf{J}_{5R}$ is given by

$$\mathbf{J}_{5R}=\mathbf{A}_{5R}^{-1}\mathbf{B}_{5R} \tag{21}$$

where

$$\mathbf{A}_{5R}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=2\begin{bmatrix} x-L_1\cos q_1+a & y-L_1\sin q_1 \\ x-L_4\cos q_2-a & y-L_4\sin q_2 \end{bmatrix} \tag{22}$$

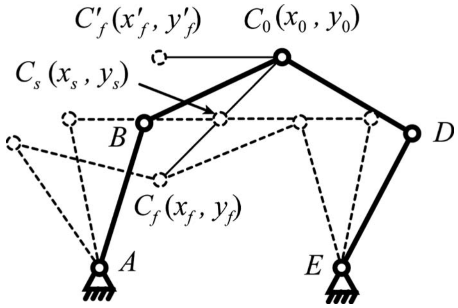

Fig. 2 Initial, singular, and final positions of the planar 5R parallel manipulator

$$\mathbf{B}_{5R} = -\begin{bmatrix} L_1(a_{11}\sin q_1 - a_{12}\cos q_1) & 0 \\ 0 & L_4(a_{21}\sin q_2 - a_{22}\cos q_2) \end{bmatrix} \tag{23}$$

For a given trajectory $\mathbf{q}_d$, the vector of the positions of the actuators $\mathbf{q}_a$ can be deduced from Eq. (12*a*), as well as the acceleration of the actuators $\ddot{\mathbf{q}}_a$

$$\mathbf{q}_a = -\frac{n}{k}(\mathbf{W}_\mathbf{b}^* + \mathbf{J}_{5R}^T\mathbf{W}_\mathbf{p}) - n\mathbf{q}_d \tag{24a}$$

$$\ddot{\mathbf{q}}_a = -\frac{n}{k}\frac{d^2}{dt^2}(\mathbf{W}_\mathbf{b}^* + \mathbf{J}_{5R}^T\mathbf{W}_\mathbf{p}) - n\ddot{\mathbf{q}}_d \tag{24b}$$

Introducing Eq. (24) into Eq. (12*b*), one can deduce the vector of actuator torques $\boldsymbol{\tau}$

$$\boldsymbol{\tau} = -I_a\left(\frac{n}{k}\frac{d^2}{dt^2}(\mathbf{W}_\mathbf{b}^* + \mathbf{J}_{5R}^T\mathbf{W}_\mathbf{p}) + n\ddot{\mathbf{q}}_d\right) - \frac{1}{n}(\mathbf{W}_\mathbf{b}^* + \mathbf{J}_{5R}^T\mathbf{W}_\mathbf{p}) \tag{25}$$

Analyzing this expression, it could be observed that, as terms $\mathbf{W}_\mathbf{b}^*$ and $\mathbf{W}_\mathbf{p}$ depend on the position, velocity, and acceleration of the input links, the input torques depend not only on these parameters, but also on the jerk and its first derivative. Therefore, on the contrary to rigid manipulators for which, in order to avoid discontinuities in the input torques, a fifth-degree polynomial is sufficient as a control law when the end-effector is not in singular configuration, for nonrigid robots, the degree of the polynomial should be increased (indeed, it should be at least a ninth-degree polynomial).

In order to avoid infinite values of the input torques when crossing a type 2 singularity, Eq. (7) has to be satisfied. From matrix $\mathbf{A}_{5R}$, one can find that the twist of the infinitesimal displacement in the singularity can be written under the form

$$\mathbf{t}_\mathbf{s} = [-\sin\psi_1, \cos\psi_1]^T \tag{26}$$

Thus, the examined manipulator can pass through the given singular positions if the wrench $\mathbf{W}_\mathbf{p}$ determined by Eq. (19) is orthogonal to the direction of the uncontrollable motion $\mathbf{t}_\mathbf{s}$ described by Eq. (26).

Let us now consider the motion planning, which makes it possible to satisfy this condition. For this purpose the following parameters of the manipulator's links are specified: $L_1=L_2=L_3=L_4=0.25$ m; $r_1=r_2=r_3=r_4=0.5$; $a=0.2$ m; $m_1=m_4=2.81$ kg; $I_1=I_4=0.02$ kg m^2; $m_2=m_3=1.41$ kg; $I_2=I_3=0.01$ kg m^2; $I_a=0.067$ kg/m^2; $k=250$ N m/rad; and $n=50$.

With regard to the prescribed trajectory generation, point C should reproduce a motion along a straight line between the initial position C_0 $((x_0,y_0)=C_0)$ (0.1,0.345) and the final point C_f $((x_f,y_f)=C_f)$ (−0.1, 0.145) in $t_f=2$ s (Fig. 2).

Thus, the given trajectory can be expressed as follows:

$$\mathbf{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 + s(t)(x_f - x_0) \\ y_0 + s(t)(y_f - y_0) \end{bmatrix} \tag{27}$$

However, the manipulator will pass by a type 2 singular position at point C_s $((x_s,y_s)=C_s)$ (0,0.245) (Fig. 2).

Developing the condition for passing through the singular position (7) for the planar 5r parallel manipulator at point C_s, we obtain

$$m_{C1}L_1(248\dot{x}^2 - 48\dot{y}^2) - 3\sqrt{6}m_{C2}\ddot{y} = 0 \tag{28}$$

Then, taking into account that the velocity and the acceleration of the end-effector in the initial and final positions are equal to zero, the following 13 boundary conditions are found:

$$s(t_0) = 0 \tag{29}$$

$$s(t_f) = 1 \tag{30}$$

$$s(t_s = 1\ \text{s}) = 0.5 \tag{31}$$

$$\dot{s}(t_0) = 0 \tag{32}$$

$$\dot{s}(t_f) = 0 \tag{33}$$

$$\dot{s}(t_s) = \dot{y}_s/(y_f - y_0) = \dot{x}_s/(x_f - x_0) = 1 \tag{34}$$

$$\ddot{s}(t_0) = \ddot{s}_0 = 0 \tag{35}$$

$$\ddot{s}(t_f) = \ddot{s}_f = 0 \tag{36}$$

$$\ddot{s}(t_s) = \ddot{s}_s = m_{C1}L_1(248\dot{x}_s^2 - 48\dot{y}_s^2)/(3(y_f - y_0)\sqrt{6}m_{C2}) \tag{37}$$

$$\dddot{s}(t_0) = \dddot{s}_0 = 0 \tag{38}$$

$$\dddot{s}(t_f) = \dddot{s}_f = 0 \tag{39}$$

$$\frac{d}{dt}\dddot{s}(t_0) = s_0^{(4)} = 0 \tag{40}$$

$$\frac{d}{dt}\dddot{s}(t_f) = s_f^{(4)} = 0 \tag{41}$$

From Eqs. (28)–(41), the following 12th-order polynomial trajectory planning is found

$$s(t) = 7.28t^5 - 14.22t^6 + 6.80t^7 + 6.41t^8 - 9.77t^9 + 5.15t^{10} - 1.28t^{11} + 0.13t^{12} \tag{42}$$

Thus the generation of the motion by the obtained 12th-order polynomial makes it possible to pass through the singularity without perturbation, and the input torques remain in the limits of finite values.

In order to compare the different cases of trajectory planning, in Table 1 are given the values of the input torques obtained using the software ADAMS for the following numerical simulations:

(1) Case A1: A trajectory between points C_0 and $C'_f(x'_f,y'_f)=C'_f$ (−0.1,0.345) (Fig. 2) without meeting any singularity. For such a case, the following fifth-order polynomial law is used $s(t)=1.25t^3-0.9375t^4+0.1875t^5$ for the trajectory planning out of the singular zone of the *rigid-link* manipulator without taking into account the flexibility in the actuated joints. In this case the values of the input torques are finite.

(2) Case A2: The fifth-order polynomial law $s(t)=1.25t^3-0.9375t^4+0.1875t^5$ for the trajectory planning between C_0 and C_f inside the singular zone for the rigid-link manipulator without taking into account the flexibility in the actuated joints. In this case the values of the input torques close to the singular positions tend to infinity.

Table 1 Variation in the input torques as a function of the polynomial law used for the trajectory

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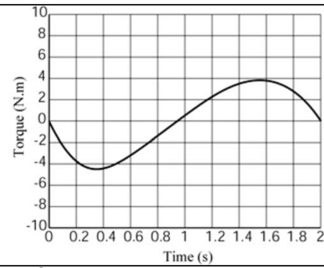
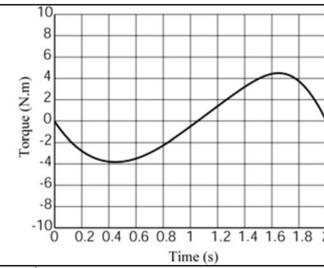
<tr><td>Case (A1)</td><td>
</td><td>
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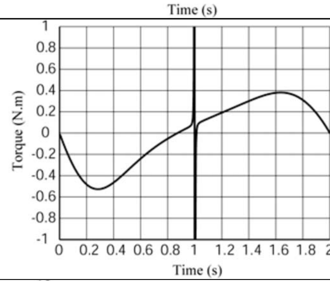
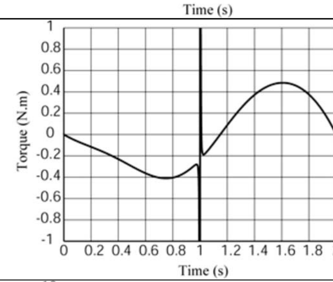
<tr><td>Case (A2)</td><td>
</td><td>
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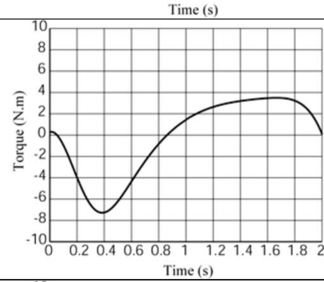
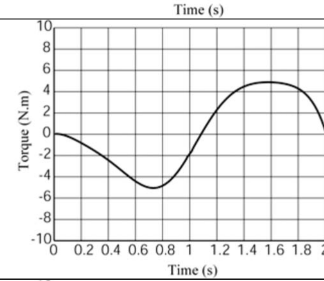
<tr><td>Case (B)</td><td>
</td><td>
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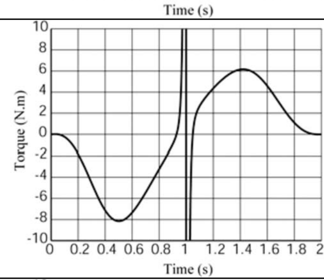
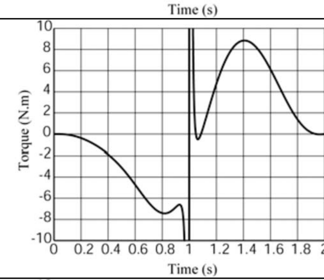
<tr><td>Case (C)</td><td>
</td><td>
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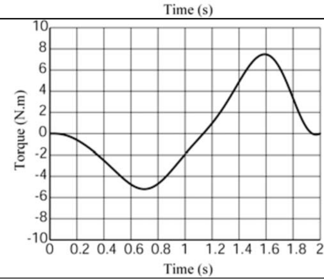
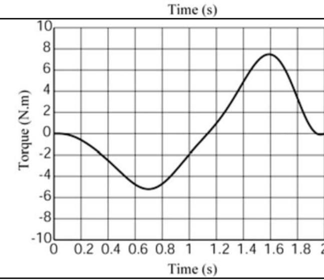
<tr><td>Case (D)</td><td>
</td><td>
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(3) Case B: The eight order polynomial law $s(t)=-0.25851t^3+3.84228t^4-5.72792t^5+3.58909t^6-1.07101t^7+0.12606t^8$ for the trajectory planning of the rigid-link manipulator without flexibility in the actuated joints inside the singular zone. The obtained results show that the values of the input torques are finite.

(4) Case C: The ninth-order polynomial law $s(t)=3.94t^5-6.56t^6+4.22t^7-1.23t^8+0.14t^9$ for the trajectory planning of the rigid-link *flexible-joint* manipulator inside the singular zone. The numerical simulation shows that the values of the input torques close to the singular positions tend to infinity.

(5) Case D: The 12th-order polynomial law (Eq. (42)) for the trajectory planning of the rigid-link flexible-joint manipulator inside of the singular zone. The values of the input torques are finite and there are no discontinuities.

Thus, the numerical simulations show that the obtained optimal dynamic conditions assume the passing of the rigid-link flexible-joint manipulator through the singular position.

4 Conclusion

At a singular configuration, in the case of an arbitrary generation of forces, a manipulator may not reproduce stable motion with the prescribed trajectory. Nevertheless it is approved that there are several motion planning techniques, which allow passing through these singular zones. These approaches are simulated by numerical examples and illustrated on several parallel structures.

However, in these studies, much more attention was focused only on the control aspects of this problem, and little attention has been paid to the dynamic interpretation, which is a crucial factor for governing the behavior of parallel manipulators at the singular zones.

In our previous work [25], the dynamic properties of parallel manipulators in the presence of type 2 singularity have been studied. It was shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts of the manipulator are orthogonal to the twist along the direction of the uncontrollable motion. This condition was applied to the rigid-link manipulators without clearance or flexibility in the joints. The obtained results showed that the planning of motion for assuming the optimal force generation can be carried out by an eight-order polynomial law.

In the present paper the rigid-link flexible-joint manipulators have been studied. It was shown that the degree of the polynomial law should be different, when the flexibility of actuated joints is introduced into conditions of the optimal force generation in the presence of singularity. The obtained results disclosed that the planning of motion for assuming the optimal force generation in the rigid-link flexible-joint manipulators must be carried out by a 12th-order polynomial law. The suggested technique was illustrated by an example, which presents a 5R planar parallel manipulator with flexible joints. The numerical simulations carried out using the software ADAMS validated the obtained theoretical results.

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