Fully Lexicalized Pregroup Grammars

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Abstract. Pregroup grammars are a context-free grammar formalism introduced as a simplification of Lambek calculus. This formalism is interesting for several reasons: the syntactical properties of words are specified by a set of types like the other type-based grammar formalisms ; as a logical model, compositionality is easy ; a polytime parsing algorithm exists.
However, this formalism is not completely lexicalized because each pregroup grammar is based on the free pregroup built from a set of primitive types together with a partial order, and this order is not lexical information. In fact, only the pregroup grammars that are based on primitive types with an order that is equality can be seen as fully lexicalized.
We show here how we can transform, using a morphism on types, a particular pregroup grammar into another pregroup grammar that uses the equality as the order on primitive types. This transformation is at most quadratic in size (linear for a fixed set of primitive types), it preserves the parse structures of sentences and the number of types assigned to a word.

Key words: Pregroups, Lambek Categorial Grammars, Simulation.

1 Introduction

Pregroup grammars (PG) [1] have been introduced as a simplification of Lambek calculus [2]. Several natural languages has been modelized using this formalism: English [1], Italian [3], French [4], German [5, 6], Japanese [7], etc. PG are based on the idea that sentences are produced from words using lexical rules. The syntactical properties of each word is characterized by a finite set of types stored in the lexicon. The categories are types of a free pregroup generated by a set of primitive types together with a partial order on the primitive types. This partial order is not independent of the language that corresponds to a PG. For instance, this order is used for English to specify that a yes-or-no question (primitive type q) is a correct sentence (primitive type s). Thus the partial order says that $q \leq s$.

This partial order is not lexicalized but corresponds to global rules that are specific to a particular PG. Is it then possible to find a PG that is equivalent to a given PG but where the partial order on primitive types is *universal*? Moreover, we hope that the computed PG is not *too big* if we compare it to the source PG and if it works in a *similar* way as the source PG.

Using mathematical words, it means that transformation must be polynomial in space (the size of a resulting grammar is polynomially bounded by the size of the source grammar) and should be a homomorphism from the source free pregroup to the resulting free pregroup that must also satisfy the converse of the monotonicity condition.

The paper defines such a transformation. The size of the resulting grammar is bounded by a second degree polynomial of the initial grammar. Moreover, for all the PG that are based on the same free pregroup, this transformation is linear. The size of the tranformed PG is bounded by the size of the source PG four times the number of primitive types of the free pregroup of this PG, plus a constant. This transformation defines a homomorphism of (free) pregroups. A pregroup grammar is transformed into another pregroup grammar that associates the same number of types to a word as the source grammar: a k-valued grammar is transformed into a k-valued grammar.

After an introduction about PG in section 2, the paper proves several lemmas that are necessary for the correctness of our transformation (the source and the resulting PG must define the same language). The transformation is defined in section 4.2, its properties are detailed in the remaining sections. Section 7 concludes.

2 Background

Definition 1 (Pregroup). *A* pregroup *is a structure* $(P, \leq, \cdot, l, r, 1)$ *such that* $(P, \leq, \cdot, 1)$ *is a partially ordered monoid*[3] *and* l, r *are two unary operations on* P *that satisfy for all primitive type* $a \in P$ $a^l a \leq 1 \leq a a^l$ *and* $a a^r \leq 1 \leq a^r a$.

Definition 2 (Free pregroup). *Let* $(P, \leq)$ *be an ordered set of primitive types,* $P^{(\mathbb{Z})} = \{p^{(i)} \mid p \in P, i \in \mathbb{Z}\}$ *is the set of atomic types and* $T_{(P,\leq)} = \left(P^{(\mathbb{Z})}\right)^* = \{p_1^{(i_1)} \cdots p_n^{(i_n)} \mid 0 \leq k \leq n, p_k \in P \text{ and } i_k \in \mathbb{Z}\}$ *is the set of types. The empty sequence in* $T_{(P,\leq)}$ *is denoted by 1. For* X *and* $Y \in T_{(P,\leq)}$, $X \leq Y$ *iff this relation is deductible in the following system where* $p, q \in P$, $n, k \in \mathbb{Z}$ *and* $X, Y, Z \in T_{(P,\leq)}$*:*

[3] We briefly recall that a *monoid* is a structure $< M, \cdot, 1 >$, such that $\cdot$ is associative and has a neutral element 1 ($\forall x \in M : 1 \cdot x = x \cdot 1 = x$). A partially ordered monoid is a monoid $< M, \cdot, 1 >$ with a partial order $\leq$ that satisfies $\forall a, b, c$: $a \leq b \Rightarrow c \cdot a \leq c \cdot b$ and $a \cdot c \leq b \cdot c$.

$$X \leq X \ (Id) \qquad \frac{X \leq Y \quad Y \leq Z}{X \leq Z}\,(Cut)$$

$$\frac{XY \leq Z}{Xp^{(n)}p^{(n+1)}Y \leq Z}\,(A_L) \quad \frac{X \leq YZ}{X \leq Yp^{(n+1)}p^{(n)}Z}\,(A_R)$$

$$\frac{Xp^{(k)}Y \leq Z}{Xq^{(k)}Y \leq Z}\,(IND_L) \quad \frac{X \leq Yq^{(k)}Z}{X \leq Yp^{(k)}Z}\,(IND_R)$$

$q \leq p$ if k is even, and $p \leq q$ if k is odd

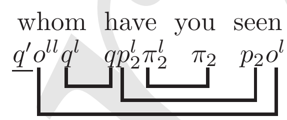
This construction, proposed by Buskowski, defines a pregroup that extends $\leq$ on primitive types P to $T_{(P,\leq)}$[4] [5].

The Cut Elimination. As for L and NL, the cut rule can be eliminated: every derivable inequality has a cut-free derivation.

Definition 3 (Simple free pregroup). *A* simple free pregroup *is a free pregroup where the order on primitive type is equality.*

Definition 4 (Pregroup grammar). *Let $(P, \leq)$ be a finite partially ordered set. A* pregroup grammar *based on $(P, \leq)$ is a lexicalized*[6] *grammar $G = (\Sigma, I, s)$ such that $s \in T_{(P,\leq)}$; G assigns a type X to a string $v_1, \ldots, v_n$ of Σ^* iff for $1 \leq i \leq n$, $\exists X_i \in I(v_i)$ such that $X_1 \cdots X_n \leq X$ in the free pregroup $T_{(P,\leq)}$.* The language *$\mathcal{L}(G)$ is the set of strings in Σ^* that are assigned s by G.*

Example 1. Our example is taken from [8] with the primitive types: π_2 = second person, p_2 = past participle, o = object, q = yes-or-no question, q' = question. This sentence gets type q' ($q' \leq s$):



Remark on Types for Correct Sentences. Usually, type s associated to correct sentences must be a primitive type ($s \in P$). However, we use here a more general definition where s can be any type in $T_{(P,\leq)}$. Our definition does not lead to a significant modification of PG (the classes of languages is the same) because proving that $X_1 \cdots X_n \leq X$ is equivalent[7] to prove that $X_1 \cdots X_n X^r \leq 1$.

[4] Left and right adjoints are defined by $(p^{(n)})^l = p^{(n-1)}$, $(p^{(n)})^r = p^{(n+1)}$, $(XY)^l = Y^l X^l$ and $(XY)^r = Y^r X^r$. We write p for $p^{(0)}$. We also iterate left and right adjoints for every $X \in T_{(P,\leq)} : X^{(0)} = X$, $X^{(n+1)} = (X^r)^{(n)}$ and $X^{(n-1)} = (X^l)^{(n)}$

[5] $\leq$ is only a preorder. Thus, in fact, the pregroup is the quotient of $T_{(P,\leq)}$ by the equivalence relation $X \leq Y \& Y \leq X$.

[6] a lexicalized grammar is a triple (Σ, I, s): Σ is a finite alphabet, I assigns a finite set of categories (or types) to each $c \in \Sigma$, s is a category (or type) associated to correct sentences.

[7] if $Y \leq X$ then $YX^r \leq XX^r \leq 1$; if $YX^r \leq 1$ then $Y \leq YX^rX \leq X$

We must notice that the transformation given in Section 4 works if we use a generalized definition of PG as above, where the type associated to correct sentences is not necessarilly a primitive type or if s is neither $\leq$ nor $\geq$ to any other primitive type and thus does not need to be changed by the transformation.

In fact, it is always possible to transform a PG using a composed type S for correct sentences into a PG using a primitive type s for correct sentences that is not related to other primitive types by the addition of a right *wall* $S^r s$ (the type can be seen as the type of the final point of a sentence) for the parsing with the second PG because $X \leq S \Leftrightarrow X S^r s \leq s$.

3 A Preliminary Fact

Proposition 1 (PG equivalence and generative capacity). *Let* $(P, \leq)$ *be an ordered set of primitive types, for any pregroup gammar* G *on* $(P, \leq)$*, we can construct a PG* G' *based on a simple free pregroup on* $(P', =)$*, a set of primitive types with a discrete order, such that* G *and* G' *have the same language.*

Proof. This is obvious because PGs define context-free languages and every context-free language corresponds to an order 1 classical categorial grammars[8] that can be easily simulated by an "order 1" simple PG [9].
Another construct is to duplicate the lexical types for each occurrence involved in an ordering.

Note. However both transformations do not preserve the size of the lexicon in general because the initial types associated to a word can correspond to an exponential number of types in the transformed PG. Moreover, the transformations do not define a pregroup homomorphism.

We look for a transformation that is polynomial in space (the size of a resulting grammar is polynomially bounded by the size of the source grammar) and that is defined using a homomorphism from the source free pregroup to the resulting free pregroup (that must also satisfy the converse of the monotonicity condition). Next section defines such a transformation. The correctness is given in the last sections and Appendix.

4 A Pregroup Morphism

4.1 Properties of Free Pregroup Morphisms

In order to simulate a free pregroup by another one with less order postulates, we consider particular mappings on posets that can be extended as a pregroup homomorphism from the free pregroup based on $(P, \leq)$ to the free pregroup based on another poset $(P', \leq')$.

[8] the order of primitive types is 0 and the order $order(A)$ of compound types is: $order(A/B) = order(B \backslash A) = max(order(A, 1 + order(B)))$

Definition 5 (Order-preserving mapping, pregroup homomorphism).

- *A mapping from a poset* $(P, \leq)$ *to a poset* $(P', \leq')$ *is said* order-preserving *iff for all* $p_i, p_j \in P : p_i \leq p_j$ *implies* $h(p_i) \leq' h(p_j)$.
- *A mapping h from the free pregroup on* $(P, \leq)$ *to the free pregroup on* $(P', \leq')$, *is a* pregroup homomorphism *iff*
  1. $\forall X \in T_{(P,\leq)} : h(X^{(n)}) = h(X)^{(n)}$
  2. $\forall X, Y \in T_{(P,\leq)} : h(XY) = h(X)h(Y)$
  3. $h(1) = 1$
  4. $\forall X, Y \in T_{(P,\leq)} :$ *if* $X \leq Y$ *then* $h(X) \leq' h(Y)$ *[Monotonicity]*

Proposition 2. *Each order-preserving mapping from* $(P, \leq)$ *to* $(P', \leq')$ *can be uniquely extended to a unique pregroup homomorphism on* $T_{(P,\leq)}$ *to* $T_{(P',\leq')}$.

Proof. The unicity comes from the three first points of the definition of pregroup homomorphism. The last point is a consequence of order-preservation which is easy by induction on a derivation $\mathcal{D}$ for $X \leq Y$, considering the last rule:
- this holds clearly for an axiom: $h(X) \leq' h(X)$;
- if the last rule introduces $p_i^{(n)} p_i^{(n+1)}$ on the left, we have $X = Z_1 p_i^n p_i^{n+1} Z_2$ and $Z_1 Z_2 \leq Y$ as previous conclusion in $\mathcal{D}$, by induction we get $h(Z_1)h(Z_2) \leq' h(Y)$, and by morphism: $h(p_i^{(n)} p_i^{(n+1)}) = h(p_i)^{(n)} h(p_i)^{(n+1)} \leq' 1$, therefore $h(X) = h(Z_1)h(p_i)^{(n)} h(p_i)^{(n+1)} h(Z_2) \leq' h(Y)$
- if the last rule uses $p_i \leq p_j$ on the left
if n is even, we have: $X = Z_1 p_i^{(n)} Z_2$ and $Z_1 p_j^{(n)} Z_2 \leq Y$ as previous conclusion in $\mathcal{D}$, by induction we get $h(Z_1)h(p_j^{(n)})h(Z_2) \leq' h(Y)$, where by morphism $h(p_j^{(n)}) = h(p_j)^{(n)}$ then: $h(X) = h(Z_1)h(p_i)^{(n)} h(Z_2) \leq' h(Y)$ since by order-preservation $h(p_i) \leq' h(p_j)$
if n is odd, we proceed similarly from $Z_1 p_i^{(n)} Z_2 \leq Y$ as previous conclusion in $\mathcal{D}$
- Rules on the right are similar (or consider $Y = 1$, then morphism properties)

In order to have a simulation, we need a converse of monotonicity.

Proposition 3 (Order-reflecting homomorphism). *Every homomorphism h from the free pregroup on* $(P, \leq)$ *to the free pregroup on* $(P', \leq')$ *is such that (1) and (2) are equivalent conditions and define order-reflecting homomorphisms:*
(1). $\forall X, Y \in T_{(P,\leq)}$ *if* $h(X) \leq' h(Y)$ *then* $X \leq Y$ *by* $(P, \leq)$.
(2). $\forall X \in T_{(P,\leq)}$ *if* $h(X) \leq' 1$ *then* $X \leq 1$ *by* $(P, \leq)$.

Proof. In fact, (2) is obviously a subcase of (1) and (1) is a corollary of (2) as follows: suppose (2) holds for a pregroup-morphism h with (3) $h(X) \leq' h(Y)$, we get (4) $h(X\,Y^r) \leq' 1$ by adding $h(Y)^r$ on the right of (3) as below:

$$h(X\,Y^r) = h(X)\;h(Y)^r \leq' h(Y)\;h(Y)^r \leq' 1$$

property (2) then gives (5) $X\,Y^r \leq 1$,
hence $X \leq Y$ (by adding Y on the right of (5): $X \leq X\,Y^r\,Y \leq Y$).

These order-reflecting properties will be shown in the simulation-morphism defined in the next subsection.

4.2 Defining a Simulation-Morphism

Preliminary Remark. The definition has to be chosen carefully as shown by the following pregroup homomorphisms, that are not order-reflecting in the simplest case of one order postulate.

Example 2. Let $P = \{p_0, p_1, \ldots, p_n\}$ and $\leq$ be reduced to one postulate $p_0 \leq p_1$, and let $\leq'$ be $=$:

- take $h(p_1) = q$, and $h(p_0) = \beta\ \beta^{(1)}\ q\ \gamma\ \gamma^{(1)}$ with $h(p_i) = p_i$ if $i \notin \{0, 1\}$ where β and γ are fixed types.
  This defines a homomorphism (using $\beta\beta^{(1)} \leq' 1$ and $\gamma\gamma^{(1)} \leq' 1$).
- If we drop β in h of this example ($\beta = 1$), the resulting translation h_γ is a homomorphism but is not order-reflecting as examplified by: $X = p_0^{(2n-2)} p_0^{(2n-1)} p_1^{(2n)} p_1^{(2n-1)}$, such that $X \not\leq 1$, whereas $h_\gamma(X) \leq' 1$; similarly, if we drop γ in h, defining h_β, $X = p_1^{(2n+1)} p_1^{(2n)} p_0^{(2n+1)} p_0^{(2n+2)}$ is a counter-example for h_β[9]

The Unrestricted Case. In this presentation, we allow to simulate either a fragment (represented as Pr below) or the whole set of primitive types ; for example, we may want not to transform isolated (i.e. not related to another type) primitive types, or to proceed incrementally.

Let $Pr = \{p_1, \ldots, p_n\}$ and $P = Pr \cup Pr'$, where no element of Pr is related by $\leq$ to an element of Pr', and each element of Pr is related by $\leq$ to another element of Pr.

We introduce new letters q and β_k, γ_k, for each element p_k of Pr. [10]

We take as poset $P' = Pr' \cup \{q\} \cup \{\beta_k, \gamma_k \mid p_k \in Pr\}$, with $\leq'$ as the restriction of $\leq$ on Pr' ($\leq'$ is reduced to identity if Pr' is empty, corresponding to the case of a unique connex component).

We then define the simulation-morphism h for Pr as follows:

Definition 6 (Simulation-morphism h for Pr).

$h(X^{(n)}) = h(X)^{(n)}$	$h(p_i) = \underbrace{\lambda_i^{(0)}\ q^{(0)} \delta_i^{(0)}}$	$h(1) = 1$
$h(X.Y) = h(X).h(Y)$	*for* $p_i \in Pr$	$h(p_i) = p_i$ *if* $p_i \in Pr'$

where λ_i *is the concatenation of* α *and all the terms* $\gamma_{k'}^{(1)}\gamma_{k'}$ *for all the indices of the primitive types* $p_{k'} \in P$ *less than or equal to* p_i *and similarly for* δ_i*:*

$$\lambda_i = \alpha \underbrace{\gamma_{k'}^{(1)} \gamma_{k'}}_{if\ p_{k'} \leq p_i} \overset{(downto\, 1)}{\cdots} \underbrace{\gamma_{k'}^{(1)} \gamma_{k'}}_{if\ p_{k'} \leq p_i}\ ,\ \delta_i = \underbrace{\beta_k\, \beta_k^{(1)}}_{if\ p_k \geq p_i} \overset{(from\, 1)}{\cdots} \underbrace{\beta_k\, \beta_k^{(1)}}_{if\ p_k \geq p_i} \alpha'$$

that we also write:

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[9] Details: $X = p_0^{(0)} p_0^{(1)} p_1^{(2)} p_1^{(1)} \overset{h_\gamma}{\rightsquigarrow} p_1 \underline{\gamma \underline{\gamma^{(1)} \gamma^{(2)}} \gamma^{(1)}} \underline{p_1^{(1)} p_1^{(2)}} p_1^{(1)} \overset{*}{\mapsto} 1$

$X = p_1^{(1)} p_1^{(0)} p_0^{(1)} p_0^{(2)} \overset{h_\beta}{\rightsquigarrow} p_1^{(1)} \underline{p_1^{(0)} . p_1^{(1)}} \underline{\beta^{(2)} \underline{\beta^{(1)} . \beta^{(2)}} \beta^{(3)}} p_1^{(2)} \overset{*}{\mapsto} 1$

[10] the symbol q can also be written q_{Pr} if necessary w.r.t. Pr

$$\lambda_i = \alpha\,(\prod_{\substack{for\ k'=n..1\\ if\ p_{k'}\leq p_i}}(\gamma_{k'}^{(1)}\,\gamma_{k'})) \quad and \quad \delta_i = (\prod_{\substack{for\ k=1..n,\\ if\ p_k\geq p_i}}(\beta_k\,\beta_k^{(1)}))\,\alpha'$$

Note the inversion by odd exponents : $h(p_i)^{(1)} = \underbrace{\delta_i^{(1)} q^{(1)}\,\lambda_i^{(1)}}$ *for* $p_i \in Pr$

Proposition 4 (h is order-preserving). *if* $p_i \leq p_j$ *then* $h(p_i) \leq' h(p_j)$.
It is then a pregroup homomorphism.

Proof. Easy by construction (if $p_i \leq p_j$, we can check that $\lambda_i \leq \lambda_j$ and $\delta_i \leq \delta_j$, using the transitivity of $\leq$ and inequalities of the form $\beta\beta^{(1)} \leq 1$ and $1 \leq \gamma^{(1)}\gamma$)

We get the first property required for a simulation, as a corollary:

Proposition 5 (Monotonicity of h).

$$\forall X, Y \in T_{(P,\leq)} : if\ X \leq Y\ then\ h(X) \leq' h(Y)$$

5 Order-reflecting Property : Overview and Lemmas

We show in the next subsection that if $h(X) \leq' 1$ then $X \leq 1$. We proceed by reasoning on a deduction $\mathcal{D}$ for $h(X) \leq' 1$. We explain useful facts on pregroup derivations when the right side is 1.

5.1 Reasoning with Left Derivations

Let $\mathcal{D}$ be the succession of sequents starting from $1 \leq' 1$ to $h(X) \leq' 1$, such that q is not related to any other primitive type by $\leq'$.

$$\underbrace{\Delta_0}_{=1} \leq' 1 \text{ (by rule } (Id)) \ldots\ \Delta_i \leq' 1 \ldots\ \underbrace{\Delta_m}_{=h(X)} \leq' 1$$

Each occurrence of q in $h(X)$ is introduced by rule (A_L). Let k denote the step of the leftmost introduction of q, let $(n), (n+1)$ be the corresponding exponents of q, we can write:

$$\begin{array}{lll}
\vdots & & \\
\Delta_{k-1} & = \Gamma_0\ \Gamma'_0 & \text{with } \Delta_{k-1} \leq' 1 \\
\Delta_k & = \Gamma_0\ q^{(n)}\ q^{(n+1)}\ \Gamma'_0 & \text{with } \Delta_k \leq' 1 \\
\vdots & & \\
\Delta_m & = \Gamma_{m-k}\ q^{(n)}\ \Gamma''_{m-k}\ q^{(n+1)}\ \Gamma'_{m-k} & \text{with } h(X) = \Delta_m \leq' 1
\end{array}$$

Γ_{m-k} is obtained from Γ_0 by successive applications of rule (A_L) or (IND_L); similarly for Γ'_{m-k} from Γ'_0:

$$\Gamma_{m-k} \leq' \Gamma_{m-k-1} \ldots \leq' \Gamma_i \leq' \ldots \Gamma_1 \leq' \Gamma_0$$

$$\Gamma'_{m-k} \leq' \Gamma'_{m-k-1} \ldots \leq' \Gamma'_i \leq' \ldots \Gamma'_1 \leq' \Gamma'_0$$

Γ''_{m-k} is also obtained from Γ_0 by successive applications of rule (A_L) or (IND_L) but from an empty sequence, in particular:

$$\Gamma''_{m-k} \leq' 1$$

We shall discuss according to some properties of the distinguished occurrence $q^{(n)}$ and $q^{(n+1)}$: the parity of n and whether they belong to images of p_i and of p_j in $h(X)$ such that $p_i \leq p_j$, $p_j \leq p_i$ or not.

5.2 Technical Lemmas

Lemma 1. *let* $X = \underbrace{X_1\, \alpha^{(2u_1)}\, Y_1\, \alpha^{(2u_1+1)}} \ldots \underbrace{X_k\, \alpha^{(2u_k)}\, Y_k\, \alpha^{(2u_k+1)}} \ldots \underbrace{X_n\, \alpha^{(2u_n)}\, Y_n\, \alpha^{(2u_n+1)}}\ X_{n+1}$

where all X_k, Y_k *have no occurrence of* α*, and* α *is not related by* $\leq$ *to other primitive types ;*

(i) if $X \leq 1$ *then :*
- *(1)* $\forall k : Y_k \leq 1$
- *(2)* $X_1\, X_2 \ldots X_k \ldots X_n\ X_{n+1} \leq 1$

(ii) if $Y_0\, \alpha^{(2v+1)}\ X\ \ \alpha^{(2v+2)} \leq 1$ *where* Y_0 *has no* α*, then :*
- *(1)* $\forall k : Y_k \leq 1$
- *(2)* $X_1\, X_2 \ldots X_k \ldots X_n\ X_{n+1} \leq 1$

The proof is given in Appendix.

Variants. A similar result holds in the odd case (called "Bis version") for $X = \underbrace{X_1\, \alpha^{(2u_1-1)}\, Y_1\, \alpha^{(2u_1)}} \ldots \underbrace{X_k\, \alpha^{(2u_k-1)}\, Y_k\, \alpha^{(2u_k)}} \ldots \underbrace{X_n\, \alpha^{(2u_n-1)}\, Y_n\, \alpha^{(2u_n)}}\ X_{n+1}$

Lemma 2. [11] *Let* I_{Pr} *denote the set of indices of elements in* Pr*. Let* $\Gamma = \underbrace{h(X_1)\, Y_1} \ldots \underbrace{h(X_k)\, Y_k} \ldots \underbrace{h(X_m)\, Y_m}\ h(X_{m+1})\ (m \geq 0)$

where all Y_k *have the following form:* $\lambda_i^{2u}\lambda_j^{2u+1}$ *or* $\delta_i^{2u-1}\delta_j^{2u}$ *with* $i, j \in I_{Pr}$ [12]
where some X_k *may be empty (then considered as 1, with* $h(1) = 1$*)*
(1) If $\Gamma \leq' 1$ *then* $\forall k_1 \leq m : (\forall k \leq k_1 : h(X_k)$ *has no* $q) \Rightarrow (\forall k \leq k_1 : Y_k \leq' 1)$
(2) If $\Gamma \leq' 1$ *then* $X_1 X_2 \ldots X_m X_{m+1} \leq 1$
(3) If $\delta_i^{(2k)}\Gamma\delta_j^{(2k+1)} \leq' 1$*, or* $\lambda_i^{(2k+1)}\Gamma\lambda_j^{(2k+2)} \leq' 1$
then $\forall k_1 \leq m : (\forall k \leq k_1 : h(X_k)$ *has no* $q) \Rightarrow (\forall k \leq k_1 : Y_k \leq' 1)$
(4) If $\delta_i^{(2k)}\Gamma\delta_j^{(2k+1)} \leq' 1$*, or* $\lambda_i^{(2k+1)}\Gamma\lambda_j^{(2k+2)} \leq' 1$ *then* $X_1 X_2 \ldots X_m X_{m+1} \leq 1$

Notation : $\sigma(\Gamma')$ will then denote $X_1 X_2 \ldots X_m X_{m+1}$, this writing is unique [13]
The proof is technical and is given in Appendix : a key-point is that some derivations are impossible.

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[11] Some complications in the formulation (1) and (3) simplify in fact the discussion in the proof
[12] we get later $p_i \not\leq p_j$ in the first form, $p_j \not\leq p_i$ in the second form
[13] due to the form of $h(p_i)$ and of δ_i, λ_i

6 Main Results

Proposition 6 (Order-reflecting property). *The simulation-morphism h from the free pregroup on $(P, \leq)$ to the free pregroup on $(P', \leq')$ satisisfies (1) and (2):*
(1). $\forall X, Y \in T_{(P,\leq)}$ *if* $h(X) \leq' h(Y)$ *then* $X \leq Y$ *by* $(P, \leq)$.
(2). $\forall X, Y \in T_{(P,\leq)}$ *if* $h(X) \leq' 1$ *then* $X \leq 1$ *by* $(P, \leq)$.

In fact, (1) can be shown from Lemma 2 (2) (case $m = 0$) and (2) is equivalent to (1) as explained before in Proposition 3.

As a corollary of monotonicity and previous proposition, we get:

Proposition 7 (Pregroup Order Simulation). *The simulation-morphism h from the free pregroup on $(P, \leq)$ to the free pregroup on $(P', \leq')$ enjoys the following property:*
$\forall X, Y \in T_{(P,\leq)}$ $h(X) \leq' h(Y)$ *iff* $X \leq Y$

Proposition 8 (Pregroup Grammar Simulation). *Given a pregroup grammar $G = (\Sigma, I, s)$ on the free pregroup on a poset $(P, \leq)$, we consider $Pr = P$ and we construct the simulation-morphism h from the free pregroup on $(P, \leq)$ to the simple free pregroup on $(P', \leq')$; we construct a grammar $G' = (\Sigma, h(I), h(s))$, where $h(I)$ is the assignment of $h(X_i)$ to a_i for each $X_i \in I(a_i)$, as a result we have : $\mathcal{L}(G) = \mathcal{L}(G')$*

This proposition applies the transformation to the whole set of primitive types, thus providing a fully lexicalized grammar G' (with no order postulate). A similar result holds when only a fragment of P is transformed as discussed in the construction of h.

7 Conclusion

In the framework of categorial grammars, pregroups were introduced as a simplification of Lambek calculus. Several aspects have been modified. The order on primitive types has been introduced in PG to simplify the calculus for simple types. The consequence is that PG is not fully lexicalized and a parser should take into account this information while performing syntactical analysis.

We have proven that this restriction is not so important because a PG using an order on primitive types can be transformed into a PG based on a simple free pregroup using a pregroup morphism whose size is bound by the size of the initial PG times the number of primitive types (times a constant which is approximatively 4). Usually the order on primitive type is not very complex, thus the size of the resulting PG is often much less that this bound. Moreover, this transformation does not change the number of types that are assigned to a word (a k-valued PG is transformed into a k-valued PG).

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8 Appendix

Proof of Lemma 1 (i) : by induction on the number n of terms $\underbrace{X_k\,\alpha^{(2u_k)}\,Y_k\,\alpha^{(2u_k+1)}}$.

∘ If $n > 1$, let $(2u_j+1)$ denote the rightmost exponent of α such that it is greater or equal than all exponents of α in X ; consider a derivation of $X \leq 1$ and the step when $\alpha^{(2u_j+1)}$ appears first in the derivation :
$Z_1\alpha^{(2u_j)}\alpha^{(2u_j+1)}Z_2 \leq 1$
Next steps lead to independent introductions in Z_1, Z_2 and between $\alpha^{(2u_j)}$ $\alpha^{(2u_j+1)}$, by construction of derivations, its ending can be written:
$\boxed{Z_1'\,\alpha^{(2u_j)}\,Z_3'\,\alpha^{(2u_j+1)}\,Z_2' \leq 1}$ where $\boxed{Z_3' \leq 1}$ and $\boxed{Z_1'\,Z_2' \leq 1}$

We now discuss whether $Z_3' = Y_j$ or not.

– If $Z_3' = Y_j$ (where Y_j has no α) then :

$$Z_1'\,Z_2' = \underbrace{X_1\,\alpha^{(2u_1)}\,Y_1\,\alpha^{(2u_1+1)}} \ldots \underbrace{X_{j-1}\,\alpha^{(2u_{j-1})}\,Y_{j-1}\,\alpha^{(2u_{j-1}+1)}}$$
$$X_j\;\underbrace{X_{j+1}\,\alpha^{(2u_{j+1})}\,Y_{j+1}\,\alpha^{(2u_{j+1}+1)}} \ldots \underbrace{X_n\,\alpha^{(2u_n)}\,Y_n\,\alpha^{(2u_n+1)}}\;X_{n+1}$$

We then apply the hypothesis to $n-1$ terms $\underbrace{X'_k\,\alpha^{(2u_k)}\,Y'_k\,\alpha^{(2u_k+1)}}$ in $X' = Z_1'\,Z_2'$, taking $X'j := X_jX_{j+1}$, $X'_{j+m} := X_{j+m-1}$ and $Y'_{j+m} := Y_{j+m-1}$:

(1)' $\forall k \neq j : Y_k \leq 1$

(2) $X_1\,X_2 \ldots X_k \ldots X_n\;X_{n+1} \leq 1$

we get full (1) for X from above : $Z_3' = Y_j$ and $Z_3' \leq 1$.

– If $Z_3' \neq Y_j$, since X_k, Y_k have no α, and $(2u_j+1)$ is a highest exponent, there must exist u_i, with $i < j$ such that $u_i = u_j$, therefore:
$Z_3' = Y_i\,\alpha^{(2u_i+1)}\,Z_4'\,X_j\,\alpha^{(2u_i)}\,Y_j$ where Z_4' is either empty or the sequence of successive terms $\underbrace{X_k\,\alpha^{(2u_k)}\,Y_k\,\alpha^{(2u_k+1)}}$, for $i < k < j$.

This is impossible, since the first $\alpha^{(2u_i+1)}$ on the left of Z_3', requires an occurrence of a greater exponent on the right in a derivation for $Z_3' \leq 1$, whereas $\alpha^{(2u_i+1)}$ is a highest exponent.

∘ If $n = 1$, the discussion starts as above, the derivation ending is:
$Z_1'\,\alpha^{(2u_j)}\,Z_3'\,\alpha^{(2u_j+1)}\,Z_2' \leq 1$ where $Z_3' \leq 1$ and $Z_1'\,Z_2' \leq 1$
which coincides with (1) and (2) for X ■

Proof of Lemma 1 (ii) : by induction on the number $n \geq 1$ of terms $\underbrace{X_k\,\alpha^{(2u_k)}\,Y_k\,\alpha^{(2u_k+1)}}$.

Let $(2u_j+1)$ denote the rightmost exponent of α in X such that it is greater or equal than all exponents of α in X; we consider a derivation $\mathcal{D}$ of $Y_0\,\alpha^{(2v+1)}X\alpha^{2v+2} \leq 1$ and the step when $\alpha^{(2u_j+1)}$ (rightmost maximum in X) appears first in the derivation.

– If the rightmost $\alpha^{(2u_j+1)}$ is introduced with the $\alpha^{(2v+2)}$ on its right, the introduction step in $\mathcal{D}$ is of the form:

$$Z_1 \alpha^{(2u_j+1)} \alpha^{(2u_j+2)} \leq 1 \text{ with } u_j = v \text{ and } Z_1 \leq 1 \text{ as antecedent in } \mathcal{D} \ .$$

The next steps in $\mathcal{D}$ are introductions in Z_1 and between $\alpha^{(2u_j)}$ $\alpha^{(2u_j+1)}$, by construction of derivations, its ending can be written:

$$\boxed{Z'_1 \, \alpha^{(2u_j+1)} \, Z'_3 \, \alpha^{(2u_j+2)} \ \leq 1} \text{ where } \boxed{Z'_3 \leq 1} \text{ and } \boxed{Z'_1 \leq Z_1 \leq 1}$$

The beginning of Z'_1 must be $Y_0 \, \alpha^{(2v+1)}$, where $2v+1$ is the highest exponent of α in Z'_1 (Y_0 having no α). This case is thus impossible, because in a derivation of $Z'_1 \leq 1$, the $\alpha^{(2v+1)}$ in the left of Z'_1 should combine with an $\alpha^{(2v+2)}$ on its right.

– The rightmost $\alpha^{(2u_j+1)}$ is thus introduced with an $\alpha^{(2u_j)}$ on its left, this step is:

$$Z_1 \alpha^{(2u_j)} \alpha^{(2u_j+1)} Z_2 \leq 1 \text{ with } Z_1 Z_2 \leq 1 \text{ as antecedent in } \mathcal{D} \ .$$

Next steps lead to independent introductions in Z_1, Z_2 and between $\alpha^{(2u_j)}$ $\alpha^{(2u_j+1)}$, by construction of derivations, the ending of $\mathcal{D}$ can be written:

$$\boxed{Z'_1 \, \alpha^{(2u_j)} \, Z'_3 \, \alpha^{(2u_j+1)} \, Z'_2 \leq 1} \text{ where } \boxed{Z'_3 \leq 1} \text{ and } \boxed{Z'_1 \, Z'_2 \leq 1}$$

The ending of Z'_3 must be Y_j. We now detail the possibilities.

If $n = 1$, by hypothesis $Y_0 \, \alpha^{(2v+1)} \underbrace{\underbrace{X_1 \, \alpha^{(2u_1)} \, Y_1 \, \alpha^{(2u_1+1)}} \, X_2}_{=X} \, \alpha^{(2v+2)} \leq 1$,

the inequality $Z'_3 \leq 1$ is $Y_1 = Z'_3 \leq 1$, that is a part of (1)
and $Z'_1 \, Z'_2 \leq 1$ is: $Y_0 \, \alpha^{(2v+1)} \ X_1 \, X_2 \ \alpha^{(2v+2)}$ where Y_0, X_1, X_2 have no α, therefore, by construction of derivations, we must have $Y_0 \leq 1$ (the end of (1)) and $X_1 \, X_2 \leq 1$ that is (2)

If $n > 1$, we first show that we must have $Z'_3 = Y_j$.

– If $Z'_3 \neq Y_j$, since X_k, Y_k have no α, and $(2u_j + 1)$ is a highest exponent in X, if it not combined with α^{2v+2}, there must exist u_i, with $i < j$ such that $u_i = u_j$, therefore:
$Z'_3 = Y_i \, \alpha^{(2u_i+1)} \, Z'_4 \, X_j \, \alpha^{(2u_i)} \, Y_j$ where Z'_4 is either empty or the sequence of successive terms $\underbrace{X_k \, \alpha^{(2u_k)} \, Y_k \, \alpha^{(2u_k+1)}}$, for $i < k < j$.

This is impossible, since the first $\alpha^{(2u_i+1)}$ on the left of Z'_3, requires an occurrence of a greater exponent on the right in a derivation for $Z'_3 \leq 1$, whereas $\alpha^{(2u_i+1)}$ is a highest exponent.

– $Z'_3 = Y_j$ (where Y_j has no α) then :
$Z'_1 \, Z'_2 =$

$$Y_0 \, \alpha^{2v+1} \ \underbrace{X_1 \, \alpha^{(2u_1)} \, Y_1 \, \alpha^{(2u_1+1)}} \ \ldots \ \underbrace{X_{j-1} \, \alpha^{(2u_{j-1})} \, Y_{j-1} \, \alpha^{(2u_{j-1}+1)}}$$

$$X_j \ \underbrace{X_{j+1} \, \alpha^{(2u_{j+1})} \, Y_{j+1} \, \alpha^{(2u_{j+1}+1)}} \ \ldots \ \underbrace{X_n \, \alpha^{(2u_n)} \, Y_n \, \alpha^{(2u_n+1)}} \ X_{n+1} \alpha^{2v+2}$$

we apply the hypothesis to $n-1$ terms $\underbrace{X'_k \, \alpha^{(2u_k)} \, Y'_k \, \alpha^{(2u_k+1)}}$ in $X' = Z'_1 \, Z'_2$, taking $X'j := X_j X_{j+1}$, $X'_{j+m} := X_{j+m-1}$ and $Y'_{j+m} := Y_{j+m-1}$:

(1)' $\forall k \neq j : Y_k \leq 1$
(2) $X_1\, X_2 \ldots X_k \ldots X_n\; X_{n+1} \leq 1$
we get full (1) for X from above : $Z'_3 = Y_j$ and $Z'_3 \leq 1$ ■

Detailed writing and properties of the auxiliary types

Lemma 2 uses the following facts on auxiliary types :

- if $p_i \leq p_j$ then $\lambda_i^{2u}\lambda_j^{2u+1} \leq' 1$
- if $p_i \not\leq p_j$ then $\lambda_i^{2u}\lambda_j^{2u+1} \not\leq' 1$
- if $p_j \leq p_i$ then $\delta_i^{2u-1}\delta_j^{2u} \leq' 1$
- if $p_j \not\leq p_i$ then $\delta_i^{2u-1}\delta_j^{2u} \not\leq' 1$

This can be summarized as : the only $Y_k \not\leq 1$ of Lemma 2 are $\lambda_i^{2u}\lambda_j^{2u+1}$ for $p_i \not\leq p_j$ and $\delta_i^{2u-1}\delta_j^{2u}$ for $p_j \not\leq p_i$
This can be checked using the following detailed writing for the auxiliary types

$$\lambda_i^{2u}\lambda_j^{2u+1}\ (i = j): \qquad \alpha^{(2u)}\,\Big(\prod_{\substack{for\ k'=n..1\\ if\ p_{k'} \leq p_i}} (\gamma_{k'}^{(2u+1)}\ \gamma_{k'}^{(2u)})\Big)\,\Big(\prod_{\substack{for\ k=1..n\\ if\ p_k \leq p_i}} (\gamma_k^{(2u+1)}\ \gamma_k^{(2u+1+1)})\Big)\,\alpha^{(2u+1)}$$

$$\delta_i^{2u-1}\delta_j^{2u}\ (i = j):\ \alpha'^{\,(2u-1)}\,\Big(\prod_{\substack{for\ k'=n..1\\ if\ p_{k'} \geq p_i}} (\beta_{k'}^{(2u-1+1)}\ \beta_{k'}^{(2u-1)})\Big)\,\Big(\prod_{\substack{for\ k=1..n,\\ if\ p_k \geq p_i}} (\beta_k^{(2u)}\ \beta_k^{(2u+1)})\Big)\,\alpha'^{\,(2u)}$$

$$\lambda_i^{2u}\lambda_j^{2u+1}\ (i \neq j): \qquad \alpha^{(2u)}\,\Big(\prod_{\substack{for\ k'=n..1\\ if\ p_{k'} \leq p_i}} (\gamma_{k'}^{(2u+1)}\ \gamma_{k'}^{(2u)})\Big)\,\Big(\prod_{\substack{for\ k=1..n\\ if\ p_k \leq p_j}} (\gamma_k^{(2u+1)}\ \gamma_k^{(2u+1+1)})\Big)\,\alpha^{(2u+1)}$$

$$\delta_i^{2u-1}\delta_j^{2u}\ (i \neq j):\ \alpha'^{\,(2u-1)}\,\Big(\prod_{\substack{for\ k'=n..1\\ if\ p_{k'} \geq p_i}} (\beta_{k'}^{(2u-1+1)}\ \beta_{k'}^{(2u-1)})\Big)\,\Big(\prod_{\substack{for\ k=1..n,\\ if\ p_k \geq p_j}} (\beta_k^{(2u)}\ \beta_k^{(2u+1)})\Big)\,\alpha'^{\,(2u)}$$

Proof of Lemma 2. We consider the *leftmost introduction* of q in a derivation of $\Gamma \leq' 1$, involving p_i^n, p_j^{n+1}. *The inductives Cases for (3)(4) can be treated similarly as (1)(2).*
If there is no q, we apply lemma1: if there is no q, each Y_k is of the form $\alpha^{(2u)} Z_k \alpha^{(2u+1)}$ where Z_k has no $\alpha^{(\cdot)}$ or of the form $\alpha'^{\,(2u-1)} Z_k \alpha'^{\,(2u)}$ where Z_k has no $\alpha'^{\,(\cdot)}$; we apply lemma 1, using $\alpha^{(\cdot)}$, then the second version of lemma 1, using $\alpha'^{\,(\cdot)}$

We now suppose there is an occurrence of q.

- $n = 2k$ even, case $q^{(n)}$ belongs to $h(p_i)^{(2k)}$, $q^{(n+1)}$ belongs to $h(p_j)^{(2k+1)}$
  We can write: $\Gamma = Z\ h(p_i^{(2k)})\ Z''\ h(p_j^{(2k+1)})\ Z'$ where Z has no q

i.e. $\Gamma = Z \underbrace{\lambda_i^{(2k)}\ q^{(2k)} \delta_i^{(2k)}}\ Z''\ \underbrace{\delta_j^{(2k+1)} q^{(2k+1)}\ \lambda_j^{(2k+1)}}\ Z'$

where $\Gamma_1, \Gamma_2, \Gamma_3$ have a form similar to that of Γ

By construction of $\mathcal{D}$, we have two sequents for which the lemma applies:

$\boxed{Z\ \lambda_i^{(2k)} \lambda_j^{(2k+1)}\ Z' \leq' 1}$ and $\boxed{\delta_i^{(2k)}\ Z''\ \delta_j^{(2k+1)} \leq' 1}$

by induction

- from (1): this entails (1) for $Y_k \in Z$, which proves (1) for Γ;
- from (2): $\sigma(Z)\sigma(Z') \leq 1$
- and from (3): $\sigma(Z'') \leq 1$.

For each $p_i \leq p_j$, we get a derivation of $\sigma(Z) p_i^{(n)} p_j^{(n+1)} \sigma(Z') \leq 1$, then of $\sigma(Z) p_i^{(n)} \sigma(Z'') p_j^{(n+1)} \sigma(Z') \leq 1$.

That is $\sigma(\Gamma) \leq 1$, when $p_i \leq p_j$.

– $n = 2k{+}1$ odd, case $q^{(n)}$ belongs to $h(p_i)^{(2k+1)}$, $q^{(n+1)}$ belongs to $h(p_j)^{(2k+2)}$

We then have: $\Gamma = Z\ h(p_i^{(2k+1)})\ Z''\ h(p_j^{(2k+2)})\ Z'$

i.e. $\Gamma = Z\ \underbrace{\delta_i^{(2k+1)} q^{(2k+1)}\ \lambda_i^{(2k+1)}}\ Z''\ \underbrace{\lambda_j^{(2k+2)}\ q^{(2k+2)} \delta_j^{(2k+2)}}\ Z'$

By construction of $\mathcal{D}$:

$\boxed{Z \delta_i^{(2k+1)} \delta_j^{(2k+2)} Z' \leq' 1}$ and $\boxed{\lambda_i^{(2k+1)}\ Z''\ \lambda_j^{(2k+2)} \leq' 1}$

by induction,

- from (1)this entails (1) for $Y_k \in Z$, which proves (1) for Γ;
- from (2): $\sigma(Z)\sigma(Z') \leq 1$
- and from (3): $\sigma(Z'') \leq 1$.

For each $p_j \leq p_i$, we get a derivation of $\sigma(Z) p_i^{(n)} p_j^{(n+1)} \sigma(Z') \leq 1$, then of $\sigma(Z) p_i^{(n)} \sigma(Z'') p_j^{(n+1)} \sigma(Z') \leq 1$. That is $\sigma(\Gamma) \leq 1$, when $p_j \leq p_i$.

– $n = 2k$, $p_i \not\leq p_j$: $\Gamma = \underbrace{Z}_{no\ q}\ \underbrace{\lambda_i^{(2k)}\ q^{(2k)} \delta_i^{(2k)}}\ Z''\ \underbrace{\delta_j^{(2k+1)} q^{(2k+1)}\ \lambda_j^{(2k+1)}}\ Z'$,

such that, by construction of $\mathcal{D}$: $Z\ \lambda_i^{(2k)} \lambda_j^{(2k+1)} Z' \leq' 1$

which is is impossible by induction (1) applied to it (key point: $\lambda_i^{(2k)} \lambda_j^{(2k+1)} \not\leq 1$)

– $n = 2k+1$, $p_j \not\leq p_i$: $\Gamma = \underbrace{Z}_{no\ q}\ \underbrace{\delta_i^{(2k+1)} q^{(2k+1)}\ \lambda_i^{(2k+1)}}\ Z''\ \underbrace{\lambda_j^{(2k+2)}\ q^{(2k+2)} \delta_j^{(2k+2)}}\ Z'$ such that,

by construction of $\mathcal{D}$: $Z\ \delta_i^{(2k+1)} \delta_j^{(2k+2)}\ Z' \leq' 1$

which is impossible by induction (1) applied to it (key point: $\delta_i^{(2k+1)} \delta_j^{(2k+2)} \not\leq 1$)