

Software Construction

The B Method - Event B

J. Christian Attiogbé

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Plan

Event-B : References

- *Modeling in Event-B: System and Software Engineering*,
J-R. Abrial, Cambridge, 2010
- *Modelling and proof of a Tree-structured File System*.
Damchoom, Kriangsak and Butler, Michael and Abrial, Jean-Raymond,
Conference ICFEM 2008.
- *Applying Event and Machine Decomposition to a Flash-Based Filestore in Event-B*.
Damchoom, Kriangsak and Butler, Michael; Conference SBMF 2009.
- *Faultless Systems: Yes We Can!*,
Jean-Raymond Abrial, Computer, vol. 42, no. 9, pp. 30-36, Sept. 2009

Event B Specification Approach

Correct-by-construction: build correctly the systems.
(abstraction, modelling, refinement, composition/decomposition, proof)

Some hints to formal methods:

- Formal methods are **rigorous engineering tools**.
- Formal methods are **means to build** executable code from software requirement documents (informal, natural language).
- **Requirement Documents** (provided by clients) **should be rewritten** after analysis and understanding into **Reference Document** (where every thing is made clear and properly labelled for traceability).

B Method and Event B

- Event-B is an **extension of the B-method** (J-R. Abrial).
- It is devoted
 - for **system engineering** (both hardware and software)
 - for **specifying and reasoning about complex systems** : concurrent and reactive systems.
- Event-B comes with a new modelling framework called Rodin.
(like Atelier B tool for the classical B)
- The **Rodin platform** is an eclipse-based open and extensible tool for B model specification and verification.
It integrates various plug-ins: **B Model editors, proof-obligation generator, provers, model-checkers, UML transformers, etc**

Event B Modelling

Yet used in various case studies and real cases:

- Train signalling system
- Mechanical press system
- Access control system
- Air traffic information system
- Filestore system
- Distributed programs
- Sequential programs
- etc

Event B Modelling

Observe the behaviour of any system; what matters?

- We see a **set of changes** of its states.
- But, the **observation distance** does matter!
(the details may be observed or not: parachutist paradigm)
- The **observation focus** does matter!
(the observed changes are not the same)
- Different point of views = **abstraction**.

Event B Specification Approach

Event B Specification \Rightarrow Abstract systems or Abstract model

An **abstract system** is a mathematical model of an **asynchronous system behaviour**

System behaviour : described by **events**

Events are guarded actions/substitutions The events occurrence involve a State-transition model.

- Abstract System (or Model) = Specification unit
- **Refinement** (data and events)
The parachutist paradigm / microscope paradigm (JR Abrial)
- **Decomposition** (of a system into sub-systems)

B Abstract System

Variables

Predicate

Events

```
SYSTEM
SETS ...
VARIABLES
...
INVARIANT
... predicate
INITIALISATION
...
EVENTS
...
END
```

but structured more efficiently using **Contexts**.

Capturing a system behaviour - Events

- The **behaviour** of a discrete system is a **sequence of changes** (transition system).
- The changes may be **internal** or enabled by **external** signals.
- Each **event describes the occurrence of a change** in the discrete system under modelisation.
event = when **Conditions** then **Effects**
- Event B uses **Guards** and **Actions** [Dijkstra]
- But, the behaviour of a system may/should be captured gradually.

Events

An event has one of the following general forms (Fig. 1)

```

name  $\widehat{=}$  /* event name */
  WHEN /* formerly SELECT*/
     $P(gcv)$ 
  THEN
     $GS(gcv)$ 
  END
  
```

(WHEN/SELECT Form)

```

name  $\widehat{=}$  /* event name */
  ANY  $bv$  WHERE
     $P(bv, gcv)$ 
  THEN
     $GS(bv, gcv)$ 
  END
  
```

(ANY Form)

Figure: General forms of events

bv denotes the local bound variables of the event;
 gcv denotes the global constants and variables of the abstract;
 $P(bv, gcv)$ a predicate.

Events

An event without guards has the following form:

```
name  $\hat{=}$  /* event name */  
  BEGIN  
    GS(gcv)  
  END
```

Abstract System (or a model, or a machine)

- The **guard** of an event with the WHEN form is: $P(gcv)$.
- The **guard** of an event with the ANY form is: $\exists(bv).P(bv, gcv)$.
- The WHEN form is a particular case of the other.
- The action associated to an event is modeled with a **generalized substitution** using the variables accessible to the event:
 $GS(bv, gcv)$.

Abstract System : Semantics and Consistency

An abstract system describes a mathematical model that simulates the behaviour of a system.

Its semantics arises from the invariant and is enhanced by proof obligations.

The consistency of the model is established by such proof obligations.

Consistency of an event B model:

- PO: the initialisation establishes the invariant
- PO: each event of the abstract system preserves the invariant of the model

$I(gcv)$ the invariant and $GS(bv, gcv)$ the generalized substitution modelling the event action.

Abstract System : Semantics and Consistency

- the **initialisation** establishes the invariant;

$$[U]Inv$$

- each event preserves the invariant** :

In the case of an event with the ANY form, the proof obligation is:

$$I(gcv) \wedge P(bv, gcv) \wedge \text{prd}_v(S) \Rightarrow [GS(bv, gcv)]I(gcv)$$

Moreover the events (e) terminate:

$$Inv \wedge eGuard \Rightarrow \text{fis}(eBody)$$

(note that Inv is $I(Gcv)$)

Abstract System : Semantics and Consistency

The predicate $\text{fis}(S)$ expresses that S does not establish *False*:

$$\text{fis}(S) \Leftrightarrow \neg [S]\text{False}$$

ie

$$\text{Inv} \wedge e\text{Guard} \Rightarrow \neg [S]\text{False}$$

The predicate $\text{prd}_v(S)$ is the *before-after predicate* of the substitution S ; it relates the values of state variables just before (v) and just after (v') the substitution S .

The $\text{prd}_v(\text{ANY } x \text{ WHERE } P(x, v) \text{ THEN } v := S(x, v) \text{ END})$ is :
 $\exists x.(P(x, v) \wedge v' = S(x, v))$

Example : producer/consumer

Features: Concurrency and synchronization

- Concurrent running of a process **consumer** which retrieves a data from a buffer filled by another process **producer**.
- The consumer cannot retrieve an empty buffer and the producer cannot fill in a buffer already full.

An event-driven model of the system is as follows:

Example : producer/consumer

```

system ProdCons /* Model */
sets
  DATA ;      STATE = {empty, full}
variables   buffer, bufferstate, bufferc
invariant
  bufferstate ∈ STATE ∧ buffer ∈ DATA ∧ bufferc ∈ DATA
initialization
  bufferstate := empty || buffer := DATA || bufferc := DATA
events
  produce  $\hat{=}$  /* if buffer empty */
    any dd where   dd ∈ DATA ∧ bufferstate = empty
  then   buffer := dd || bufferstate := full
  end ;
  consume  $\hat{=}$  /* if buffer is full */
    select  bufferstate = full
  then   bufferc := buffer || bufferstate := empty
  end
end

```

Refinement

- Data refinement
(as usually: new variables + properties; binding invariant)
- Event Refinement (**extended**):
 - **Strengthening guards** (unlike with Classical B)
More variables are introduced with their properties.
 - **Each event of the concrete system refines an event of the abstraction.**
 - Introduction of **new events** which refine **skip**, and use new variables.

Refinement

Let A with Invariant: $I(av)$

```

evta ≡ /* Abs. ev. */
  when P(av)
  then GS(av)
  end
  
```

avec $\text{prd}_v(\dots) = \text{Ba}(av, av')$

Refined with: Invariant $J(av, cv)$

```

evtr ≡ /* Conc. ev. */
  when Q(cv)
  then GS(cv)
  end
  
```

avec $\text{prd}_v(\dots) = \text{Bc}(cv, cv')$

Proof obligation:

$$I(av) \wedge J(av, cv) \wedge Q(cv) \wedge Bc(cv, cv') \Rightarrow \exists cv'. (Ba(av, av') \wedge J(av', cv'))$$

Tools

- First generation tools
 - Translation into classical B
 - B4free
- New generation tools: DataBase, Eclipse Plugins, ...
 - Rodin (Deploy Project)

Refinement: structuring models

Refinement= development technique.

Various refinement strategies.

- **Vertical refinement:** From abstract to concrete models.
Details are gradually introduced in an abstract model in order to make it **more concrete**.
The specifier introduces new variables and takes some choices (design), etc
Events may be split, machines may be split too: event decomposition /machine decomposition
- **Horizontal refinement:** From a small and abstract to a larger abstract.
Details are gradually introduced in an abstract model in order to make it more precise
(wrt to requirements ==> adding more features, gradually).

Vertical Refinement: event decomposition

A coarse grain event is analysed and described in a more detailed (fine grain) way.

Think about the transfer of a file via a network.

- A given change consists of:
 - start by sub-change...;
 - follow by sub-change...;
 - end by sub-change...;
- Hence, at least one sub-change (an event), refines the abstract event.

Machine Decomposition: structuring models

A coarse grain model is analysed and described in a more detailed (fine grain) way.

Think about a system involving software and physical devices.

- A given model is made of variables that model purely physical devices, and events are associated only to these variables
- The splitting is based on variables splitting (but not always straightforward).
- Divide and conquer: a small model is more tractable than a huge one.

Decomposition enables one to break complexity, to structure and develop more easily.

Machine Decomposition: structuring models

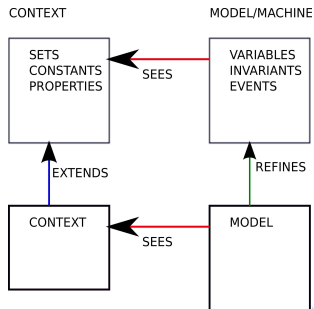
- **Decomposition with Abrial's style (shared variables):** Machine variables and events are partitioned into sub-machines. The sub-machines may interact with each other via **shared variables**.
Shared variables are duplicated, new external-events are introduced in each machine that has a shared variable in order to ensure consistency of changes.
- **Decomposition with Butler's style:** Machine variables and events are partitioned into sub-machines. The variables are not shared; an event which uses variables in separate machines, is shared (then separated-duplicated). The sub-machines may interact with each other via **synchronisation over shared parameterised events**.

Event-B Model Decomposition, Carine Pascal(Systemel) and Renato Silva(University of Southampton)

Structuring Event-B Models

An event-B model is structured with

- **Contexts** that contain carrier **sets, axioms and theorems** (seen by various machine)
- **Machines** which sees the contexts and defines a **state space** (static part: **variables + labelled invariants**) and a dynamic part made of some events.
- A **context may be extended**; A **machine may be refined**.



Event-B Model Example: File transfer protocol

```

MODEL Transfer
SETS DATA
CONSTANTS n
PROPERTIES n : NAT
VARIABLES
sf /* sender file */
, rf /* receiver file */
, nr /* number of records in the file f */

INVARIANT
nr : NAT
& sf : 1..nr -> DATA /* all records of sf */
& rf : 1..nr +-> DATA /* probably part of
records of sf */
INITIALISATION
sf := {} || rf := {} || nr := 0

```

```

EVENTS
transf = /* instantaneous transfer, from far
way */
BEGIN
rf := sf
END

/* the following events are introduced by
anticipation of the forthcoming gradual
refinement*/
; sendta = skip
; recdta = skip
; sendac = skip
; recvac = skip
/* the followings are events that simulate the
non-releiability of channels */
; rmvData = skip
; rmvAck = skip
END

```

Event-B Model Example: File transfer protocol

```

REFINEMENT
Transfer_R1

REFINES Transfer

VARIABLES

cs /* current record to be sent */
, cr /* current record received */
, rf
, sf /* sender file */
, erf /* effectively received file */
, nr /* number of records in the file f */
, dataChan /* data channel */
, ackChan /* ack channel */

INVARIANT
cs : 1..nr+1 /* current to be sent */
& cr : 0..nr /* current received */
& cr <= cs /* current received is <= current
sent */
& cs <= cr+1 /* cr <= cs <= cr+1 */
& erf = (1..cr) <| sf
& dataChan <: (1..cs) <| sf
& ackChan <: 1..cr

```

```

INITIALISATION
cs := 1
|| cr := 0
|| rf := {}
|| sf := {}
|| erf := {}
|| nr := 0
|| dataChan := {}
|| ackChan := {}

EVENTS
transf =
WHEN
cs = (nr + 1) /* that is all cs are received
(last ack received) */
THEN
rf := erf /* not necessary, effective copy of
the received file in the receiver */
END

... (continued)
END

```

Event-B Model Example: File transfer protocol

```

/* new events introduced (ie. we "forget" the
anticipation in the abstract model) */
; sendta =
WHEN
cs <= nr
THEN
dataChan(cs) := sf(cs)
/* now wait for the ack, before updating cs */
END

; recdta =
WHEN cr+1 : dom(dataChan)
THEN
erf(cr+1) := dataChan(cr+1)
|| cr := cr + 1 /* the next data to be received
*/
END

; sendac =
WHEN cr /= 0 /* send ack for the received cr
data */
/* may be observed repeatedly until the next
data */
THEN
ackChan := ackChan {cr}
END

```

```

recvac =
WHEN cs : ackChan /* ack for the already sent
cs */
THEN
cs := cs + 1 /* now the next to be sent */
END
/* Simulating non-reliability of channels,
data/ack may be loss */
; rmvData =
ANY i, d WHERE
i|->d : dataChan
THEN
dataChan := dataChan - {i|->d}
END
;
rmvAck =
ANY i WHERE
i : ackChan
THEN
ackChan := ackChan - {i}
END

```

Case Study : Multiprocess specification (Readers/writers)

- Description
 - Multiple processes: **readers, writers**
 - Shared resources between the processes
 - Several readers may read the resource
 - **Only one writer at a time**
- Property:
Mutual exclusion between readers and writers
- Improvement:
no starvation → as a new property
(using refinements)

Multiprocess specification

```
MODEL
readWrite2
SETS
WRITER /* set of writer processes */
; READER /* set of reader processes */

VARIABLES
writers /* current writers */
, activeWriter
, waitingWriters
, readers /* current readers */
, waitingReaders
, activeReaders /* we may have svrl readers simultan. */
```

Multiprocess specification

```

INVARIANT
writers <: WRITER
& activeWriter <: WRITER
& card(activeWriter) <= 1
& waitingWriters <: WRITER
& writers /\ waitingWriters = {}
& activeWriter /\ waitingWriters = {}
& activeWriter /\ writers = {}
/* merge */
& readers <: READER
& waitingReaders <: READER
& activeReaders <: READER
& card(activeReaders) >= 0
& readers /\ waitingReaders = {}
& activeReaders /\ waitingReaders = {}
& activeReaders /\ readers = {}
/*-----safety properties -----*/
& not((card(activeWriter) = 1)&(card(activeReaders) >= 1))

```


Multiprocess specification

```
INITIALISATION
activeWriter := {}
|| waitingWriters := {}
|| activeReaders := {}

|| readers :: POW(READER)
|| writers :: POW(WRITER)
|| waitingReaders := {}
```

Multiprocess specification

```
want2write = /* observed when a process wants to write */
ANY ww WHERE
ww : writers
& ww /: waitingWriters
& ww /: activeWriter
THEN
waitingWriters := waitingWriters \ / {ww}
|| writers := writers - {ww}
END
;
writing =
ANY ww WHERE
ww : waitingWriters
& activeReaders = {} & activeWriter = {}
THEN
activeWriter := {ww}
|| waitingWriters := waitingWriters - {ww}
END
```

Multiprocess specification

```
endWriting =
ANY ww WHERE
ww : activeWriter
THEN
writers := writers \ / {ww}
|| activeWriter := {}
END
;
want2read =
ANY rr WHERE
rr : readers
& rr /: waitingReaders
& rr /: activeReaders
THEN
waitingReaders := waitingReaders \ / {rr}
|| readers := readers - {rr}
END
```

Multiprocess specification

```
reading =
  ANY rr WHERE
  rr : waitingReaders
  & activeWriter = {}
  THEN
  activeReaders := activeReaders \/ {rr}
  || waitingReaders := waitingReaders - {rr}
  END
;
endReading =
/* one of the active readers finishes and leaves
the competition to the shared resources */
  ANY rr WHERE
  rr : activeReaders
  THEN
  activeReaders := activeReaders - {rr}
  || readers := readers \/ {rr}
  END
```

Multiprocess specification

```
newWriter = /* a new Writer */
ANY ww
WHERE ww : WRITER
& ww /\: (writers \/ waitingWriters \/ activeWriter)
THEN
writers := writers \/ {ww}
END
; leaveWriters = /* a writer leaves the group */
ANY ww
WHERE
ww : writers
THEN
writers := writers - {ww}
END
```

Multiprocess specification

```
newReader = /* a new reader joins the readers */
ANY rr WHERE
rr : READER
& rr /: (readers\waitingReaders \/activeReaders)
THEN
readers := readers \/ {rr}
END
; leaveReader =
ANY rr WHERE
rr : readers & card(readers) > 1
THEN
readers := readers - {rr}
END
```

Example: Flash-based Filestore in Event-B