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Torque minimisation of the 2-DOF serial manipulators based on minimum energy consideration and optimum mass redistribution

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ABSTRACT

This paper deals with the analytically tractable solution for input torques minimisation of two degrees of freedom serial manipulators based on minimum energy control and optimal redistribution of movable masses. The minimisation problem is carried out in two steps: at first, the optimal trajectory of the manipulator is defined as a function, which leads to the minimisation of energy consumption. Then, by introducing the obtained trajectory into dynamic equations, the torques are reduced by using the optimal redistribution of movable masses, which is carried out via an adaptive counterweight system. For this purpose, the torques due to the dynamic loads of the counterweights are presented as a function of the counterweight positions. The conditions for optimal dynamic balancing are formulated by minimisation of the root-mean-square value of the input torque including the dynamic loads of the unbalanced manipulator and counterweights. The suggested approach is illustrated by numerical simulations carried out using ADAMS software.

1. Introduction

The loading of manipulator actuators depends on the distribution of mass in the links as well as the efficient motion generation. The first problem can be partially solved via gravity compensation, i.e. by static balancing. This means that potential energy is constant for all possible configurations, i.e. zero actuator torques due to the static loads are required. Previous works on the static balancing of robot mechanisms can be arranged in the following groups:

(a) Balancing by counterweights mounted on the links of the initial system [1–4]. Such balancing is very simple to realize. However, it leads to the important increase of the moving masses of the manipulator.
(b) Balancing by counterweights mounted on the auxiliary linkage connected with the initial system [5–9]. Articulated dyads [5–7] are used as the added systems for optimal displacements of the counterweights. In [9] the pantograph mechanism is used as an auxiliary linkage, which allows the generation of a vertical force applied to the manipulator platform. The balanced platform becomes a weightless link, and it can be displaced by low-power actuators.
(c) Balancing by springs jointed directly with manipulator links [10–15]. It was shown that complete static balancing can be achieved when the zero free length spring is applied and partial balancing for the non-zero free length spring. In the work [13] it was shown that the mass of the balancing spring increases the unbalanced moment and it cannot be neglected. Therefore a study to gravity balance considering the spring mass was developed.
(d) Spring balancing by using a cable and pulley arrangement [16–20]. Such an approach allows zero free length springs to be used, which is more favourable for realisation of complete balancing.
(e) Spring balancing by using an auxiliary linkage [21–31]. In these studies the articulated dyads, pantograph mechanism and parallelogram structure are used as the added systems for optimal displacements of the springs. It should be noted that many balancing methods carried out by springs are only applicable for planar manipulators.
(f) Spring balancing by using a cam mechanism [32–35]. In [33] was shown a balanced technique which uses springs in addition to the cam with Archimedean spiral curve.
(g) Spring balancing by using gear train [36–39].

It is obvious that such a balancing is very useful for static mode of operation of the manipulator. However, with the increase of the accelerations of moving links, the inertia forces become important and the complete static balancing in dynamic operation cannot be optimal. In this context another problem may be formulated: to

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find such a distribution of movable masses, which allows the minimisation of input torques of the actuators in dynamic mode of operation. This problem was stated and solved for the PArallel Manipulator of the INSA (PAMINSA) [40]. It was shown via numerical simulations and validated by experimental tests that in the case of the dynamic mode of operation, the complete static balancing is not always optimal in terms of input torques.

With regard to the second problem, i.e. the efficient motion generation, the solutions result in torque saturation. Several offline planning algorithms have been proposed for the minimum-time trajectory, considering manipulator dynamics with torque limits of the actuators [41–46].

In robot design, these two problems are considered separately, i.e. the efficient motion generation is studied as a control problem and the optimal mass redistributions as a balancing problem.

In this paper above mentioned problems are considered together. In a first part, the minimisation of the input torques of the 2-DOF serial manipulator is carried out by an optimal motion execution based on the energy minimisation. Then, by using the obtained optimal motion laws, the minimisation of the torques due to the inertia forces is carried out by optimal redistribution of movable masses. The last one is achieved by optimal placement of counterweights.

2. Minimum energy control

The dynamics of a two degrees of freedom serial manipulator (Fig. 1), which are highly nonlinear, coupled differential equations can be written as:

$$\boldsymbol{\tau} = \boldsymbol{A}(\theta)\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\theta,\dot{\theta})\dot{\boldsymbol{\theta}} + \boldsymbol{Q}(\theta) \tag{1}$$

where τ is the 2×1 torque vector applied to the joints of the manipulator, $\boldsymbol{\theta}$, $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$ are 2×1 vectors representing the angular positions, velocities and accelerations, respectively, $\boldsymbol{A}(\theta)$ is the 2×2 inertia matrix, $\boldsymbol{C}(\theta,\dot{\theta})$ and $\boldsymbol{Q}(\theta)$ are 2×1 vectors of Coriolis/centripetal forces and gravity loading.

2.1. Feedback linearization

The notion of feedback linearization of nonlinear systems is a relatively recent idea in control theory [47]. The practical realizations are implemented using digital signal processors (rapid development of microprocessor technology). In the robotics context, feedback linearization is known as inverse dynamics. The main idea in joint space inverse dynamics is to exactly compensate the highly nonlinear and highly coupled dynamics of Lagrange's equations (1).

The nonlinear feedback control law (feedforward-computed torque) can be computed as:

$$\boldsymbol{\tau} = \widehat{\boldsymbol{A}}(\theta)\ddot{\boldsymbol{v}} + \widehat{\boldsymbol{C}}(\theta,\dot{\theta})\dot{\boldsymbol{\theta}} + \widehat{\boldsymbol{Q}}(\theta) \tag{2}$$

where $\widehat{\boldsymbol{A}}(\theta)$, $\widehat{\boldsymbol{C}}(\theta,\dot{\theta})$ and $\widehat{\boldsymbol{Q}}(\theta)$ are the estimated values of the matrixes $\boldsymbol{A}(\theta), \boldsymbol{C}(\theta,\dot{\theta})$ and $\boldsymbol{Q}(\theta)$.

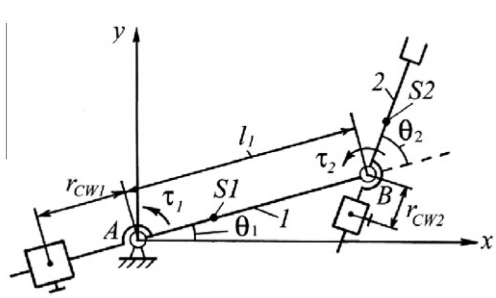

Fig. 1. 2-DOF serial manipulator.

Now, we can implement the joint space inverse dynamics in a so-called feedback linearization architecture as shown in Fig. 2 (the carets signify the estimated matrix).

If, $\widehat{\boldsymbol{A}}(\theta) = \boldsymbol{A}(\theta),\ \widehat{\boldsymbol{C}}(\theta,\dot{\theta}) = \boldsymbol{C}(\theta,\dot{\theta})\ et\ \widehat{\boldsymbol{Q}}(\theta) = \boldsymbol{Q}(\theta)$, the feedback linearization of the robot drives to the decoupled linear double integrators and we have:

$$\ddot{\boldsymbol{\theta}}(t) = \boldsymbol{v}(t) \tag{3}$$

2.2. Closed-loop controller of double integrators

Given a joint space trajectory, $\theta_j(t)$, an obvious choice for the controller, which produces $v(t)$ is a proportional-derivative plus feedforward acceleration control

$$v(t) = \ddot{\theta}_j + K_P(\theta_j - \theta) + K_D(\dot{\theta}_j - \dot{\theta}) \tag{4}$$

where the proportional and derivative matrices K_P and K_D are positives and diagonals.

Substituting Eq. (4) into Eq. (3), we obtain:

$$\ddot{\theta} = \ddot{\theta}_j + K_P(\theta_j - \theta) + K_D(\dot{\theta}_j - \dot{\theta}) \tag{5}$$

Then, if we define, $\tilde{\theta} = \theta_j - \theta$, we have the linear and decoupled closed-loop system

$$\ddot{\tilde{\theta}} + K_D\dot{\tilde{\theta}} + K_P\tilde{\theta} = 0 \tag{6}$$

The linear and decoupled closed-loop system is presented in Fig. 3.

The separation between the feedback linearization architecture and the closed-loop controller is important for several reasons. The feedback linearization architecture is fixed by Lagrange's equations. The closed-loop controller given in Eq. (5) is merely the simplest choice and achieves asymptotic tracking of joint space trajectories in the ideal case of perfect knowledge of the model given by Eq. (1). However, we have complete freedom to modify closed-loop controller to achieve various other goals (for example, to enhance the robustness to parametric uncertainty, external disturbances, etc.) without the need to modify the dedicated feedback linearization architecture.

2.3. Minimum energy of a double integrator

The system of double integrator

$$\underbrace{\begin{bmatrix} \ddot{\theta}(t) \\ \dot{\theta}(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u(t) \tag{7}$$

is completely controllable.

We seek $u(t)$ that steers $x(0) = [0\,\theta_I]^{\mathrm{T}}$ to $x(T) = [0\,\theta_F]^{\mathrm{T}}$ and minimizes

$$\int_0^{\mathrm{T}} u^2(t)dt = \int_0^{\mathrm{T}} \ddot{\theta}^2(t)dt \to \min \tag{8}$$

where θ_I and θ_F are the initial and final positions.

The least-norm continuous input for $0 \leqslant t \leqslant T$ gives:

$$u(t) = B^T e^{A(T-t)} \left[\int_0^T \left(e^{A \cdot t} B B^T e^{A^T t} \right) dt \right]^{-1} [x(T) - e^{A \cdot T} x(0)] \tag{9}$$

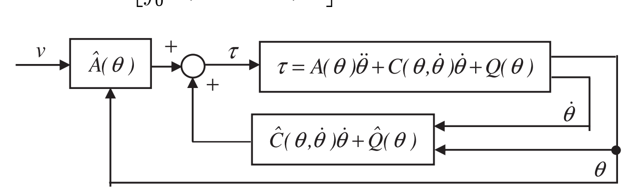

Fig. 2. Feedback linearization architecture.

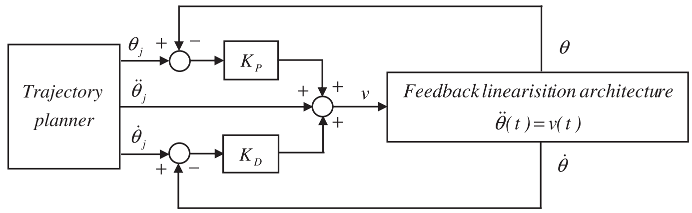

Fig. 3. Linear and decoupled closed-loop system.

from which we find:

$$u(t)=\frac{6}{T^2}(\theta_F-\theta_I)\left[1-2\frac{t}{T}\right] \tag{10}$$

Thus, for a 2-DOF serial manipulator, the angular accelerations, angular velocities and angular positions determined from minimisation of condition (8) are

$$\ddot{\theta}_j(t)=\frac{6}{T^2}(\theta_{jF}-\theta_{jI})\left[1-2\frac{t}{T}\right],\quad (j=1,2) \tag{11}$$

$$\dot{\theta}_j(t)=\frac{6}{T}(\theta_{jF}-\theta_{jI})\left[\frac{t}{T}-\left(\frac{t}{T}\right)^2\right],\quad (j=1,2) \tag{12}$$

$$\theta_j(t)=\theta_{jI}+(\theta_{jF}-\theta_{jI})\left[3\left(\frac{t}{T}\right)^2-2\left(\frac{t}{T}\right)^3\right],\quad (j=1,2) \tag{13}$$

It should be noted that these relationships, which ensure the execution of the motions between the initial and final positions, provide the minimal energy consumption.

3. Torque minimisation via an adaptive counterweight system

Taking the obtained joint angles and their derivatives (11)–(13), the dynamic Eq. (1) of a two degrees of freedom unbalanced serial manipulator can be rewritten as:

$$\tau_1=\left(k_{11}^0+k_{12}^0\cos\theta_2\right)\ddot{\theta}_1+\left(k_{31}^0+k_{32}^0\cos\theta_2\right)\ddot{\theta}_2-k_{12}^0\dot{\theta}_1\dot{\theta}_2 \times\sin\theta_2-k_{32}^0\dot{\theta}_2^2+p_1^0\cos\theta_1+p_2^0\cos(\theta_1+\theta_2) \tag{14}$$

$$\tau_2=\left(k_{31}^0+k_{32}^0\cos\theta_2\right)\ddot{\theta}_1+2k_2^0\ddot{\theta}_2+0.5k_{12}^0\dot{\theta}_1^2\sin\theta_2+p_2^0 \times\cos(\theta_1+\theta_2) \tag{15}$$

with

$$k_{11}^0=I_A+I_{S2}+m_2\left(l_1^2+l_{S2}^2\right) \tag{16}$$

$$k_{12}^0=2m_2l_1l_{S2} \tag{17}$$

$$k_2^0=0.5\left(I_{S2}+m_2l_{S2}^2\right) \tag{18}$$

$$k_{31}^0=I_{S2}+m_2l_{S2}^2 \tag{19}$$

$$k_{32}^0=m_2l_1l_{S2} \tag{20}$$

$$p_1^0=m_2gl_1+m_1gl_{S1} \tag{21}$$

$$p_2^0=m_2gl_{S2} \tag{22}$$

where m_1 and m_2 are the masses of links 1 and 2; l_1 is the distance between the joint centres A and B; θ_1 is the angular displacement of link 1 relative to the base; θ_2 is the angular displacement of link 2 relative to link 1; $\dot{\theta}_1$ is the angular velocity of link 1 relative to the base; $\dot{\theta}_2$ is the angular velocity of link 2 relative to link 1; I_A is the axial moment of inertia of link 1 relative to A; l_{S1} is the distance between the centre of mass $S1$ of link 1 and joint centre A; I_{S2} is the axial moment of inertia of link 2 relative to the centre of mass $S2$ of link 2; $l_{S2}$is the distance between the centre of mass $S2$ of link 2 and joint centre B; g is the gravitational acceleration.

Let us now consider the input torques due to the counterweights:

$$\tau_1^{CW}=(k_{11}+k_{12}\cos\theta_2)\ddot{\theta}_1+(k_{31}+k_{32}\cos\theta_2)\ddot{\theta}_2-k_{12}\dot{\theta}_1\dot{\theta}_2 \times\sin\theta_2-k_{32}\dot{\theta}_2^2+p_1\cos\theta_1+p_2\cos(\theta_1+\theta_2) \tag{23}$$

$$\tau_2^{CW}=(k_{31}+k_{32}\cos\theta_2)\ddot{\theta}_1+2k_2\ddot{\theta}_2+0.5k_{12}\dot{\theta}_1^2\sin\theta_2+p_2 \times\cos(\theta_1+\theta_2) \tag{24}$$

with

$$k_{11}=m_{CW1}r_{CW1}^2+m_{CW2}\left(l_1^2+r_{CW2}^2\right) \tag{25}$$

$$k_{12}=-2m_{CW2}l_1r_{CW2} \tag{26}$$

$$k_2=0.5m_{CW2}r_{CW2}^2 \tag{27}$$

$$k_{31}=m_{CW2}r_{CW2}^2 \tag{28}$$

$$k_{32}=-m_{CW2}l_1r_{CW2} \tag{29}$$

$$p_1=-m_{CW1}gr_{CW1}+m_{CW2}gl_1 \tag{30}$$

$$p_2=-m_{CW2}gr_{CW2} \tag{31}$$

where m_{CW1} and m_{CW2} are the masses of the counterweights; r_{CW1} is the rotation radius of the centre of mass of the counterweight with respect A; r_{CW2} is the rotation radius of the centre of mass of the counterweight with respect B.

Eqs. (23) and (24) can be rewritten to show the influence of the rotation radiuses of the centres of the counterweight masses on the input torques:

$$\tau_1^{CW}=c_{11}r_{CW1}^2+c_{12}r_{CW1}+c_{13} \tag{32}$$

where

$$c_{11}=m_{CW1}\ddot{\theta}_1 \tag{33}$$

$$c_{12}=-m_{CW1}g\cos\theta_1 \tag{34}$$

$$c_{13}=m_{CW2}\left[\left(l_1^2+r_{CW2}^2-l_1r_{CW2}\cos\theta_2\right)\ddot{\theta}_1+(r_{CW2}^2-l_1r_{CW2}\cos\theta_2)\ddot{\theta}_2 +2l_1r_{CW2}\dot{\theta}_1\dot{\theta}_2\sin\theta_2+l_1r_{CW2}\dot{\theta}_2^2\sin\theta_2+l_1g\cos\theta_1 -r_{CW2}g\cos(\theta_1+\theta_2)\right] \tag{35}$$

and

$$\tau_2^{CW}=c_{21}r_{CW2}^2+c_{22}r_{CW2} \tag{36}$$

where

$$c_{21}=m_{CW2}(\ddot{\theta}_1+\ddot{\theta}_2) \tag{37}$$

$$c_{22}=m_{CW2}[l_1\ddot{\theta}_1\cos\theta_2+l_1\dot{\theta}_1^2\sin\theta_2+g\cos(\theta_1+\theta_2)] \tag{38}$$

Thus, the problem can be formulated as follows: *to find such rotation radiuses of the centres of the counterweight masses which will allow to minimise the root-mean-square (RMS) values of the input torques.*

It should be noted that in this case the conditions of the static balancing cannot be respected that the disposition of counterweights are modified but their masses are not.

To obtain a minimum of the RMS values it is necessary to minimise the sum:

$$\Delta\tau_j = \sum_{i=1}^{N}\left(\tau_{ji} + \tau_{ji}^{CW}\right)^2 \to \min_{r_{CWj}}, \quad (j = 1, 2) \tag{39}$$

where i and N are, respectively, the index and the number of calculated positions of the manipulator.

For this purpose, it is necessary to ensure the conditions:

$$\frac{\partial\Delta\tau_j}{\partial r_{CWj}} = 0 \quad , \quad (j = 1, 2) \tag{40}$$

from which we obtain two cubic equations:

$$z_j^3 + a_j z_j^2 + b_j z_j + c_j = 0, \quad (j = 1, 2) \tag{41}$$

where

$$z_1 = r_{CW1} \tag{42}$$

$$a_1 = 3\sum_{i=1}^{N} c_{11i}c_{12i} \Big/ 2\sum_{i=1}^{N} c_{11i}^2 \tag{43}$$

$$b_1 = \left(2\sum_{i=1}^{N} c_{11i}\tau_{1i} + 2\sum_{i=1}^{N} c_{11i}c_{13i} + \sum_{i=1}^{N} c_{12i}^2\right) \Big/ 2\sum_{i=1}^{N} c_{11i}^2 \tag{44}$$

$$c_1 = \left(\sum_{i=1}^{N} c_{12i}\tau_{1i} + \sum_{i=1}^{N} c_{12i}c_{13i}\right) \Big/ 2\sum_{i=1}^{N} c_{11i}^2 \tag{45}$$

$$z_2 = r_{CW2} \tag{46}$$

$$a_2 = 3\sum_{i=1}^{N} c_{21i}c_{22i} \Big/ 2\sum_{i=1}^{N} c_{21i}^2 \tag{47}$$

$$b_2 = \left(2\sum_{i=1}^{N} c_{21i}\tau_{2i} + \sum_{i=1}^{N} c_{22i}^2\right) \Big/ 2\sum_{i=1}^{N} c_{21i}^2 \tag{48}$$

$$c_2 = \sum_{i=1}^{N} c_{22i}\tau_{2i} \Big/ 2\sum_{i=1}^{N} c_{21i}^2 \tag{49}$$

The solutions of cubic Eq. (41) with real coefficients can be expressed in algebraic form by means of Viette–Cordano method [48].

For determination of roots, first of all, we shall calculate:

$$Q_j = \left(a_j^2 - 3b_j\right)/9 \tag{50}$$

$$R_j = \left(2a_j^3 - 9a_jb_j + 27c_j\right)/54 \tag{51}$$

When $R_j^2 < Q_j^3$, cubic equation has three real roots, determined by the following expressions:

$$z_{1j} = -2\sqrt{Q_j}\cos(t_j) - a_j/3 \tag{52}$$

$$z_{2j} = -2\sqrt{Q_j}\cos(t_j + 2\pi/3) - a_j/3 \tag{53}$$

$$z_{3j} = -2\sqrt{Q_j}\cos(t_j - 2\pi/3) - a_j/3 \tag{54}$$

$$t_j = \cos^{-1}\left[R_j\sqrt{Q_j^3}/3\right] \tag{55}$$

When $R_j^2 \geqslant Q_j^3$, general cubic equation case has one real root and two real roots for confluent case.

For determination of the complex roots, it is necessary to calculate:

$$A_j = -sign(R_j)\sqrt[3]{|R_j| + \sqrt{R_j^2 - Q_j^3}} \tag{56}$$

$$B_j = Q_j/A_j(\text{if } A_j \neq 0) \text{ and } B_j = 0(\text{if } A_j = 0) \tag{57}$$

The real root is

$$z_{1j} = A_j + B_j - a_j/3 \tag{58}$$

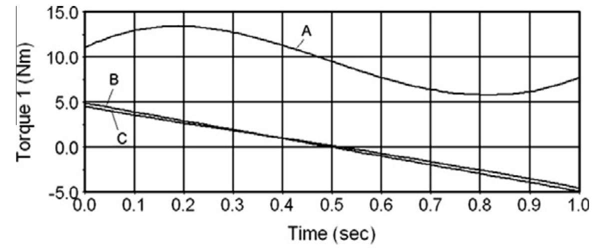

Fig. 4. Torque of the first actuator ($j = 1$).

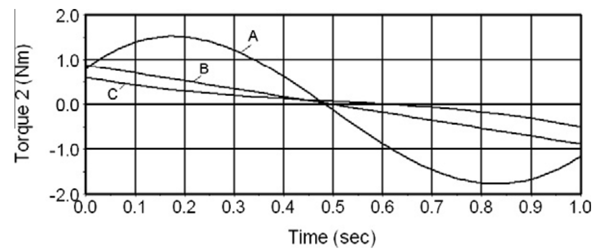

Fig. 5. Torque of the second actuator ($j = 2$).

In the case, when $A_j = B_j$, the complex roots become the real roots:

$$z_{2j} = -A_j - a_j/3 \tag{59}$$

4. Illustrative example

For illustration of the suggested approach let us consider a numerical example. Numerical simulations were carried out for the 2-DOF serial manipulator with parameters: $m_1 = 2.5$ kg; $m_2 = 2$ kg; $l_1 = 0.4$ m; $l_{AS1} = l_{BS2} = 0.1$ m; $l_{S1} = l_{S2} = 0.1$ m; $I_{S1} = 0.15$ kgm^2; $I_{S2} = 0.1$ kgm^2.

In order to compare the manipulator's behaviour for different cases, in Figs. 4 and 5 are given the values of the input torques obtained using the software ADAMS for the following numerical simulations.

A. The generation of motions for an unbalanced manipulator between the initial and final positions of links :$\theta_{1I} = 0$; $\theta_{1F} = 0.5236$; $\theta_{2I} = 1.1526$ and $\theta_{2F} = 1.6762$, are carried out by the following fifth order polynomial laws: $\theta_j = 5.2360t^3 - 7.8540t^4 + 3.1416t^5$, $(j = 1,2)$, $0 \leqslant t \leqslant 1$ s.
B. The generation of motions between the same initial and final positions are carried out by laws $\theta_j = 0.5236\ (3t^2 - 2t^3)$, determined from Eq. (13).
C. For the same manipulator with $m_{CW1} = 7.4$ kg, $m_{CW2} = 2$ kg and laws $\theta_j = 0.5236\ (3t^2 - 2t^3)$, we obtain from (58) and (59) the rotation radiuses of the counterweights $r_{CW1} = 0.25$ m, $r_{CW2} = 0.147$ m and compute again the torques.

The obtained results showed that after first step of minimisation the torque root-mean-square sum minimisation was reduced up to 71% and 59%, after second step: 78% and 74%, respectively. In more practical terms, the maximum actuator torque required after first step of minimisation was reduced up to 63% and 50%, after second step: 67% and 66%, respectively.

5. Conclusion

This paper deals with the analytically tractable solution for input torques minimisation of two degrees of freedom serial manipulators via minimum energy control and optimal redistribution of movable masses. In the first part, the minimisation of the input torques of the 2-DOF serial manipulator is carried out by an optimal motion execution based on the energy minimisation. Then, the optimal dispositions of the counterweights are obtained by minimisation of the root-mean-square values of the input torques due to the dynamic and static loads. Consequently, two cubic equations are deduced, making possible the determination of the rotation radiuses of the centres of the counterweight masses.

The suggested approach is illustrated by numerical simulations carried out using ADAMS software. The obtained results showed that for examined 2-DOF serial manipulator the significant reduction of torques was achieved.

Finally, it should be noted that the main advantage with such an approach concerns the fact that optimal motion laws and counterweights dispositions are obtained in a symbolic form and they can be implemented easily on the manipulator controllers.

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