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Design of planar 3-DOF 3-RRR reactionless parallel manipulators

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ABSTRACT

This paper discusses the development of reactionless 3-RRR planar parallel manipulators, which apply no reaction forces or moments to the mounting base during motion. Design equations and techniques are proposed which allow for the dynamic substitution of the mass of the moving platform of a parallel manipulator by three concentrated masses. The dynamic model of the moving platform consequently represents a weightless link with three concentrated masses. This allows for the transformation of the problem of the design of a reactionless manipulator into a problem of balancing pivoted legs carrying concentrated masses. The total angular momentum of the manipulator can be reduced to zero using two approaches: (i) on the basis of counter-rotations and (ii) using an inertia flywheel rotating with a prescribed angular velocity. The suggested solutions are illustrated through computer simulations and the results verified by showing that the manipulator is indeed reactionless, there being no forces or moments transmitted to the base during motion of the moving platform.

1. Introduction

In high-speed mechanical systems, mass balancing of the moving links brings about a reduction of the variable dynamic loads on the frame and, as a result, a reduction of vibrations. Different approaches and solutions have been developed and documented [1–3] but, despite its long history, mechanism balancing theory continues to develop and new approaches and solutions are constantly being reported. A new field for their application is the design of fast parallel manipulators, which are very efficient for advanced robotic applications. Previous work on the problem of balancing of parallel manipulators may be arranged in the following groups:

(a) Shaking force balancing by counterweights mounted on the movable links of the parallel manipulator [4–8]. The aim of these balancing methods is the redistribution of movable masses by adding counterweights to the links, which allows the fixation of the common centre of mass of the moving links of the manipulator. After such a redistribution of the masses, the gravitational and inertia forces are cancelled.
(b) Gravitational force balancing by springs mounted on the movable links of the parallel manipulator [7–10]. Such a balancing can be defined as when the weights of the links do not produce any force on the actuators for any configuration of the manipulator, i.e. potential energy of the parallel manipulator is constant for all possible configurations. It should be noted that many results in the field of balancing of robotic arms and linkages [11–17] can be successfully applied to the balancing problems of parallel manipulators.
(c) Gravitational force balancing by secondary mechanisms coupled with the parallel manipulator [18–22]. In this case the balancing element, which can be a spring [18], a counterweight [19] or an actuating power cylinder [20–22], is mounted on the links of the secondary mechanism. In these studies the added system is a pantograph linkage which allows the gravitational forces to be balanced.

These approaches have been developed for inertia or gravitational force balancing of parallel manipulators. In the case of shaking force balancing the mentioned methods allow the cancellation of the resultant of all reaction forces at the frame. However, the unbalanced angular moments create a moment on the frame, which can also be significant.

Among several works on this subject, studies devoted to the design of reactionless parallel manipulators [23,24] should be highlighted. These manipulators are of interest because the inertia forces are cancelled together with the total angular momentum of the manipulator. Such a design enables the cancellation of the reaction forces and torques at the frame of the parallel manipulator.

In this paper, the design of reactionless 3-DOF 3-RRR planar parallel manipulators is considered and design equations and techniques are developed.

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2. 3-DOF 3-RRR planar parallel manipulator and dynamic model with concentrated masses (a special shape of the moving platform)

The moving platform of a planar 3-DOF 3-RRR parallel manipulator is connected to its legs by three revolute joints P_i ($i = 1,2,3$) (Fig. 1) [25]. Each leg comprises two links connected by revolute joints A_i ($i = 1,2,3$) and they are mounted on the frame by revolute joints O_i ($i = 1,2,3$). The input parameters of such a manipulator are defined by the joint angles θ_i ($i = 1,2,3$) of each leg and the output parameters by the pose of the moving platform, i.e. its orientation ϕ and position of one point of the moving platform, by example, the centre of mass of the moving platform (x_O, y_O).

Note that all axes of revolute joints are parallel, i.e. this is a mechanism in which all points of the links describe paths located in parallel planes.

Thus the conditions for dynamic substitution of the mass of the platform (Fig. 2) by three concentrated masses situated on the axis of joints P_i ($i = 1,2,3$) are the following [26–28]:

$$\sum_{i=1}^{3} m_i = m_{\mathrm{pl}} \tag{1}$$

$$\sum_{i=1}^{3} m_i x_i = 0 \tag{2}$$

$$\sum_{i=1}^{3} m_i y_i = 0 \tag{3}$$

$$\sum_{i=1}^{3} m_i (x_i^2 + y_i^2) = I_{\mathrm{pl}(zz)} \tag{4}$$

where m_i are point masses located on the joint axis; x_i and y_i are coordinates of point masses with respect to the platform frame xOy; m_{pl} is the mass of the moving platform and $I_{\mathrm{pl}(zz)}$ is the axial moment of inertia of the moving platform with respect to the centre of mass.

Thus, if we have a platform with such a redistribution of masses, when the conditions (1)–(4) are fulfilled, the mass of the platform can be dynamically substituted by three concentrated masses, i.e. the platform can be considered as a weightless link with three point masses attached.

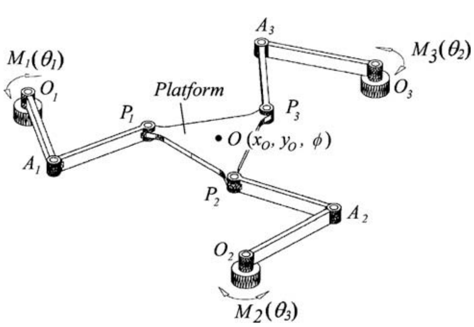


Fig. 1. Planar 3-DOF 3-RRR parallel manipulator.

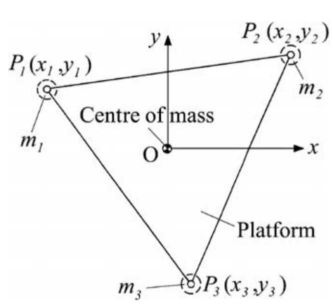


Fig. 2. Moving platform of the parallel manipulator and the point masses m_i ($i = 1,2,3$).

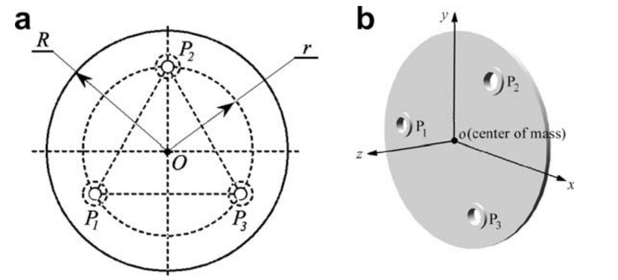


Fig. 3. A special shape of the moving platform: (a) drawing in 2D and (b) CAD model in 3D.

A design example of such a platform is now considered. Fig. 3 shows a platform of a parallel manipulator, which represents a cylinder of radius R. The axial moment of inertia of this platform with respect to the centre of the mass is equal to $J_{\mathrm{pl}(zz)} = m_{\mathrm{pl}}R^2/2$. If it is desired to substitute dynamically the mass of the platform by three point masses disposed on the vertices of an isosceles triangle $P_1P_2P_3$, it is necessary to situate the revolute joints P_i at distances $r = R/\sqrt{2}$ from the centre O. In this case, the axial moment of inertia of the three point masses and the moving platform are identical. This is an example of conceivable shape, but it is obviously possible to find several examples which allow the dynamic substitution of the mass of the platform by three concentrated masses.

Thus, we can replace the effect of mass and inertia of the moving platform by three point masses, which are at the end of each leg. This model allows the transformation of the manipulator balancing problem into one of balancing the legs. The latter is much simpler than the former.

3. Balancing of legs

3.1. Balancing by counter-rotations

The suggested balancing method is based on balancing of the inertia forces by means of counterweights mounted on the links and balancing of the total angular momentum by means of counterweights with planetary gear trains to generate the counter-rotations.

3.1.1. Shaking force balancing

In order to achieve the dynamic balancing of the manipulator, we first have to ensure that it is force balanced, i.e. statically balanced. As mentioned above, the mass of the moving platform is substituted by three equivalent point masses located at the legs, i.e. each leg of the manipulator can be balanced independently.

The centre of mass of each leg relative to its base O_i (Fig. 4) can be found by the expressions:

$$x_{Si} = (m_{2i}x_{2i} + m_{3i}x_{3i} + m_i x_{Pi})/(m_{2i} + m_{3i} + m_i) \tag{5}$$

$$y_{Si} = (m_{2i}y_{2i} + m_{3i}y_{3i} + m_i y_{Pi})/(m_{2i} + m_{3i} + m_i) \tag{6}$$

where

$$x_{2i} = r_{S2i}\cos\theta_i \tag{7}$$

$$y_{2i} = r_{S2i}\sin\theta_i \tag{8}$$

$$x_{3i} = l_{2i}\cos\theta_i + r_{S3i}\cos\theta_{i+3} \tag{9}$$

$$y_{3i} = l_{2i}\sin\theta_i + r_{S3i}\sin\theta_{i+3} \tag{10}$$

$$x_{Pi} = l_{2i}\cos\theta_i + l_{3i}\cos\theta_{i+3} \tag{11}$$

$$y_{Pi} = l_{2i}\sin\theta_i + l_{3i}\sin\theta_{i+3} \tag{12}$$

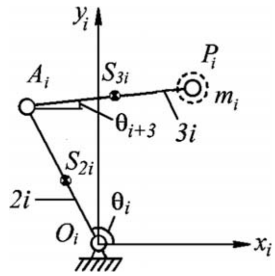


Fig. 4. Modeling of leg i of the planar parallel manipulator (i = 1,2,3).

m_{2i} and m_{3i} are the masses of links 2i and 3i; m_i is the point mass obtained from dynamic substitution of masses of the platform; $r_{S2i} = l_{O_iS_{2i}}$ is the distance of the centre of mass S_{3i} of the link 3i from the joint centre A_i; $r_{S3i} = l_{A_iS_{3i}}$ is the distance of the centre of mass S_{2i} of the link 2i from the joint centre O_i; $l_{2i} = l_{O_iA_i}$ and $l_{3i} = l_{A_iP_i}$ are the lengths of the links 2i and 3i.

It is clear that the motion of the centre of mass of each leg is generated by the two angles θ_i and θ_{i+3}. Thus, for the position of the centre of mass to remain constant and located at the axis O_i, it is sufficient that the coefficients of the variables θ_i and θ_{i+3} be equal to zero, i.e.

$$m_{3i}r_{S3i} + m_i l_{3i} = 0 \tag{13}$$

$$m_{2i}r_{S2i} + (m_i + m_{3i})l_{2i} = 0 \tag{14}$$

The conditions (13) and (14) can be satisfied by adding two counterweights mounted on links 2i and 3i, which produce negative values of radii r_{S2i} and r_{S3i}.

After such a redistribution of masses, all moving masses of the manipulator can be replaced by three fixed masses $m_{Oi} = m_{2i} + m_{3i} + m_i$ (i = 1,2,3) located at the axis O_i and the centre of mass of the manipulator is located at the centre of these three fixed masses. Thus, the centre of mass of the manipulator remains motionless for any motion of links and hence, the manipulator transmits no inertia loads to its base.

3.1.2. Shaking force and shaking moment balancing

Now that the inertia force balancing is achieved, we have to consider the cancellation of the shaking moment. As in the first case, we consider the balancing of the manipulator legs. There are several approaches for complete shaking moment balancing of articulated dyads with two revolute joints [29–33]. The balancing method applied in this case is based on the shaking moment balancing by means of counterweights with planetary gear trains carrying out the counter-rotations [31]. The dynamic balancing scheme of each leg is designed in the following manner. The gear 3_{GRi} (Fig. 5) is mounted on the rotation axis O_i of input link 2i and is linked kinematically with 3i through belt transmission 6i. It meshes also with gear 4i mounted on the base. The gear 2_{GRi} is mounted on input link 2i and meshes with gear 5i mounted on the rotation axis B_i. It should be noted that the joint between the gear 3_{GRi} and the frame is different from the joint O_i and it will be designated O_{3GRi}.

Thus the shaking moment may be balanced by the moment of inertia of gears 4i and 5i taking into account that the angular velocities of links are the following: $\dot{\theta}_{4i} = -\dot{\theta}_{i+3}$ and $\dot{\theta}_{5i} = -\dot{\theta}_i$.

After shaking force balancing, the shaking moment applied on the base is constant relative to any point, i.e. for a given position of the manipulator it has the same value for any point of the base and can be expressed as

$$M^{\mathrm{sh}} = \sum_{i=1}^{3} M_i^{\mathrm{sh}} = \sum_{i=1}^{3} \frac{\mathrm{d}H_{Oi}}{\mathrm{d}t} \quad (i = 1,2,3) \tag{15}$$

where H_{Oi} is the angular momentum of the moving links of each leg with respect to O_i.

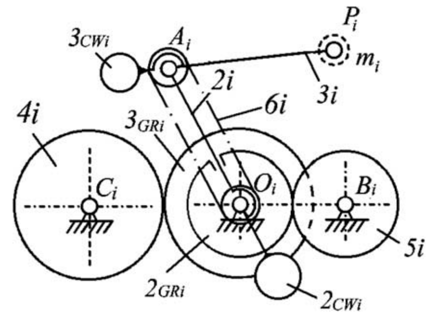


Fig. 5. Dynamic balancing scheme of legs (i = 1,2,3).

In order to have a shaking moment equal to zero for all trajectories, the sum of the angular momentum of the legs must be constant over time.

The angular momentum for each leg with added planetary gear trains can be written as

$$\begin{aligned} H_{Oi} = {} & m_{2i}(x_{2i}\dot{y}_{2i} - y_{2i}\dot{x}_{2i}) + I_{2i}\dot{\theta}_i + I_{2GRi}\dot{\theta}_i + I_{5i}\dot{\theta}_{5i} \\ & + m_{3i}(x_{3i}\dot{y}_{3i} - y_{3i}\dot{x}_{3i}) + I_{3i}\dot{\theta}_{i+3} + I_{3GRi}\dot{\theta}_{i+3} + I_{4i}\dot{\theta}_{4i}] \\ & + m_i(x_{Pi}\dot{y}_{Pi} - y_{Pi}\dot{x}_{Pi}) \quad (i = 1,2,3) \end{aligned} \tag{16}$$

where I_{2i} and I_{3i} are the moments of inertia of links 2i and 3i about the centres of mass of the links (axial moment of inertia), I_{2GRi} and I_{3GRi} are the axial moments of inertia of gears 2GRi and 3GRi, I_{4i} and I_{5i} are the axial moments of inertia of the added gears.

We substitute Eqs. (7)–(12) and their derivatives into Eq. (17) and taking into account conditions (13) and (14), we obtain the following expression of the angular momentum for each leg:

$$\begin{aligned} H_{Oi} = {} & \left(I_{3GRi} + I_{3i} + m_{3i}r_{3Si}^2 + m_i l_{3i}^2\right)\dot{\theta}_{i+3} + I_{4i}\dot{\theta}_{4i} \\ & + \left(I_{2GRi} + J_{2i} + m_{2i}r_{2Si}^2 + (m_{3i} + m_i)l_{2i}^2\right)\dot{\theta}_i + I_{5i}\dot{\theta}_{5i} \\ & (i = 1,2,3) \end{aligned} \tag{17}$$

from which we obtain the conditions of shaking moment balancing

$$I_{4i} = I_{3GRi} + I_{3i} + m_{3i}r_{3Si}^2 + m_i l_{3i}^2 \tag{18}$$

$$I_{5i} = I_{2GRi} + I_{2i} + m_{2i}r_{2Si}^2 + (m_{3i} + m_i)l_{2i}^2 \tag{19}$$

Hence, any 3-DOF 3-RRR parallel manipulator satisfying Eqs. (1)–(4), (13), (14), (19) and (20) will be dynamically balanced, i.e. reactionless.

The disadvantage of the suggested balancing scheme is the need for the connection of gears to the oscillating links. The oscillations of the links of the manipulator will create noise unless expensive anti-backlash gears are used.

Anti-backlash gears are devices that bias the gear always to favour one side of the tooth through spring action. Regardless of the direction of movement, they should always 'push' up against the same side of the tooth. They are basically comprised of two gears that are spring-loaded in opposing directions. One gear is attached to the mechanism being moved, the other simply floats to provide the bias.

3.2. Numerical example and simulation results

We shall now examine the ground bearing forces and the ground bearing moments of a 3-DOF 3-RRR parallel manipulator which is fully force and moment balanced. The geometry and mass distribution parameters of the links are listed in Table 1. (Parameters x_{P_i} and y_{P_i} show the selected configuration of the manipulator.)

The platform of the examined parallel manipulator represents a cylinder with radius R = 0.082 m and m = 3 kg. As a result, $I_{\mathrm{pl}(zz)}$ = 0.01 kg m^2 (so that m_i = 1 kg and r_i = 0.058 m (i = 1,2,3)).

Table 1
Parameters of the balanced manipulator

Parameter	Legs		
	1	2	3
X_{O_i}	0	0.46	0.22
Y_{O_i}	0	0	0.4
l_{2i} (m)	0.18	0.18	0.18
l_{3i} (m)	0.18	0.18	0.18
x_{P_i} $(t=0)$ (m)	0.18	0.28	0.22
y_{P_i} $(t=0)$ (m)	0	0	0.087
m_i (kg)	1	1	1
r_i (m)	0.058	0.058	0.058
m_{2i} (kg)	7.2	7.2	7.2
r_{2Si} (m)	0.09	0.09	0.09
m_{3i} (kg)	2.6	2.6	2.6
r_{3Si} (m)	0.07	0.07	0.07
I_{2i} (kg m^2)	0.02	0.02	0.02
I_{3i} (kg m^2)	0.017	0.017	0.017
I_{4i} (kg m^2)	0.068	0.068	0.068
I_{5i} (kg m^2)	0.2	0.2	0.2

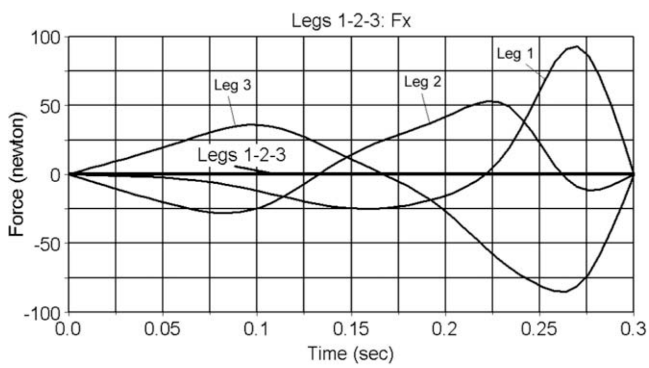


Fig. 6. Variations of the ground bearing forces of the balanced manipulator along the *X*-axis.

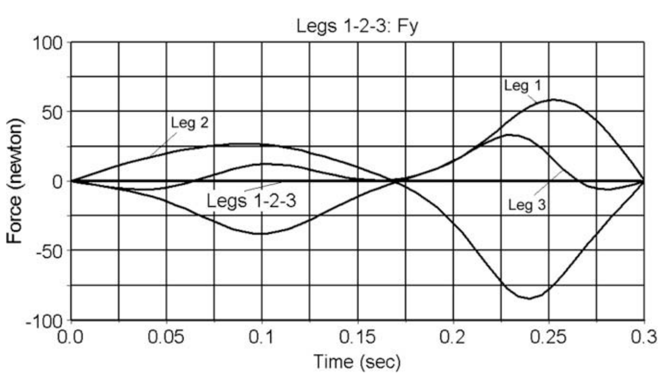


Fig. 7. Variations of the ground bearing forces of the balanced manipulator along the *Y*-axis.

The drivers are given by the expressions [34,35]: $\theta_i = a_i\pi + b_i(2\pi t/T - \sin(2\pi t/T))$, where $a_1 = 1/3$, $a_2 = 4/3$, $a_3 = 10/3$, $b_1 = 1/6$, $b_2 = -1/6$, $b_3 = 1/12$, $T = 0.3$ s. The angles of the input links θ_i are measured with respect to the global *X*-axis. The driver functions give zero velocity and acceleration at the start and end of the motion.

Figs. 6 and 7 show the resultant bearing forces of the balanced planar 3-DOF 3-RRR manipulator along the *X* and *Y* axes.

In Fig. 8 are presented the variations of the moment of the ground bearing forces and the reactions of the input torques.

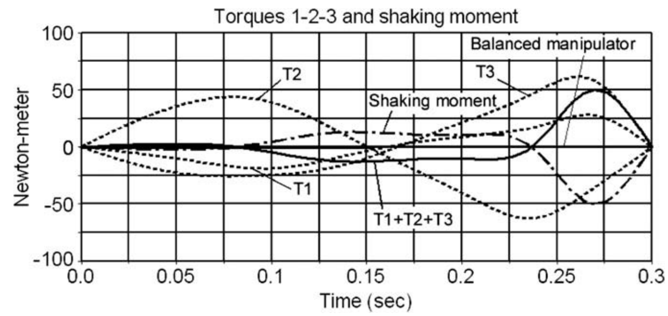


Fig. 8. Variations of the moment of the ground bearing forces and the reactions of the input torques of the balanced manipulator.

4. Balancing by inertia flywheel

In this section we consider the shaking moment cancellation of the fully force balanced 3-DOF 3-RRR parallel manipulator by an inertia flywheel with prescribed rotation. It is evident that this solution is constructively more efficient.

Fig. 9 shows the fully force balanced 3-DOF 3-RRR parallel manipulator and balancing inertia flywheel, which is mounted on the base of the manipulator. The conditions for balancing the shaking moment of the manipulator are determined from the following consideration.

Note that in this case the platform has an axial inertia moment which cannot be dynamically substituted by three concentrated masses, i.e. $\sum_{i=1}^{3} m_i(x_i^2 + y_i^2) \neq I_{\mathrm{pl}(zz)}$.

Thus, the angular momentum for the fully force balanced manipulator can be written as

$$
\begin{aligned}
H &= \sum_{i=1}^{3} H_{0i} + \Delta I_{\mathrm{pl}}\dot{\phi} \\
&= \sum_{i=1}^{3} \left[(J_{2i} + m_{2i}r_{2Si}^2 + (m_{3i} + m_i)l_{2i}^2)\dot{\theta}_i + (J_{3i} + m_{3i}r_{3Si}^2 + m_i l_{3i}^2)\dot{\theta}_{i+3} \right] \\
&\quad + \Delta I_{\mathrm{pl}}\dot{\phi}
\end{aligned}
\tag{20}
$$

where $\Delta I_{\mathrm{pl}(zz)} = I_{\mathrm{pl}(zz)} - \sum_{i=1}^{3} m_i(x_i^2 + y_i^2)$.

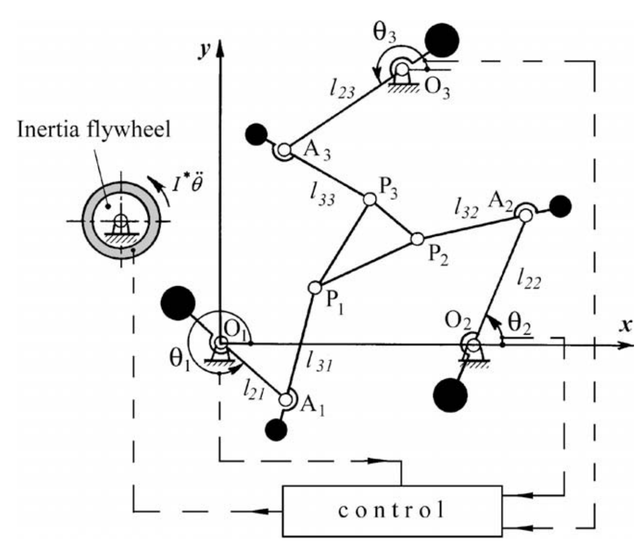


Fig. 9. Shaking moment balancing of fully force balanced 3-DOF 3-RRR parallel manipulator by an inertia flywheel.

Hence, the shaking moment is the following:

$$\begin{aligned} M^{\mathrm{sh}} &= \sum_{i=1}^{3} M^{\mathrm{sh}}_{0i} + M^{\mathrm{sh}}_{\mathrm{pl}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{3} H_{Oi} + \Delta I_{\mathrm{pl}}\dot{\phi}\right) \\ &= \sum_{i=1}^{3}(K_i\ddot{\theta}_i + K_{i+3}\ddot{\theta}_{i+3}) + \Delta I_{\mathrm{pl}}\ddot{\phi} \end{aligned} \tag{21}$$

where $K_i = I_{2i} + m_{2i}r^2_{2Si} + (m_{3i} + m_i)l^2_{2i}$ and $K_{i+3} = I_{3i} + m_{3i}r^2_{3Si} + m_i l^2_{3i}$.

To balance the shaking moment, an inertia flywheel with axial inertia moment I^* can be used. The angular acceleration of this inertia flywheel driven by a complementary actuator is the following:

$$\ddot{\theta} = M^{\mathrm{sh}}/I^* = \sum\nolimits_{i=1}^{3}(K_i\ddot{\theta}_i + K_{i+3}\ddot{\theta}_{i+3}) + \Delta I_{\mathrm{pl}(zz)}/I^* \tag{22}$$

It should be noted that the axial inertia moment of the flywheel must be selected in such a manner that its rotation with prescribed acceleration will be feasible. Therefore, the reaction of the balancing inertia flywheel on the frame cancels the shaking moment due to the parallel manipulator. In other words, the actuator, which rotates the balancing inertia flywheel with a prescribed angular acceleration $\ddot{\theta}$ has a reaction on the frame which is similar but opposite to the shaking moment of the parallel manipulator. Thus, full shaking moment is annulled.

The angular velocity $\dot{\theta}(t)$ and angular displacements $\theta(t)$ can be determined by simple integration of the obtained values of $\ddot{\theta}(t)$.

4.1. Numerical example and simulation results

Let us consider a numerical example for computer simulation. As a model we could use the previous example with the link parameters given in Table 1. However, for the best illustration of the suggested balancing approach, we change the value of the axial moment of inertia of the platform: $J_{\mathrm{pl}(zz)} = 0.015$ kg m^2, i.e. the mass of the platform cannot be dynamically substituted by three concentrated masses. It should be noted that in this case, however, we do not need this condition. Thus, by substituting statically the mass of the platform by three concentrated masses, the shaking force balancing is carried out in the same way as in the previous case. Then the angular accelerations of the movable links are determined (Table 2) taking into account that the drivers are given by the expressions [34,35]: $\theta_i = a_i\pi + b_i(2\pi t/T - \sin(2\pi t/T))$ $(i = 1,2,3)$, where $a_1 = 1/3$, $a_2 = 4/3$, $a_3 = 10/3$, $b_1 = 1/6$, $b_2 = -1/6$, $b_3 = 1/12$, $T = 0.3$ s.

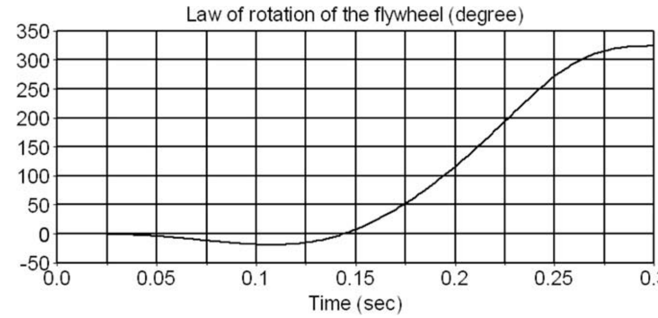


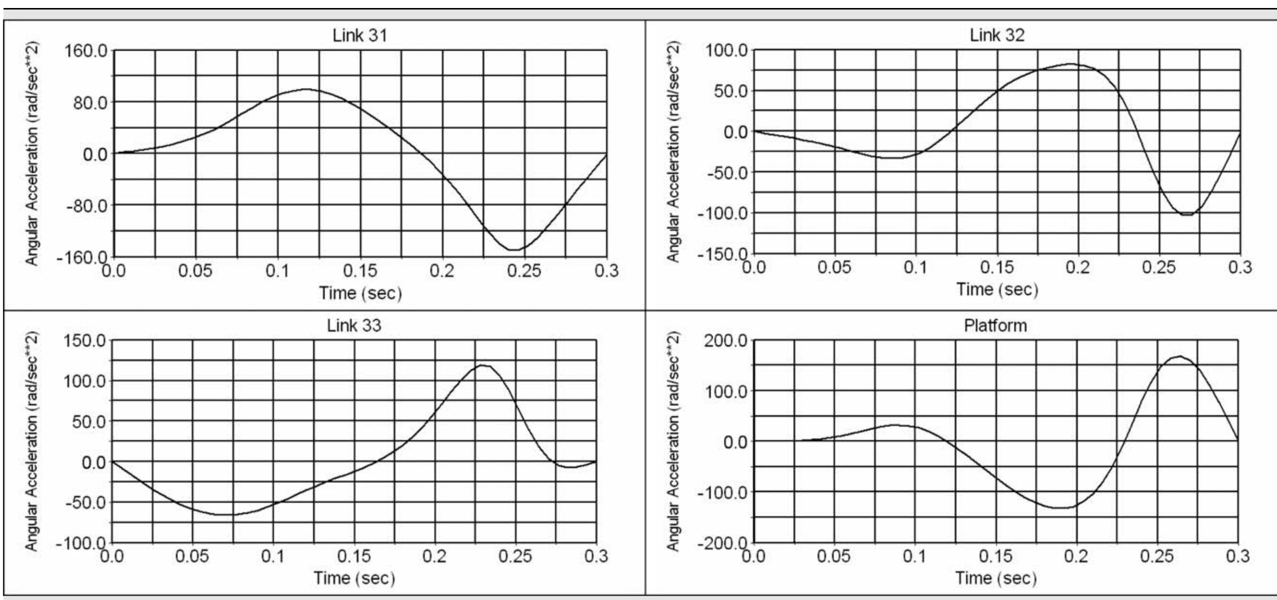
Fig. 10. Law of rotation of the balancing flywheel.

Now, by determining the shaking moment from (22) and taking as the axial moment inertia of the flywheel $I^* = 0.01$ kg m^2, we determine the angular acceleration of the balancing flywheel, which gives complete shaking moment balancing of the manipulator. Fig. 10 shows the obtained law of rotation of the flywheel, which produces complete shaking moment balancing.

5. Conclusions

A new field for shaking force and shaking moment balancing is the design of fast parallel manipulators, which are very efficient for advanced robotic applications. In this paper, the shaking force and shaking moment balancing approach is developed for planar 3-DOF 3-RRR parallel manipulators. It is based on the dynamic substitution of the mass of the platform by three concentrated masses situated at the axes of the revolute joints of the legs. By application of this approach the dynamic model of the platform represents a weightless link with three concentrated masses attached. This allows for the transformation of the problem of shaking force and shaking moment balancing of the manipulator into a problem of balancing legs carrying concentrated masses. A design example of a platform with point masses is examined. However, this

Table 2
Angular accelerations of links



approach requires the use of counter-rotations which increase the mass and inertia of the system. For this reason, a second approach is proposed, which allows the cancellation of the shaking moment of the system by means of a flywheel. Numerical examples confirm that after such a balancing, the manipulator transmits no inertia loads to its surroundings, i.e. the sum of all ground bearing forces and their moments are eliminated.

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