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Comparative analysis and synthesis of six-bar mechanisms formed by two serially connected spherical and planar four-bar linkages

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ABSTRACT

This paper deals with the comparative analysis and synthesis of the six-bar mechanisms formed by two spherical and planar four-bar linkages. The comparative analysis is carried out for a class of mechanical systems with the same output angles of oscillation. In order to obtain a consistent comparison, the input and output axes of the spherical linkages are placed parallel (in this way, the spatial mechanisms may be compared to planar four-bar mechanisms). The obtained results are compared using three generalized non-dimensional indexes, which characterize velocity, acceleration and dynamic power properties. On the basis of numerical simulations the correlation expressions are deducted. The comparative analysis has shown that for the studied class of mechanisms the spherical linkages have better properties than planar linkages.

1. Introduction

Mechanical systems with one degree of freedom, otherwise known as spherical four-bar linkages, are defined in mechanism and machine theory terminology (Terminology for the Mechanism and Machine Science, 2003) as mechanisms in which all points of its links describe paths located on concentric spheres.

Several works related to spherical four-bar linkages have been carried out. Freudenstein (1965) presented Grashof conditions for spherical 4R linkages. A notation for these linkages and a classification scheme based in Grashof's law was proposed by Savage and Hall (1970). Later on, Soni (1970) extended this classification. Chiang (1984) and Gupta (1986) provided early classifications for the spherical four-bar mechanism, which was analysed by means of the Grashof's law. Gosselin and Angeles (1987) developed a graphical representation of mobility regions for spherical 4R linkages, which was extended later on by Liu (1988). Continuing with the mobility of the spherical 4R linkage, Mallik (1994) and Gupta and Ma (1995) developed mobility conditions for its input and output link. Lu and Hwang (1996) proposed a classification of the spherical 4R linkage using symmetrical curves of the coupler link. Other classifications based on the limit positions of the input and output link were proposed by Ruth and McCarthy (1999) and McCarthy (2000). Recently, Jesus Cervantes-Sanchez and Hugo Medellin-Castillo (2002) and Medellin-Castillo and Jesus Cervantes-Sanchez (2005) proposed a classification scheme and an improved motion analysis for spherical 4R linkages. Tong et al. (1992) discussed the syntheses of planar and spherical four-bar path generators by using the pole method. The approach of spherical four-bar linkages function generation for the entire motion cycle has been discussed in Farhang and Zarqar (1999). In recent years, several authors have made significant contribution in this field and many mathematical techniques have been developed for the synthesis and analysis of spherical four-bar mechanisms (Shen et al., 2008; Brunnthaler et al., 2006; Hong and Erdman, 2005; Chablat and Angeles, 2003).

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The 4R spherical mechanism, as well as the planar four-bar linkages contain only revolute joints. The serially connected four-bar planar linkages have been well studied in mechanism design. However, until now, no study has been carried out about serially connected spherical mechanisms. The spherical mechanisms with parallel input and output axis generate same motions as planar linkages.

In this paper, we propose an comparative analysis of six-bar mechanisms formed by two serially connected spherical and planar four-bar mechanisms. The obtained results show that the spherical linkages have better kinematic and dynamic properties than planar linkages.

2. Kinematics of the examined mechanisms

Figs. 1 and 2 show six-bar mechanisms formed by serially connected spherical and planar four-bar linkages. The output links axes of both mechanisms are parallel. This condition is attained if $\alpha_4 + \alpha_8 = 180^\circ$ (Fig. 2).

Let us define the input and output links of the first linkages by the angles φ_1 and φ_3.

The position of the output link of the first planar four-bar linkage can be expressed as

$$\varphi_3(\varphi_1) = \pi - [\theta(\varphi_1) + k \cdot \beta(\varphi_1)] \tag{1}$$

where φ_1 is the rotation angle of the input link; φ_3 is the rotation angle is the output link; L is the distance between the centers of joints B_1 and D_1 (Fig. 3); L_j is the length of the jth link of the linkage ($j = 1,2,3,4$);

$$tg\theta = L_1 \sin\varphi_1/(L_4 - L_1\cos\varphi_1), \quad \cos\beta = \frac{L^2 + L_3^2 - L_2^2}{2LL_3}, \quad L^2 = L_4^2 + L_1^2 - 2L_4L_1\cos\varphi_1$$

$k = \pm 1$ is a factor that depends on the mechanism configuration.

Differentiating Eq. (1) twice with respect φ_1, we obtain

$$\varphi_3' = (d\varphi_3/dt)/(d\varphi_1/dt) = -(\theta' + k\beta')$$
$$\varphi_3'' = (d^2\varphi_3/dt^2)/(d\varphi_1/dt)^2 = -(\theta'' + k\beta'')$$

The spherical mechanism are defined by angles α_1, α_2, α_3 and α_4. Its output link position can be determined by the following expression:

$$\varphi_3(\varphi_1) = \theta(\varphi_1) + k\beta(\varphi_1) \tag{2}$$

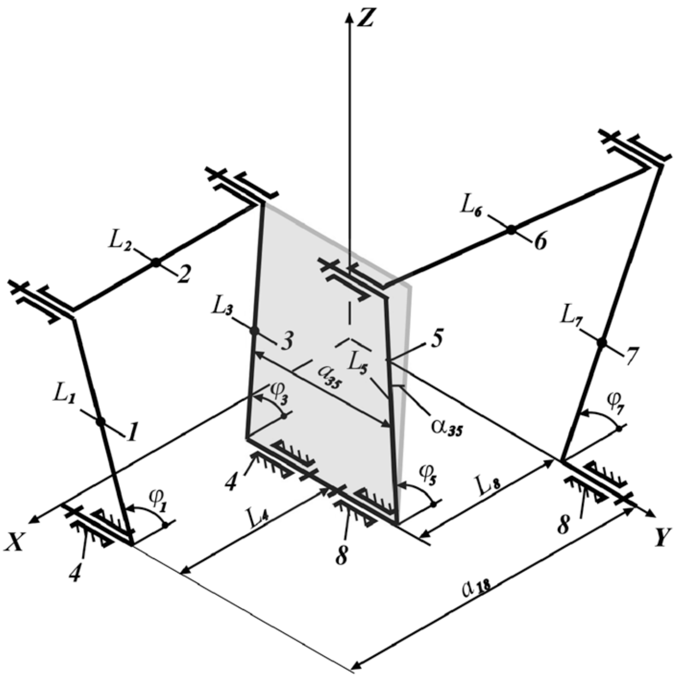

Fig. 1. Six-bar mechanisms formed by two serially connected planar four-bar linkages.

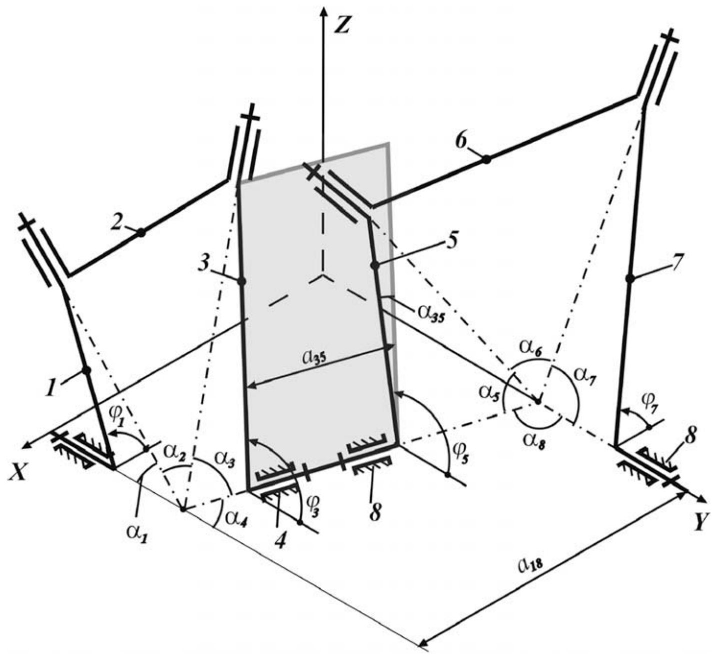

Fig. 2. Six-bar mechanisms formed by two serially connected spherical linkages.

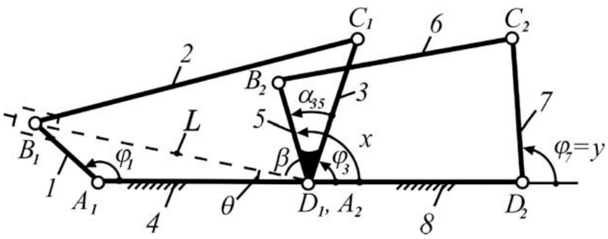

Fig. 3. 2D view of the six-bar mechanisms formed by two serially connected planar four-bar linkages.

where

$$\cos\theta = \frac{(\cos\alpha_1 \sin\alpha_4 + L_1 \cos\alpha_4 \cos\varphi_1)}{\sqrt{(\cos\alpha_1 \sin\alpha_4 + L_1 \cos\alpha_4 \cos\varphi_1)^2 + (L_1 \sin\varphi_1 - L_4)^2}},$$

$$\cos\beta = \frac{\cos\varphi_3(\cos\alpha_1 \sin\alpha_4 + L_1 \cos\alpha_4 \cos\varphi_1) + \sin\varphi_3(L_1 \sin\varphi_1 - L_4)}{\sqrt{(\cos\alpha_1 \sin\alpha_4 + L_1 \cos\alpha_4 \cos\varphi_1)^2 + (L_1 \sin\varphi_1 - L_4)^2}}$$

Differentiating Eq. (2) twice with respect φ_1, we obtain

$$\varphi_3' = \theta' + k\beta'$$
$$\varphi_3'' = \theta'' + k\beta''$$

where

$$\theta' = -\frac{1 - \lambda\cos\varphi_1}{1 + \lambda^2 - 2\lambda\cos\varphi_1}$$

$$\theta'' = -\frac{\lambda(\lambda^2 - 1)\sin\varphi_1}{(1 + \lambda^2 - 2\lambda\cos\varphi_1)^2}$$

$$\beta'' = \frac{L''(L_3\cos\beta - L) - (L')^2 - L_3\beta'(2L'\sin\beta + L\beta'\cos\beta)}{LL_3\sin\beta}$$

$$L = \sqrt{L_4^2 + L_1^2 - 2L_4L_1\cos\varphi_1}$$

$$L' = L_4L_1\sin\varphi_1/L$$

$$L'' = [L_4L_1\cos\varphi_1 - (L')^2]/L$$

where, $\lambda = L_4/L_1$.

With regard to the second linkages, they may be presented using the same expressions, taking into account the following index modifications: $\varphi_5 = x = \varphi_3 + \alpha_{35}, L_1 \to L_5, L_2 \to L_6, L_3 \to L_7, L_4 \to L_8, \alpha_1 \to \alpha_5, \alpha_2 \to \alpha_6, \alpha_3 \to \alpha_7, \alpha_4 \to \alpha_8, \varphi_7^{(n)}(\varphi_1) = y^{(n)}(\varphi_1), (n = 0, 1, 2)$.

For examined mechanisms for Figs. 1–3, we have

$$y = y(x)$$
$$\varphi_3 = \varphi_3(\varphi_1)$$
$$x = \varphi_3 + \alpha_{35}$$

relationships, which define the position function of the six-bar mechanism:

$$y = y(x, \varphi_1) \tag{3}$$

Differentiating Eq. (3), we obtain

$$\dot{y} = \dot{x}(\varphi_1) \cdot y'(x) = \mathrm{d}\varphi_3/\mathrm{d}\varphi_1 \cdot \mathrm{d}\varphi_7/\mathrm{d}\varphi_5,$$
$$\ddot{y} = \ddot{x} \cdot y' + (\dot{x})^2 \cdot y'' = \mathrm{d}^2\varphi_3/\mathrm{d}\varphi_1^2 \cdot \mathrm{d}\varphi_7/\mathrm{d}\varphi_5 + (\mathrm{d}\varphi_3/\mathrm{d}\varphi_1)^2 \cdot \mathrm{d}^2\varphi_7/\mathrm{d}\varphi_5$$
$$y' = \mathrm{d}\varphi_7/\mathrm{d}\varphi_5, \quad y'' = \mathrm{d}^2\varphi_7/\mathrm{d}\varphi_5^2$$

3. Synthesis and comparative analysis

Optimal parameter determination of the examined six-bar spherical mechanism is carried out by the following stages:

(a) For the examined six-bar mechanisms, the following α_i angles of the first spherical mechanism are selected:
Case 1:
$\alpha_1 = \alpha_3 = 90°$, $\alpha_2 = 60°$, $\alpha_4 = 45°$.
Case 2:
$\alpha_1 = 85°$, $\alpha_2 = 65°$, $\alpha_3 = 80°$, $\alpha_4 = 45°$.

(b) With regard to the second spherical mechanism, the following values of α_i angles are defined:
$15° \leqslant \alpha_5 \leqslant 25°, \quad \alpha_6 = 85°, \quad 85° \leqslant \alpha_7 \leqslant 95°, \quad \alpha_8 = 135°$

(c) For the first planar four-bar linkage, the following non-dimensional lengths of links are selected:
$L_1 = L_3 = 3, L_2 = 4$ and $L_4 = 1$

(d) With regard to the second planar four-bar linkage, the following values of link lengths are defined:
$L_5 = 1, \quad L_6 = 7, \quad 5 \leqslant L_7 \leqslant 6$

(e) The length L_8 of the second planar four-bar linkage is determined from the following expression:

$$\cos^{-1}\left[\frac{L_7^2 + L_8^2 - (L_5 + L_6)^2}{2L_7L_8}\right] - \cos^{-1}\left[\frac{L_7^2 + L_8^2 - (L_6 - L_5)^2}{2L_7L_8}\right] - \psi = 0$$

In this way we obtain the same oscillation angle $\psi = \psi_{(\mathrm{plan})} = \psi_{(\mathrm{sph})}$ for both examined six-bar mechanisms. Please note that $\psi = |\varphi_{7(\max)} - \varphi_{7(\min)}|$, i.e. it is the absolute value between the maximal and minimal values of the output angle φ_7.

Table 1
Principal characteristics of the examined six-bar spherical mechanisms

Double-crank: $\alpha_4 = 45°$, $\alpha_1 = \alpha_3 = 90°$, $\alpha_2 = 60°$								
α_5	$\max\varphi_7'$	$\min\varphi_7'$	$\max\varphi_7''$	$\min\varphi_7''$	$\max K_\omega$	$\max K_\varepsilon$	$\max K_p$	Ψ
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 85°$								
15°	0.43	−0.33	0.64	−1.03	1.59	10.47	9.39	0.75
20°	0.63	−0.43	0.75	−1.41	1.63	9.57	8.72	1.01
25°	0.92	−0.531	0.8	−1.82	1.75	8.46	8.3	1.28
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 90°$								
15°	0.48	−0.36	0.62	−1.06	1.74	10.52	11.02	0.75
20°	0.7	−0.47	0.71	−1.49	1.78	9.67	10.28	1.01
25°	1.01	−0.57	0.81	−1.96	1.87	8.58	9.64	1.29
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 95°$								
15°	0.54	−0.39	0.59	−1.11	1.89	10.42	12.43	0.77
20°	0.8	−0.51	0.66	−1.59	1.917	9.53	11.51	1.04
25°	1.17	−0.61	1.17	−2.17	1.98	8.27	10.46	1.33

Table 2
Principal characteristics of the examined six-bar spherical mechanisms

Double-crank: $\alpha_4 = 45°$, $\alpha_1 = 85°$, $\alpha_2 = 65°$, $\alpha_3 = 80°$								
α_5	$\max \varphi'_7$	$\min \varphi'_7$	$\max \varphi''_7$	$\min \varphi''_7$	$\max K_\omega$	$\max K_\varepsilon$	$\max K_p$	Ψ
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 85°$								
15°	0.45	−0.43	0.98	−0.84	1.52	8.23	8.51	0.75
20°	0.58	−0.67	1.22	−1.30	1.58	6.29	5.96	1.01
25°	0.71	−1.03	1.49	−2.01	1.76	5.08	5.89	1.28
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 90°$								
15°	0.49	−0.45	0.95	−0.87	1.64	7.8	8.88	0.75
20°	0.63	−0.71	1.19	−1.37	1.63	5.8	5.96	1.01
25°	0.76	−1.07	1.45	−2.13	1.81	5.18	6.17	1.2
$\alpha_8 = 135°$, $\alpha_6 = 85°$, $\alpha_7 = 95°$								
15°	0.53	−0.49	0.93	−0.91	1.72	7.29	8.92	0.77
20°	0.67	−0.76	1.16	−1.48	1.72	5.79	6.58	1.04
25°	0.81	−1.18	1.58	−2.36	2.03	6.05	8.54	1.33

The comparison of these mechanisms is established by the following generalized non-dimensional indexes, which are defined for the mechanical systems with oscillating links as follows (Levitsky, 1979; Levitsky, 1964):

(i) velocity index: $(K_\omega)_7 = K_\omega = \varphi'_5 \cdot \varphi_\psi/\psi$,
(ii) acceleration index: $(K_\varepsilon)_7 = K_\varepsilon = \varphi''_5 \cdot \varphi^2_\psi/\psi$,
(iii) dynamic power index: $(K_p)_7 = K_p = K_\omega \cdot K_\varepsilon$,

where φ_ψ is the angle of rotation of the input link corresponding to the oscillation angle ψ.

The obtained results for six-bar spherical mechanisms corresponding to cases 1 and 2 are given in Tables 1 and 2. The results of similar simulations carried out for planar six-bar mechanisms are shown in Table 3. On the basis of the obtained numerical simulations, the correlation expressions are deducted, which allow us to carry out the synthesis of the examined class of mechanisms. The convergence of correlation is characterized by coefficients r_{ab} and r_{cd} given in Table 4.

Table 3
Principal characteristics of the examined six-bar planar mechanisms

Double-crank: $L_4 = 1$, $L_1 = L_3 = 3$, $L_2 = 4$								
L_8	$\max \varphi'_7$	$\min \varphi'_7$	$\max \varphi''_7$	$\min \varphi''_7$	$\max K_\omega$	$\max K_\varepsilon$	$\max K_p$	Ψ
$L_5 = 1$, $L_6 = 7$, $L_7 = 5$								
3.953	0.257	−0.746	1.216	−1.011	1.848	5.591	6.86	0.75
3.309	0.36	−1.146	2.58	−1.53	1.915	7.283	8.934	1.01
3.052	0.494	−1.654	6.933	−2.102	1.983	12.778	14.375	1.282
$L_5 = 1$, $L_6 = 7$, $L_7 = 5.5$								
3.534	0.243	−0.764	1.222	−1.076	1.846	5.369	6.708	0.753
2.877	0.336	−1.195	2.585	−1.693	1.923	6.792	8.667	1.015
2.586	0.456	−1.778	6.371	−2.509	2.012	10.53	13.004	1.29
$L_5 = 1$, $L_6 = 7$, $L_7 = 6$								
3.13	0.231	−0.799	1.254	−1.174	1.854	5.194	6.662	0.77
2.465	0.315	−1.281	2.678	−1.955	1.945	6.425	8.602	1.042
2.137	0.422	−1.992	6.426	−3.227	2.064	9.187	12.514	1.332

Table 4
Principal characteristics of the examined six-bar spherical crank-and-rocker mechanisms

Correlation expressions	a	b	r_{ab}	c	d	r_{cd}
$\alpha_8 = 135°$, $\alpha_5 = 15° \cdots 25°$, $\alpha_6 = 85°$, $\alpha_7 = 85° \cdots 95°$						
$\max K_\omega = (a\alpha_7 + b)\alpha_5^3 + (c\alpha_7 + d)$	−5.37	10.41	0.99	1.81	−1.13	1
$\max K_\varepsilon = (a\alpha_7^4 + b)\alpha_5^2 + (c\alpha_7 + d)$ (4)	−0.82	−11.29	0.96	0.3	11.08	0.99
$\max K_p = (a\alpha_7^4 + b)\alpha_5 + (c\alpha_7^1 + d)$ (5)	−1.87	3.053	0.99	25.3	−26.7	1
Planar crank-and-rocker mechanisms						
$L_5 = 1$, $L_6 = 7$, $L_7 = 5 \cdots 6$						
$\max K_\omega = (aL_7 + b)L_8^{-3} + (cL_7 + d)$	−4.002	26.91	0.99	0.02	1.62	0.94
$\max K_\varepsilon = (aL_7^{-7} + b)L_8^{-7} + (cL_7^{-1} + d)$ (6)	$2.28\overline{E}10^9$	$-8.6\overline{E}10^3$	0.98	−39.6	11.5	0.99
$\max K_p = (aL_7^{-2} + b)L_8^{-3} + (cL_7 + d)$ (7)	$2.43\overline{E}10^4$	−606.3	0.99	3.52	−17.4	0.99

The comparative analysis is shown in Fig. 4. As can be seen in this figure, obviously the best results are obtained for spherical mechanisms. It should be noted that the obtained correlation expressions allow us to carry out the synthesis of these mechanisms. Let us consider two examples.

Example 1. For given initial parameters $\alpha_8 = 135°$, $\alpha_6 = 85°$, $\max K_\varepsilon = 10$, $\max K_p = 9$, one must carry out the synthesis of spherical six-bar mechanism.

Thus, by means of the expressions (4) and (5) (see Table 4), we determine $\alpha_5 = 18.503°$, $\alpha_7 = 84.976°$. It should be noted that the analysis of the spherical six-bar mechanism by obtained angles α_5 and α_7 provides maximum values of generalized indexes: $\max K_\varepsilon = 10$ and $\max K_p = 8.998$. We can see that the errors do not exceed 0.022%.

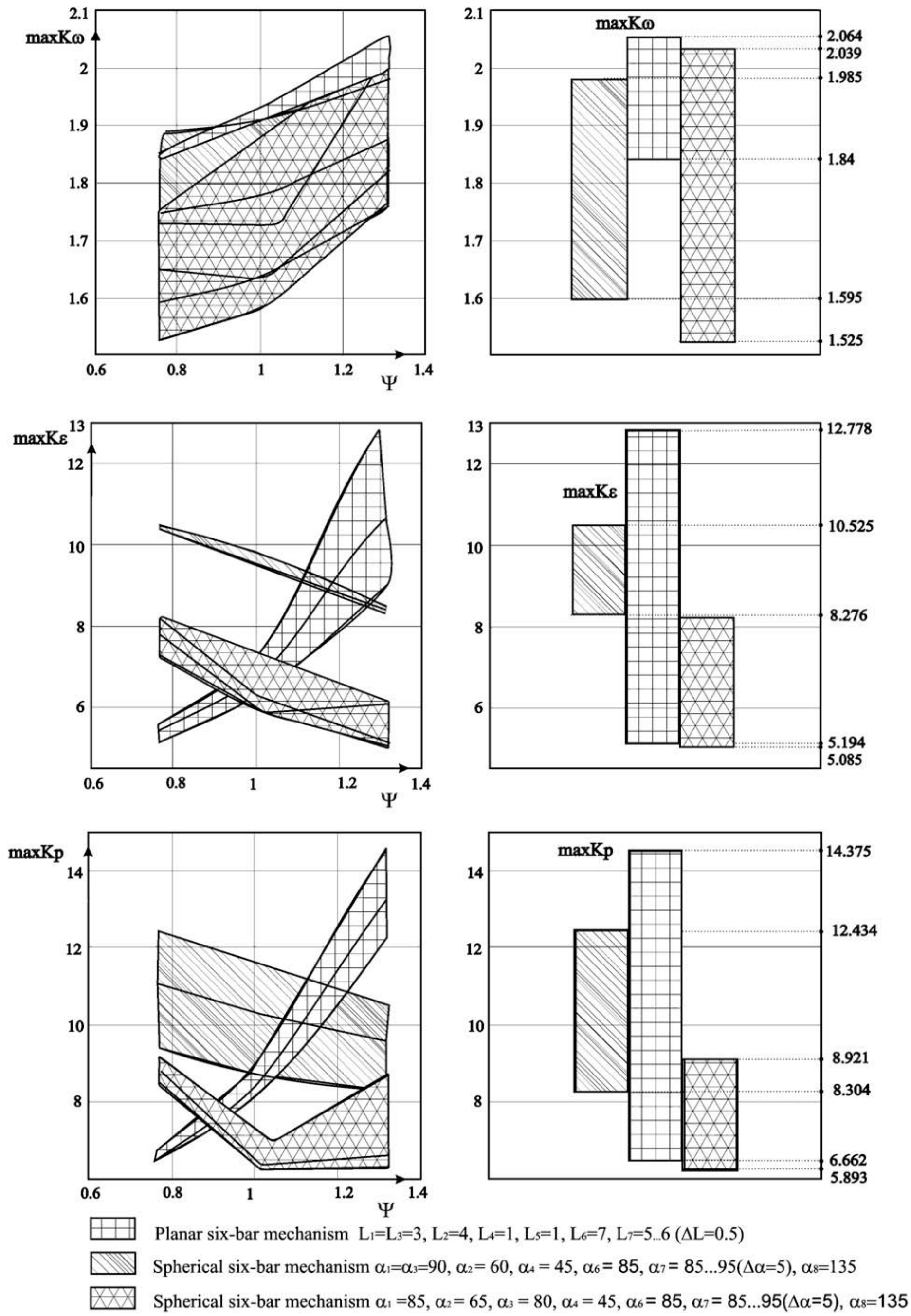

Fig. 4. Variations of the generalized non-dimensional indexes.

Example 2. For given initial parameters $L_7 = 5.5$, $\max K_\varepsilon = 7$, max it $K_p = 10$, one must carry out the synthesis of planar six-bar mechanism.

In this case, by means of the expressions (6) and (7) (see Table 4), we determine $L_7 = 5.817$, $L_8 = 2.548$. With the obtained parameters, the synthesised mechanism assumes maximum values of generalized indexes: $\max K_\varepsilon = 6.994$ and $\max K_p = 9.994$. The maximal errors for the present example do not exceed 0.086%.

4. Conclusion

The 4R spherical mechanism, as well as the planar four-bar linkages contain only revolute joints. The serially connected four-bar planar linkages have been well studied in mechanism design. However, until now, no study has been carried out in the field of serially connected spherical mechanisms. The comparative analysis and synthesis of six-bar mechanisms formed by two spherical and planar four-bar linkages have been discussed. By placing input and output links of the spatial mechanism parallel and by assuming the same angle of oscillation of the output links of both mechanical systems, the similar output conditions has been chosen for comparative analysis. The obtained results was analyzed using three generalized non-dimensional indexes: velocity, acceleration and dynamic power. It was shown that obviously the best results are obtained for the spherical mechanisms. Based on the obtained numerical results, the correlation expressions have been deduced, which allow the synthesis of such mechanisms. The suggested synthesis has been illustrated using two examples.

Finally, it should be noted that for examined class of mechanisms, the obtained general analytical expressions for synthesis of six-bar linkages provide a good convergence of correlation.

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