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Partial shaking moment balancing of fully force balanced linkages

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Abstract

This paper deals with a solution of the shaking force and shaking moment balancing of planar and spatial linkages. The conditions for balancing are formulated by the minimization of the root-mean-square value of the shaking moment. There are two cases considered: mechanism with the input link by constant angular velocity and mechanism with the input link by variable angular velocity. The method is realized by displacement of the axis of rotation of the input link connected with the counterweight. The efficiency of the suggested method is illustrated by two numerical examples: planar four-bar linkage and RSS'R spatial linkage. © 2001 Published by Elsevier Science Ltd.

Résumé

Dans cet article est développée une méthode d'équilibrage dynamique des mécanismes plans et spatiaux fondée sur la minimisation de la valeur moyenne quadratique du couple résultant des forces d'inertie. La méthode est réalisée par le déplacement de la position de l'axe de rotation du contrepoids. Deux cas sont étudiés: un mécanisme avec un élément d'entrée à vitesse angulaire constante et un mécanisme avec un élément d'entrée à vitesse angulaire variable. L'efficacité de la solution présentée est illustrée par deux exemples numériques: un mécanisme articulé à quatre barres et un mécanisme spatial du type *RSS'R*. © 2001 Published by Elsevier Science Ltd.

1. Introduction

The vibrations in the machines can, in certain cases, be useful if the operation of the machine is based on the effect of oscillations (vibropress, oscillating conveyers, vibrohammers, etc.).

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However, these are regarded mainly as undesirable due to their influence on the reliability of the machines, of the noises which they cause and of the negative action on human health. In these cases, it is necessary to ensure the complementary means of protection against the vibrations.

One of the most effective means used for the reduction of the vibratory activity of the high speed machines is the balancing of shaking force and shaking moment of linkages, full or partial, by internal mass redistribution or counterweight addition [1].

Balancing of planar linkages. The first study of complete shaking force and shaking moment balancing of planar crank-rocker mechanism is the method of ''Mass redistribution'' proposed by Berkof [2]. In this work the total elimination of the shaking moment based on the use of such a full force balanced mechanism is achieved by the introduction of counter-rotating disks and by giving the floating links the shape of a physical pendulum. This method has been developed in [3] by a new application. The other solution for complete dynamic balancing of mechanisms is presented in [4,5]. This is realized by connecting to the mechanism to be balanced by a two-link group forming a pantograph. In his studies Kochev treats the balancing of mechanical systems with double crank linkages as symmetrically opposed [6] and the method of active balancing of shaking moment in planar linkages [7].

However, the complete shaking force and shaking moment balancing problem is very complicated. Often in practice, the problem of mass balancing is limited by full force balancing and partial moment balancing [8–13]. The methods of optimization based on the non-linear programming are treated in the studies of Sadler [8] and Smith [9]. Based on the studies [10,11], Wienderich and Roth [12] considered the shaking moment reduction in a fully force balanced four-bar linkage by way of decreasing the angular momentum fluctuations. Carson and Stephens [13] extended the theory given in [10,11] by showing that full force balance and root-mean-square (RMS) moment balance could be maintained for certain ranges of coupler and output link mass distribution, even if the prescribed link length ratios were held.

Balancing of spatial linkages. Balancing the shaking force and shaking moment of spatial linkages are developed in [14–26].

One of the first methods for balancing the spatial linkages is proposed by Kaufman and Sandor [14]. In this work a complete force balancing of the RSSR and RSSP spatial mechanisms is obtained. The balancing of Bennett mechanism is treated in the studies of Chen and Zhang [15]. A method of partial force balancing of the RCCC linkage is developed in the study of Chen [16]. Yu [17–19] proposes to balance the shaking force and shaking moment using additional groups connected with the counterweights. For example, to the balancing of the RSS'R mechanism, the author proposes to add an SSR complementary group.

The studies of Wawrzecki [20–22] are devoted to the study of spatial mechanisms of sowing machines. In the works [3,23] are developed the principles for the construction of self-balanced systems. The objective of these studies is to create the mechanical systems at double-modules which realize identical but opposite movements.

The balancing of the shaking force by the method of ''substitution of the masses'' [24] is proposed in the work [25]. Chiou and Tsai [26] examined the balancing of the spatial mechanisms by three rotary counterweights arranged on the perpendicular planes.

However, the majority of spatial mechanism balancing works have been concentrated on shaking force balancing. Research on shaking moment balancing has been less productive. Often the balancing of shaking moment is carried out by a substantial complication of the initial mechanism. One such approach is not always reasonable and applicable.

In this work we propose a method for complete shaking force and partial shaking moment balancing that is applicable to any planar or spatial linkage with an input rotating link. This method is based, on the one hand, on the known principle of the independence of the properties of static balancing of the linkage from the position of the axis of rotation of the counterweight [27,28] and, on the other hand, on the methods of the RMS balancing [29]. In the study [27] the first harmonic of the shaking moment is eliminated by attaching the required input link counterweight, not to the input shaft itself, but to a suitably offset one which rotates with the same angular velocity. In the present study the conditions for such optimum balancing are formulated by the minimization of the RMS value of the shaking moment [29].

The object of the work presented here is to develop the principle of the independence of the properties of static balancing of the linkage from the position of the axis of rotation of the counterweight and to provide the optimum conditions of the moment balancing of planar and spatial linkages.

2. Complete shaking force and partial shaking moment balancing of planar linkages

2.1. Balancing of linkage with constant input angular velocity

Let us consider an arbitrary n-bar planar linkage with constant input angular velocity $\dot{\varphi}$ (Fig. 1).

After the balancing of shaking force of this linkage, the shaking moment relative to the center of rotation of the input link can be expressed as [27]

$$M_O^{\text{int}} = \sum_{i=1}^{n-1} M_O(F_k^{\text{int}}) - \sum_{i=1}^{n-1} I_{S_k} \ddot{\varphi}_k, \tag{1}$$

where for the k^{th} link: F^{int} is the shaking force, I_S is the moment of inertia relative to the center of mass S of the link, and $\ddot{\varphi}$ is the angular acceleration.

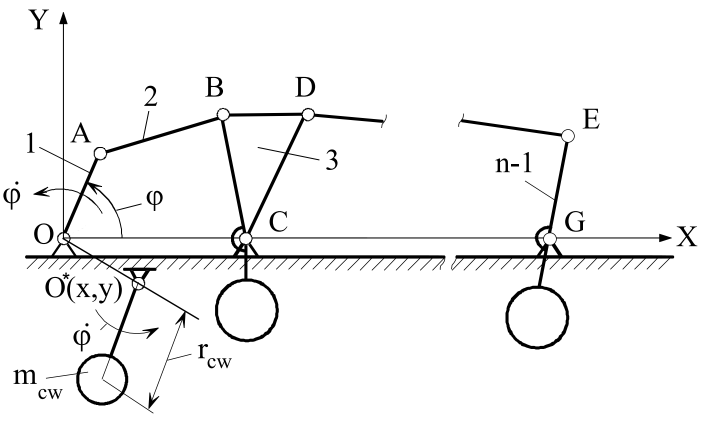

Fig. 1. Arbitrary n-bar planar linkage.

In the case of parallel displacement of the axis of rotation of the counterweight (Fig. 1) from center O to the center $O^*(x, y)$, the balancing of shaking force of the mechanism will occur, but a supplementary moment is therewith added [27]

$$M^* = S(\dot{\varphi})^2(x \sin\varphi - y\cos\varphi), \tag{2}$$

where φ and $\dot{\varphi}$ are the angle of rotation and the angular velocity of the input link, and $S = m_{cw}r_{cw}$ is the static moment of the counterweight relative to the axis of rotation.

In the study [27], the shaking moment $M_O^{int}(\varphi)$ is approximated by the trigonometric series and the first harmonic of this series is balanced by the supplement moment M^*. The efficiency of such balancing will be improved by using the approximation method [29] based on the minimization of the RMS value:

$$\mathrm{RMS} = \sqrt{\sum_{i=1}^{N}(M_i^{int} + M_i^*)^2/N}, \tag{3}$$

where N is the number of calculated positions of the mechanism.

For the minimization of the RMS, it is necessary to minimize the sum:

$$\Delta = \sum_{i=1}^{N}(M_i^{int} + M_i^*)^2 \rightarrow \min_{x,y}. \tag{4}$$

For this purpose, we shall achieve the conditions:

$$\partial\Delta/\partial x = 0 \quad \text{and} \quad \partial\Delta/\partial y = 0, \tag{5}$$

from where

$$S(\dot{\varphi})^2\left(y\sum_{i=1}^{N}\cos^2\varphi_i - x\sum_{i=1}^{N}\sin\varphi_i\cos\varphi_i\right) = -\sum_{i=1}^{N}M_i^{int}\cos\varphi_i, \tag{6}$$

$$S(\dot{\varphi})^2\left(x\sum_{i=1}^{N}\sin^2\varphi_i - y\sum_{i=1}^{N}\sin\varphi_i\cos\varphi_i\right) = -\sum_{i=1}^{N}M_i^{int}\sin\varphi_i. \tag{7}$$

From these expressions and taking into account the condition $\sum_{i=1}^{N}\sin\varphi_i\cos\varphi_i = 0$ for $\varphi \in [0; 2\pi]$, we determine the unknowns:

$$x = \sum_{i=1}^{N}M_i^{int}\sin\varphi_i \bigg/ S(\dot{\varphi})^2\sum_{i=1}^{N}\sin^2\varphi_i, \tag{8}$$

$$y = -\sum_{i=1}^{N}M_i^{int}\cos\varphi_i \bigg/ S(\dot{\varphi})^2\sum_{i=1}^{N}\cos^2\varphi_i. \tag{9}$$

2.2. Balancing of linkage with variable input angular velocity

With the variable angular velocity $\ddot{\varphi}$ of the input link taken into consideration, the supplementary moment (2) that balances a shaking moment will be equal to:

$$M^* = S((\dot{\varphi})^2(x\sin\varphi - y\cos\varphi) - \ddot{\varphi}(r - x\cos\varphi - y\sin\varphi)). \tag{10}$$

In this case, from the system of linear equations:

$$\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \tag{11}$$

where

$$A_1 = \sum_{i=1}^{N}((\dot{\varphi}_i)^2\sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i)^2, \tag{12}$$

$$A_2 = \sum_{i=1}^{N}((\dot{\varphi}_i)^2\sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i)((\dot{\varphi}_i)^2\cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i), \tag{13}$$

$$B_1 = -\sum_{i=1}^{N}((\dot{\varphi}_i)^2\cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i)((\dot{\varphi}_i)^2\sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i), \tag{14}$$

$$B_2 = -\sum_{i=1}^{N}((\dot{\varphi}_i)^2\cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i)^2, \tag{15}$$

$$C_1 = \sum_{i=1}^{N}\ddot{\varphi}_i r((\dot{\varphi}_i)^2\sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i) - \sum_{i=1}^{N}(M_i^{\mathrm{int}}/S)((\dot{\varphi}_i)^2\sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i), \tag{16}$$

$$C_2 = \sum_{i=1}^{N}\ddot{\varphi}_i r((\dot{\varphi}_i)^2\cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i) - \sum_{i=1}^{N}(M_i^{\mathrm{int}}/S)((\dot{\varphi}_i)^2\cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i), \tag{17}$$

we determinate the coordinates of the axis of rotation of the counterweight:

$$x = D_x/D \quad \text{and} \quad y = D_y/D, \tag{18}$$

where D_x, D_y and D are the determinants obtained from system (11).

3. Numerical example and comparative analysis

For the four-bar linkage (Fig. 2) with parameters: $\ell_{OA} = 0.1$ m, $\ell_{AB} = 0.3$ m, $\ell_{BC} = 0.2$ m, $\ell_{OC} = 0.3$ m, $\ell_{OS_1} = 0.05$ m, $\ell_{AS_2} = 0.15$ m, $\ell_{CS_3} = 0.1$ m, $m_1 = 1$ kg, $m_2 = 3$ kg, $m_3 = 2$ kg, $I_{S_2} = 0.0225$ kg m^2, $I_{S_3} = 0.01$ kg m^2, $\dot{\varphi} = 10$ s^{-1}, taken into account: $r_{\mathrm{cw}_1} = 0.05$ m and $r_{\mathrm{cw}_3} = 0.1$ m, we realize the complete shaking force balancing of the mechanism [27] and by means of the formulae (8) and (9) we determine the values $x = 0.189$ m and $y = -0.071$ m.

In Fig. 3 are presented the variations of the shaking moments $M_j(\varphi)$ for a four-bar linkage:
1. non-balanced ($j = 1$);
2. only shaking force balanced ($j = 2$);
3. complete shaking force and partial shaking moment balanced by the known method [27] ($j = 3$);
4. complete shaking force and partial shaking moment balanced by the proposed method ($j = 4$).

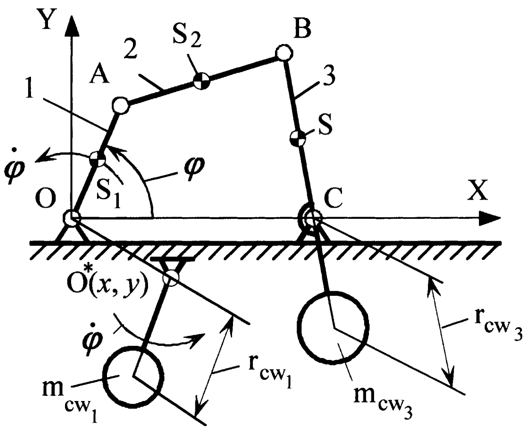

Fig. 2. Shaking moment balancing of a four-bar linkage.

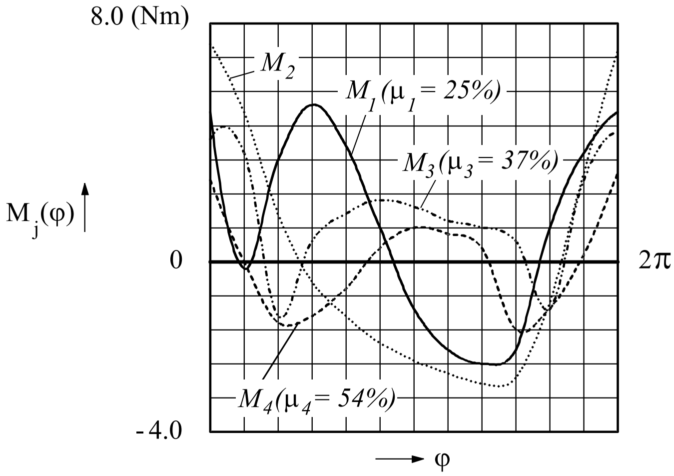

Fig. 3. Variations of the shaking moments for different balancing methods.

In Fig. 3, with the diagrams of shaking moment are presented the coefficients μ_j which characterize the efficiency of balancing methods. These coefficients are calculated by the following expression:

$$\mu_j = \frac{\max|M_m(\varphi)| - \max|M_j(\varphi)|}{\max|M_m(\varphi)|} 100\% \quad (j = 1, \ldots, 4;\ j \neq m), \tag{19}$$

where

$$\max|M_m(\varphi)| = \max\{|M_1(\varphi)|, |M_2(\varphi)|, |M_3(\varphi)|, |M_4(\varphi)|\}. \tag{20}$$

It should be noted that in most cases: $m = 2$, i.e., the high value of the shaking moment is obtained for the mechanism which is balanced statically.

4. Complete shaking force and partial shaking moment balancing of spatial linkages

4.1. Balancing of linkage with constant input angular velocity

In Fig. 4 is presented a spatial *n*-bar linkage with constant input angular velocity $\dot{\varphi}$.

It is considered that this linkage is balanced statically (the shaking force is cancelled). The shaking moment of the linkage can be represented by the expression:

$$\overline{M}^{\text{int}} = \overline{M}_X^{\text{int}} + \overline{M}_Y^{\text{int}} + \overline{M}_Z^{\text{int}} = \sum_{i=1}^{n-1} \overline{M}_{X_K}^{\text{int}} + \sum_{i=1}^{n-1} \overline{M}_{Y_K}^{\text{int}} + \sum_{i=1}^{n-1} \overline{M}_{Z_K}^{\text{int}}, \tag{21}$$

where $\overline{M}_X^{\text{int}}$, $\overline{M}_Y^{\text{int}}$, $\overline{M}_Z^{\text{int}}$ are the components of the shaking moment of the linkage; $\overline{M}_{X_K}^{\text{int}}$, $\overline{M}_{Y_K}^{\text{int}}$, $\overline{M}_{Z_K}^{\text{int}}$ are the components of the shaking moment of the k^{th} link of the linkage; $n-1$ is the number of moving links.

In the case of parallel displacement of the axis of rotation of the counterweight (Fig. 4) from center O to the center $O^*(x, y, z)$, the balancing of shaking force of the linkage will occur, but a supplementary moment is therewith added.

The values of the components of this additional moment are the following:

$$M_X^* = S(\dot{\varphi})^2 z \cos\varphi, \tag{22}$$

$$M_Y^* = -S(\dot{\varphi})^2 z \sin\varphi, \tag{23}$$

$$M_Z^* = S(\dot{\varphi})^2 (x \sin\varphi - y \cos\varphi), \tag{24}$$

where φ and $\dot{\varphi}$ are, respectively, the angle of rotation and the angular velocity of the input link 1 (Fig. 4), and $S = m_{\text{cw}} r_{\text{cw}}$ is the static moment of the counterweight relative to the axis of rotation.

The problem is the following: to find such coordinates x, y, z of the counterweight displacement which will allow to minimize the shaking moment of the linkage. For it is necessary to minimize the RMS value of the shaking moment:

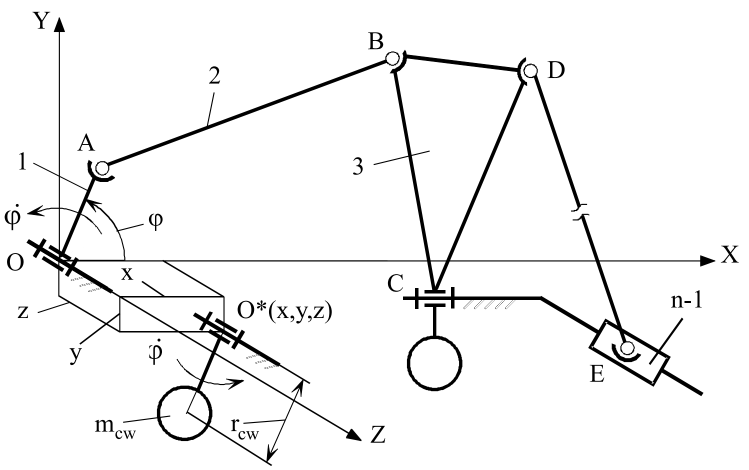

Fig. 4. Arbitrary *n*-bar spatial linkage.

$$\text{RMS} = \sqrt{\sum_{i=1}^{N} \left|\overline{M}_i^{\text{int}} + \overline{M}_i^*\right|^2 \Big/ N}, \tag{25}$$

where i and N are, respectively, the index and the number of calculated positions of the linkage; $\overline{M}^* = \overline{M}_X^* + \overline{M}_Y^* + \overline{M}_Z^*$ is the additional moment.

To obtain a minimum of the RMS it is necessary to minimize the sum:

$$\Delta = \sum_{i=1}^{N} |\overline{M}_i^{\text{int}} + \overline{M}_i^*|^2 \rightarrow \min_{x,y,z}, \tag{26}$$

or

$$\Delta = \sum_{i=1}^{N} ((M_{X_i}^{\text{int}} + M_{X_i}^*)^2 + (M_{Y_i}^{\text{int}} + M_{Y_i}^*)^2 + (M_{Z_i}^{\text{int}} + M_{Z_i}^*)^2) \rightarrow \min_{x,y,z}, \tag{27}$$

with this intention it is necessary to ensure the conditions:

$$\partial\Delta/\partial x = 0, \quad \partial\Delta/\partial y = 0 \quad \text{and} \quad \partial\Delta/\partial z = 0, \tag{28}$$

from where

$$S(\dot{\varphi})^2 \left(y \sum_{i=1}^{N} \cos^2\varphi_i - x \sum_{i=1}^{N} \sin\varphi_i \cos\varphi_i \right) = -\sum_{i=1}^{N} M_{Z_i}^{\text{int}} \cos\varphi_i, \tag{29}$$

$$S(\dot{\varphi})^2 \left(x \sum_{i=1}^{N} \sin^2\varphi_i - y \sum_{i=1}^{N} \sin\varphi_i \cos\varphi_i \right) = -\sum_{i=1}^{N} M_{Z_i}^{\text{int}} \sin\varphi_i, \tag{30}$$

$$S(\dot{\varphi})^2 z \left(\sum_{i=1}^{N} \cos^2\varphi_i + \sum_{i=1}^{N} \sin^2\varphi_i \right) = \sum_{i=1}^{N} M_{Y_i}^{\text{int}} \cos\varphi_i - \sum_{i=1}^{N} M_{X_i}^{\text{int}} \sin\varphi_i. \tag{31}$$

From these expressions and taking into account the condition $\sum_{i=1}^{N} \sin\varphi_i \cos\varphi_i = 0$ for $\varphi \in [0; 2\pi]$, we determine the unknowns:

$$x = \sum_{i=1}^{N} M_{Z_i}^{\text{int}} \sin\varphi_i \Big/ S(\dot{\varphi})^2 \sum_{i=1}^{N} \sin^2\varphi_i, \tag{32}$$

$$y = -\sum_{i=1}^{N} M_{Z_i}^{\text{int}} \cos\varphi_i \Big/ S(\dot{\varphi})^2 \sum_{i=1}^{N} \cos^2\varphi_i, \tag{33}$$

$$z = \frac{\sum_{i=1}^{N} M_{Y_i}^{\text{int}} \cos\varphi - \sum_{i=1}^{N} M_{X_i}^{\text{int}} \sin\varphi}{S(\dot{\varphi})^2 \left(\sum_{i=1}^{N} \cos^2\varphi_i + \sum_{i=1}^{N} \sin^2\varphi_i \right)}. \tag{34}$$

4.2. Balancing of linkage with variable input angular velocity

If for the studied mechanism the angular velocity of the input crank 1 (Fig. 4) is regarded as variable: $\dot{\varphi} = \dot{\varphi}(t)$, the additional moment which balances the shaking moment can be represented by the expressions:

$$M_X^* = Sz((\dot{\varphi})^2 \sin\varphi - \ddot{\varphi}\cos\varphi), \tag{35}$$

$$M_X^* = -Sz((\dot{\varphi})^2 \cos\varphi + \ddot{\varphi}\sin\varphi), \tag{36}$$

$$M_Z^* = S((\dot{\varphi})^2 (x\sin\varphi - y\cos\varphi) - \ddot{\varphi}(r - x\cos\varphi - y\sin\varphi)). \tag{37}$$

In this case, from the system of linear equations:

$$\begin{bmatrix} A_1 & B_1 & 0 \\ A_2 & B_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}, \tag{38}$$

where

$$A_1 = \sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i)^2, \tag{39}$$

$$A_2 = \sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i)((\dot{\varphi}_i)^2 \cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i), \tag{40}$$

$$B_1 = -\sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i)((\dot{\varphi}_i)^2 \sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i), \tag{41}$$

$$B_2 = -\sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i)^2, \tag{42}$$

$$C_1 = \sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \sin\varphi_i - (\ddot{\varphi}_i)\cos\varphi_i)^2 + \sum_{i=1}^{N} ((\dot{\varphi}_i)^2 \cos\varphi_i + (\ddot{\varphi}_i)\sin\varphi_i)^2, \tag{43}$$

$$E_1 = \sum_{i=1}^{N} \ddot{\varphi}_i r((\dot{\varphi}_i)^2 \sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i) - \sum_{i=1}^{N} (M_{Z_i}^{\text{int}}/S)((\dot{\varphi}_i)^2 \sin\varphi_i + (\ddot{\varphi}_i)\cos\varphi_i), \tag{44}$$

$$E_2 = \sum_{i=1}^{N} \ddot{\varphi}_i r((\dot{\varphi}_i)^2 \cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i) - \sum_{i=1}^{N} (M_{Z_i}^{\text{int}}/S)((\dot{\varphi}_i)^2 \cos\varphi_i - (\ddot{\varphi}_i)\sin\varphi_i), \tag{45}$$

$$E_3 = \sum_{i=1}^{N} (M_{Y_i}^{\text{int}}/S)((\dot{\varphi}_i)^2 \cos\varphi_i + (\ddot{\varphi}_i)\sin\varphi_i) - \sum_{i=1}^{N} (M_{X_i}^{\text{int}}/S)((\dot{\varphi}_i)^2 \sin\varphi_i - (\ddot{\varphi}_i)\cos\varphi_i), \tag{46}$$

we determinate the coordinates of the center of rotation of the counterweight:

$$x = D_x/D, \quad y = D_y/D \quad \text{and} \quad z = D_z/D, \tag{47}$$

where D_x, D_y, D_y and D are the determinants obtained from system (38).

5. Numerical example

Let us examine the shaking moment balancing of the RSS'R spatial linkage (Fig. 5) with the following parameters: $OA = 0.05$ mm; $AB = 0.2$ mm; $BC = 0.6$ mm; $CD = 0.4$ mm;

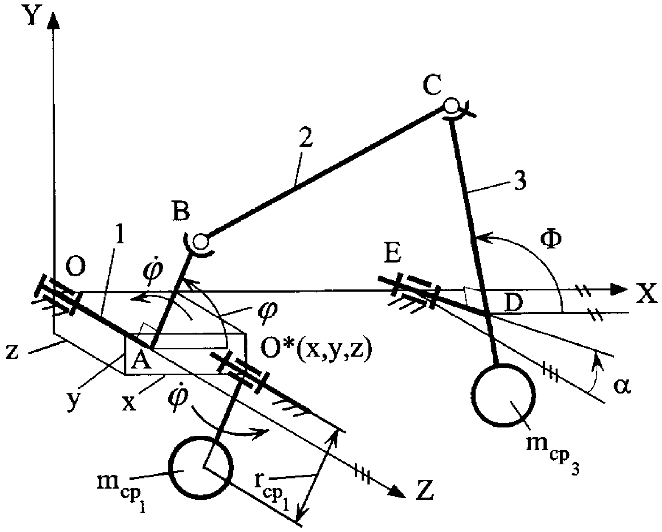

Fig. 5. Complete shaking force and partial shaking moment balancing of the RSS'R spatial linkage.

$ED = 0.1$ mm; $OE = 0.5$ mm; $\alpha = 15°$; $m_1 = 3$ kg; $m_2 = 1.8$ kg; $m_3 = 2$ kg; $I_{S_{ED}} = 0.025$ kg m^2; $\dot{\varphi} = 10$ s^{-1}; $OA \perp AB$; $ED \perp CD$. The parameters of the mass centers of links 1–3 are the following: $r_1 = AB/4$, $r_2 = BC/2$, $r_3 = DC/2.5$. The rotation of link 2 about the axis BC is cancelled by a pin and it has a symmetry relative to the center of the masses S_2: $I_{S_2} = 0.0135$ kg m^2 – moment of inertia of the link 2 relative to the plan which is perpendicular to the axis BC.

For simplicity we establish the fixed coordinate system with X-axis along E, Z-axis along A and Y-axis determined by the right-hand rule. We consider that this linkage is balanced statically and that the static moments of the counterweights are the following: $S_1 = m_{cw_1} r_{cw_1} = 0.33$ kg m and $S_1 = m_{cw_3} r_{cw_3} = 0.5$ kg m.

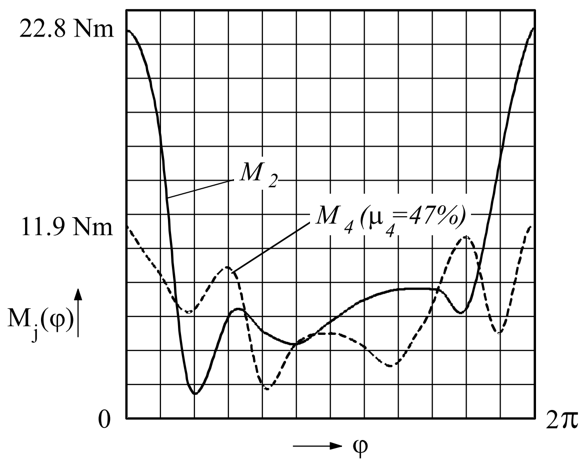

Fig. 6. Variations of the shaking moments for statically balanced linkage ($j = 2$) and for the linkage balanced by the suggested method ($j = 4$).

The parameters of the displacement of the counterweight, calculated by the expressions (32)–(34), are the following: $x = 0.056$ m, $y = -0.350$ m and $z = 0.047$ m.

In Fig. 6 are presented the variations of the shaking moments $M_j(\varphi)$ for an RSS'R four-bar spatial linkage:

1. only shaking force balanced ($j = 2$);
2. complete shaking force and partial shaking moment balanced by the proposed method ($j = 4$).

In this case, the coefficients μ_j (see Eq. (19)), which characterize the efficiency of suggested balancing method, are equal to 47%.

6. Conclusions

This paper deals with a method for shaking force and shaking moment balancing of linkages. It is universal; i.e., applicable to any planar or spatial linkage with input link rotating at constant or variable angular velocity. The conditions for balancing are formulated by the minimization of the RMS value of the shaking moment.

The suggested method has two principal advantages: a simple realization, without essential change in the construction of the initial mechanism (i.e., for a shaking moment balancing of the mechanism it is enough to transfer the axis of rotation of the input link counterweight) and a minimization of the shaking moment on the frame of the linkage without an increase in the total mass of the counterweights (i.e., only by the use of the counterweight masses designed for the static balancing of the linkage). Mathematical means for the realization of such a balancing is compact and comprehensible, which allows engineers and technicians to apply the method in short time and without much difficulty.

The efficiency of the suggested method is illustrated by two numerical examples: for the four-bar planar linkage the reduction of the shaking moment is 54% and for the RSS'R spatial linkage the reduction of the shaking moment is 47%.

References

[1] V. Arakelian, M. Dahan, M. Smith, in: Proceedings of the International Symposium on History of Machines and Mechanisms – HMM2000, Dordrecht/Boston/London, 2000, p. 291.
[2] R.S. Berkof, Mech. and Mach. Theory 8 (1973) 397.
[3] I. Esat, H. Bahai, Mech. and Mach. Theory 34 (6) (1999) 903.
[4] V. Arakelian, Mech. and Mach. Theory 33 (4) (1998) 425.
[5] V. Arakelian, M.R. Smith, Mech. and Mach. Theory 34 (9) (1999) 1141.
[6] I.S. Kochev, Mech. and Mach. Theory 25 (4) (1990) 467.
[7] I.S. Kochev, Mech. and Mach. Theory 27 (1) (1992) 53.
[8] J.P. Sadler, Mech. Eng. 1 (1975) 139.
[9] M.R. Smith, Comput. Aided Design 7 (1) (1975) 15.
[10] G.G. Lowen, R.S. Berkof, Trans. ASME B 93 (1971) 39.
[11] R.S. Berkof, G.G. Lowen, Trans. ASME B 93 (1971) 53.
[12] J.L. Wienderich, B. Roth, Trans. ASME B 98 (1976) 1289.
[13] W.L. Carson, J.M. Stephens, Mech. and Mach. Theory 13 (1978) 649.
[14] R.E. Kaufman, G.N. Sandor, Trans. ASME B 93 (2) (1971) 620.

[15] N.X. Chen, Q.X. Zhang, in: Proceedings of the Sixth World Congress on Theory of Machines and Mechanisms, New Delhi, vol. 1, 1983, p. 329.
[16] N.X. Chen, Mech. and Mach. Theory 19 (2) (1984) 257.
[17] Y.Q. Yu, Mech. and Mach. Theory 22 (1) (1987) 27.
[18] Y.Q. Yu, Mech. and Mach. Theory 22 (1) (1987) 39.
[19] Y.Q. Yu, Mech. and Mach. Theory 23 (4) (1988) 279.
[20] J. Wawrzecki, in: Proceedings of the Ninth World Congress on Theory of Machines and Mechanisms, Milan, Italy, vol. 4, 1995, p. 2838.
[21] J. Wawrzecki, Mech. and Mach. Theory 33 (8) (1998) 1195.
[22] J. Wawrzecki, in: Proceedings of the Tenth World Congress on Theory of Machines and Mechanisms, Oulu, Finland, vol. 4, 1999, p. 1589.
[23] I.D. Belonovskaya, F.M. Dimentberg, L.B. Maysuk, Mashinovedeniye, Moscow 1 (1987) 3.
[24] X.W. Kong, T.L. Yang, in: Proceedings of the 1998 ASME Design Engineering Technical Conferences, Atlanta, USA, 1998 (DETC/MECH-5846).
[25] X.W. Kong, T.L. Yang, R.Q. Guo, in: Proceedings of the Tenth World Congress on Theory of Machines and Mechanisms, Oulu, Finland, vol. 4, 1999, p. 1447.
[26] S.T. Chiou, R.J. Tsai, Mech. and Mach. Theory 30 (3) (1995) 405.
[27] V.A. Schepetilnikov, Balancing of mechanisms, Mashinostroeniya, Moscow (1982) 256.
[28] T.T. Gappoev, D.B. Tabouev, in: Proceedings of the Fifth World Congress on Theory of Machines and Mechanisms, Canada, vol. 2, 1979, p. 1420.
[29] V. Arakelian, in: Proceedings of the Ninth World Congress on the Theory of Machines And Mechanisms, Italy, vol. 1, 1995, p. 205.