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# Complete shaking force and shaking moment balancing of linkages

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## Abstract

This paper proposes a new solution to the problem of complete shaking force and shaking moment balancing of linkages. The method involves connecting to the mechanism to be balanced a two-link group forming a pantograph with the crank and coupler. Three versions of sub-linkages are considered: (1) the articulation dyad; (2) the asymmetric link with three rotational pairs; (3) the crank-slider mechanism. The mathematical basis for the realisation of this method is the well-known method of static and dynamic substitution of distributed masses by concentrated point masses. The method is illustrated by new balancing schemes for the Stephenson and Watt linkages. © 1998 Elsevier Science Ltd. All rights reserved.

## Résumé

Dans cet article est proposée une nouvelle méthode d'équilibrage dynamique complet des mecanismes. La méthode proposée est réalisée par l'addition au mécanisme à équilibrer de groupe articulée à deux barres formant avec la manivelle et la bielle du mécanisme initial un pantographe. On considere trois versions des sub-mécanismes: (1) la groupe articulée à deux barres; (2) l'élément asymétrique à trois couples de rotation; (3) le mécanisme à manivelle et tiroir. Le moyen mathématique pour la réalisation de cette méthode est basé sur les méthodes connues de substitution statique et dynamique de masses des éléments du mécanisme par les masses-points concentrées. La méthode proposée est illustrée par les nouveaux schémas d'équilibrage des mécanismes de Stephenson et de Watt. © 1998 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

In high-speed machines, mass balancing of the moving links brings about a reduction of vibration that considerably improves their performance. However, complete shaking force and shaking moment balancing of linkages is a complicated problem. The need to satisfy the necessary conditions of balancing brings about a great increase of the masses of the movable links of the linkages. This results in a rise of the forces in the kinematic pairs and an increase of the input moment.

The first study of complete shaking force and shaking moment balancing of planar crank–rocker mechanism was the method of "mass redistribution" presented by Berkof [1]. However, the full mass balancing problem is very complicated especially for such a widespread mechanism as the crank–slider mechanism. The most efficient method for solving this problem is considered to be the "duplicating mechanism" method [2, 3] by adding to the initial mechanism an identical mechanism which is a revolved mirror reflection of the initial mechanism. The disadvantages of such an approach are a partial balancing due to the shaking moment of inertia forces of the slider, as well as the greater friction losses due to the additional sliding (prismatic) pair.

Other workers [4–6] have proposed methods for the full balancing of mass of linkages by counterweights with planetary gear trains. The disadvantage of these balancing schemes is the fact that the gear inertia counterweights needed for balancing the shaking moment are mounted on the movable links that are not connected directly to the frame.

Another approach is applied by Kochev [7]. In his study it is proposed to balance shaking moment (in the force balanced mechanism) by a prescribed input speed fluctuation. However, in practice, the approximation of the motion of the input link is very difficult and needs a special type of drive generator.

In a study by Bagci [8], a method of full balancing is presented involving the addition of "balancing idler loops" which form a parallelogram with the initial links of the mechanism. The method is based on the well-known principle of the "independence of the mechanism balanced state from parallel transfer of counterweight rotation axis". The balancing is realised by using the general balancing conditions elaborated by Lowen and Berkof [9] known as the "method of linearly independent vectors".

Hilpert [10] has successfully used the pantograph mechanism for complete shaking force balancing of four-bar linkages. This idea has been developed by Arakelian [11] for complete shaking force and shaking moment balancing of the in-line crank–slider mechanism.

The object of the study presented here is to provide the conditions for a complete shaking force and shaking moment balancing of linkages with a relatively small increase of the total mass of movable links by mounting the gear inertia counterweights on the base of the mechanism, in addition to further developing the work in Ref. [11] for the balancing of off-set crank–slider mechanisms.

This has been achieved by the addition of a supplementary link. Such a solution provides an improvement in the known methods of balancing, rendering them more suitable for practical application.

A quite different approach and solution are applied to the balancing of the crank–slider mechanism. In this case, the added articulation dyad forms a pantograph with the crank and coupler of the initial linkage. The solution permits the balancing of the linkage with a relatively small increase in the total mass of the movable links.

Finally, by application of this new approach together with the principle of dynamic substitution of mass, new schemes for the complete balancing of both the Stephenson and Watt six-bar linkages are presented. In these cases, the mass of the coupler is replaced by three dynamically equivalent point masses located at the pin joints. This method, first applied by Berkof [1] for the full balancing of the four-bar linkage, has the advantage that the dynamic characteristics of the initial mechanism are unchanged and the point masses may be considered part of the mass of the adjacent links which are then balanced in turn.

## 2. Complete shaking force and shaking moment balancing of sub-linkages

Let us consider three versions of sub-linkages.

### 2.1. Articulation dyad

The well-known scheme of complete shaking force and shaking moment balancing of an articulation dyad [5] is shown in Fig.1(a).

The principle of such an approach is as follows. To link 2 is added a counterweight which permits the displacement of the centre of mass of link 2 to joint A. Then, by means of a counterweight with mass $m_{\mathrm{cw}_1}$ [see Fig. 1(a)] a complete balancing of shaking force is achieved. A complete shaking moment balance is realised through four gear inertia counterweights 3–6, one of them being of the planetary type and mounted on link 2 [5].

The scheme suggested here [Fig. 1(b)] is distinguished from the earlier scheme by the fact that gear 3 is mounted on the base and is linked kinematically with link 2 through link 1′.

To have a more illustrative representation of the advantages of such a balancing, let us consider application of the new system with the mass of link 1′ not taken into account.

In this case (compared to the usual method Fig. 1(a), the mass of the counterweight of link 1 will be reduced by an amount

$$\Delta m_{\mathrm{CW}_1} = m_3 l_{\mathrm{OA}} / r_{\mathrm{CW}_t}, \tag{1}$$

where $m_3$ is the mass of the gear 3; $l_{\mathrm{OA}}$ is the distance between the centres of hinges O and A; $r_{\mathrm{CW}_1}$ is the rotation radius of the centre of mass of the counterweight.

It is obvious that the moment of inertia of the links is correspondingly reduced. If the gear inertias are made in the form of heavy rims in order to obtain a large moment of inertia, the moments of inertia of the gear inertia counterweights may be presented [2] as $I_i = m_i D_i^2/4$ ($i = 3, \ldots, 6$). Consequently, the mass of gear 6 will be reduced by an amount

$$\Delta m_6 = 4(m_3 l_{\mathrm{OA}}^2 + \Delta m_{\mathrm{CW}_1} r_{\mathrm{CW}_1}^2) Z_6 / D_6^2 Z_5, \tag{2}$$

where $Z_5$ and $Z_6$ are the numbers of teeth of the corresponding gears. Thus, the total mass of the system will be reduced by an amount

$$\Delta m = \Delta m_{CW_1} + \Delta m_6 \tag{3}$$

Let us now consider the complete shaking force and shaking moment balancing of the articulation dyad with the mass and inertia of link 1′ taken into account. For this purpose,

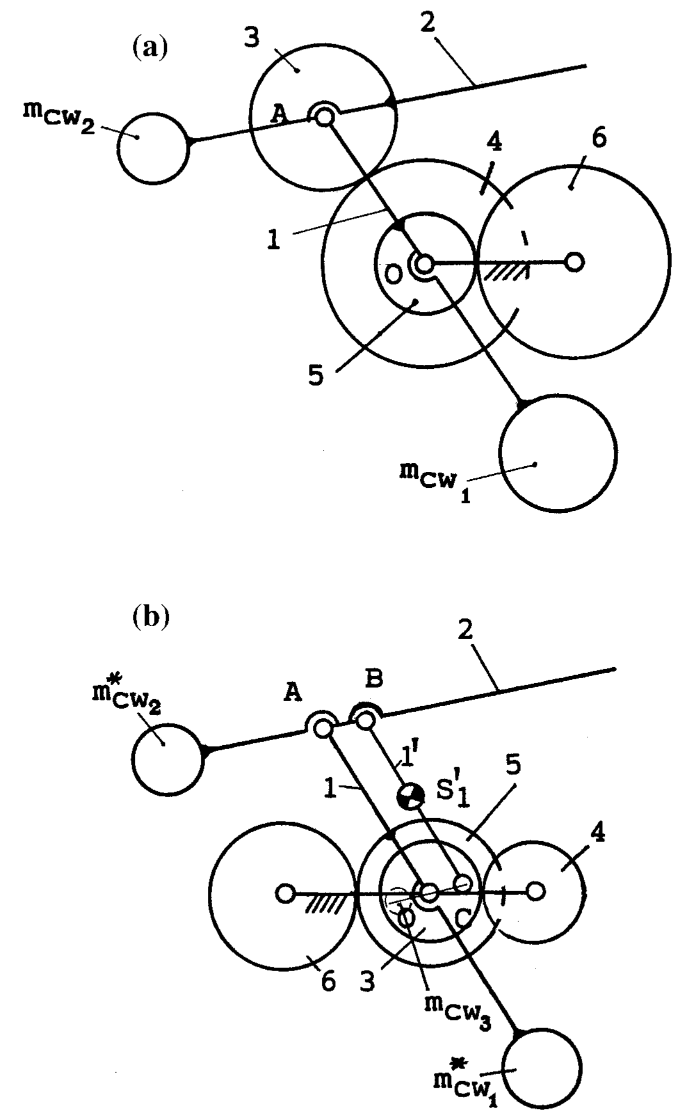


Fig. 1. (a) Complete shaking force and shaking moment balancing of articulation dyad. (b) Complete shaking force and shaking moment balancing of articulation dyad by gear inertia counterweights mounted on the base.

initially, we shall statically replace mass $m'_1$ of link 1′ by two point masses $m_B$ and $m_C$ at the centres of the hinges B and C:

$$m_B = m'_1 l_{CS'_1} / l_{BC}, \tag{4}$$

$$m_c = m'_1 l_{BS'_1} / l_{BC}, \tag{5}$$

where $l_{BC}$ is the length of link 1; $l_{CS'_1}$ and $l_{BS'_1}$ are the distances between the centres of the joints C and B and the centre of mass $S'_1$ of link 1′, respectively.

After such an arrangement of masses the moment of inertia of link 1′ will be equal to [1].

$$I^{*}_{S'_1} = I_{S_1'} - m'_1 l_{BS'_1} l_{CS'_1}, \tag{6}$$

where $I_{S'_1}$ is the moment of inertia of link 1′ about the centre of mass $S'_1$ of the link.

Thus, we obtain a new dynamic model of the system where the link 1′ is represented by two point masses $m_B$, $m_C$ and has a moment of inertia $I^{*}_{S'_1}$.

This fact allows for an easy determination of the parameters of the balancing elements as follows:

$$m^{*}_{CW_2} = (m_2 l_{AS_2} + m_B l_{AB})/r_{CW_2}, \tag{7}$$

where $m_2$ is the mass of link 2; $l_{AB}$ is the distance between the centres of hinges A and B; $l_{AS_2}$ is the distance of the centre of hinge A from the centre of mass $S_2$ of link 2; $r_{CW_2}$ is the rotation radius of the centre of mass of the counterweight with respect to A, and

$$m^{*}_{CW_1} = [(m_2 + m^{*}_{CW_2} + m_B)l_{OA} + m_1 l_{OS_1}]/r_{CW_1}, \tag{8}$$

where $m_1$ is the mass of link 1; $l_{OS_1}$ is the distance of the joint centre 0 from the centre of mass $S_1$ of link 1.

Also,

$$m_{CW_3} = m_C l_{OC}/r_{CW_3}, \tag{9}$$

where $l_{OC} = l_{AB}$; $r_{CW_3}$ is the rotation radius of the centre of mass of the counterweight.

Taking into account the mass of link 1′ brings about the correction in Eq. (3). In this case, $\Delta m = \Delta m_{CW_1} + \Delta m_6 - \Delta m'_1$, where $\Delta m'_1$ is the value characterising the change in the distribution of the masses of the system links resulting from the addition of link 1′.

### 2.2. Asymmetric link with three rotational pairs (Fig. 2)

In previous work [12, 13] relating to balancing of linkages with a dynamic substitution of the masses of the link by three rotational pairs (see Fig. 2) two replacement points A and B are considered. This results in the need to increase the mass of the counterweight. However, such a solution may be avoided by considering the problem of dynamic substitution of link masses by three points. Usually, the centre of mass of such an asymmetric link is located inside a triangle formed by these points. The conditions for dynamic substitution of masses are the following:

$$\begin{bmatrix} 1 & 1 & 1 \\ l_A e^{i\theta_A} & l_B e^{i\theta_B} & l_C e^{i\theta_C} \\ l_A^2 & l_B^2 & l_C^2 \end{bmatrix} \begin{bmatrix} m_A \\ m_B \\ m_C \end{bmatrix} = \begin{bmatrix} m_i \\ 0 \\ I_{S_i} \end{bmatrix} \tag{10}$$

where $m_A$, $m_B$ and $m_C$ are point masses; $l_A$, $l_B$ and $l_C$ are the moduli of radius-vectors of corresponding points; $\theta_A$, $\theta_B$ and $\theta_C$ are angular positions of radius-vectors; $m_i$ is the mass of

[1] After the static substitution of the masses of the link by point masses, it is necessary to take into account the change of the moment of inertia of the link.

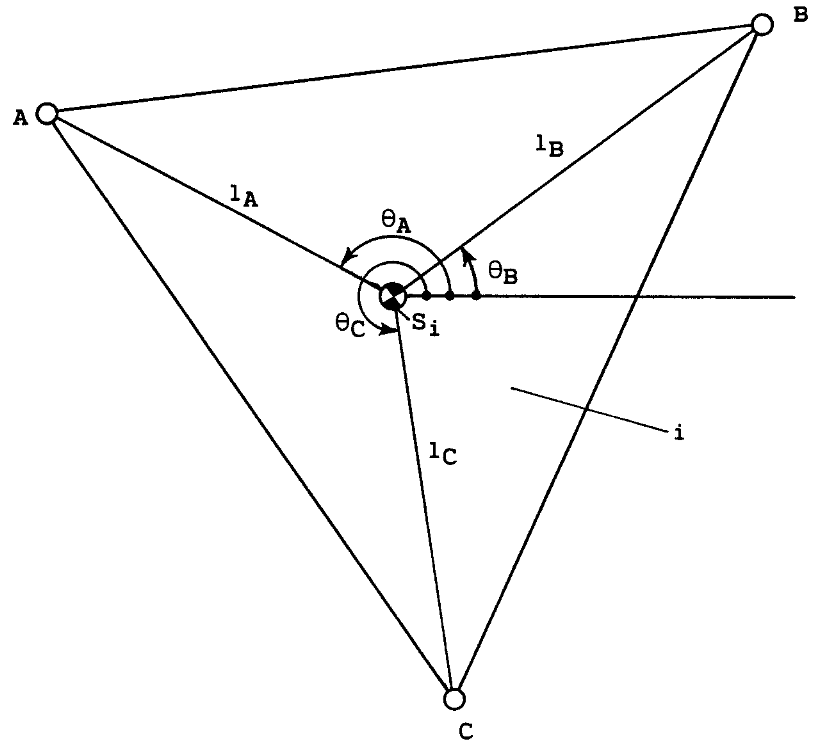


Fig. 2. Dynamic substitution of the masses of the link by three rotational pairs.

link; $I_{S_i}$ is the moment of inertia of the link about an axis through $S_i$ (axial moment of inertia of link).

From these we determine

$$m_A = D_A/D_i, , \quad m_B = D_B/D_i, \quad m_C = D_C/D_i, \tag{11}$$

where $D_A$, $D_B$, $D_C$ and $D_i$ are determinants of the third order obtained from the above system of equations.

### 2.3. Crank–slider mechanism (Fig. 3)

Complete shaking force and shaking moment balancing of the off-set crank–slider mechanism is shown in Fig. 3(a). The principle of such an approach resides in the following. On coupler 2 is added a counterweight which transfers the centre of mass of coupler 2 and slider 3 into the centre of joint A. Then, by means of a counterweight of mass $m_{CW_1}$, the general centre of mechanism mass is brought to the centre of pivot 0. Complete balancing of the shaking moment is realised by means of the four gear inertia counterweights 4–7.

Three possible solutions are examined here.

*2.3.1. Complete shaking force and shaking moment balancing of the mechanism by mounting the gear inertia counterweights on the links connected directly to the frame*

The balancing scheme illustrated in Fig. 3(b) differs from the traditional scheme by the fact that gear 4 is mounted on the base and linked kinematically with link 2 through an additional link 1′ and gear 5.

The complete balancing conditions for the shaking force and shaking moment of the mechanism are similar to those in the previous case when the scheme shown in Fig. 1(b) was considered.

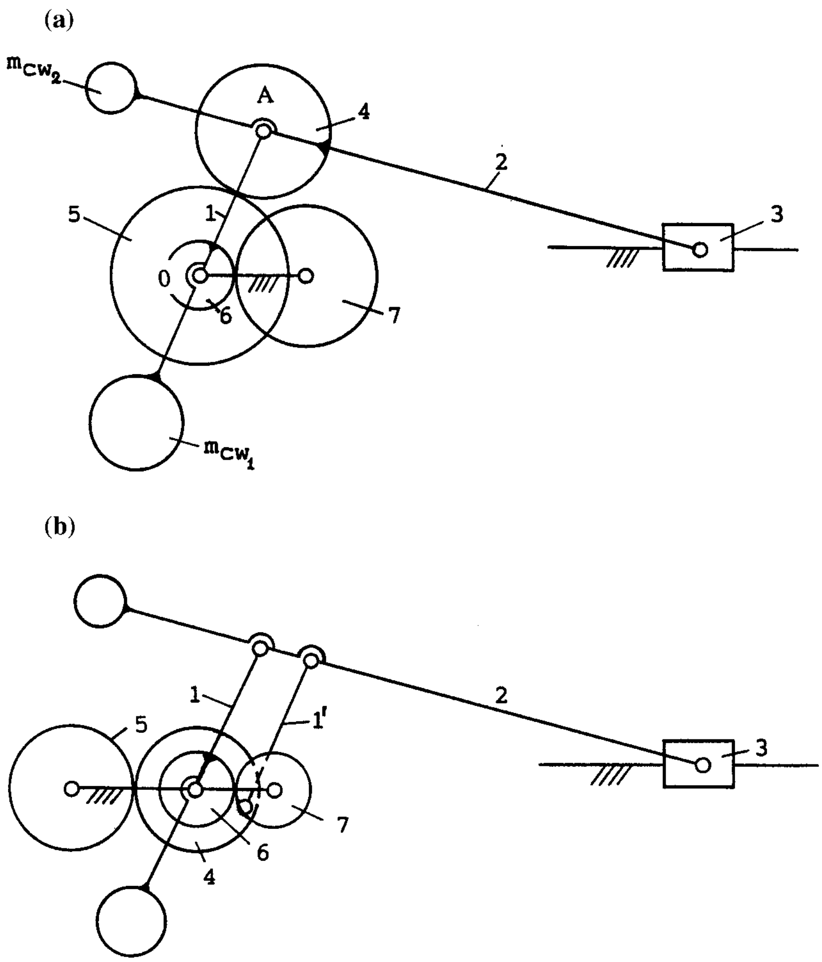


Fig. 3. (a) Complete shaking force and shaking moment balancing of crank–slider mechanism. (b) The suggested scheme for complete shaking force and shaking moment balancing of crank–slider mechanism. (c) Complete shaking force and shaking moment balancing of crank–slider mechanism based on the copying properties of the pantograph. (d) Optimum balancing of the crank–slider mechanism based on the properties of the parallelogram.

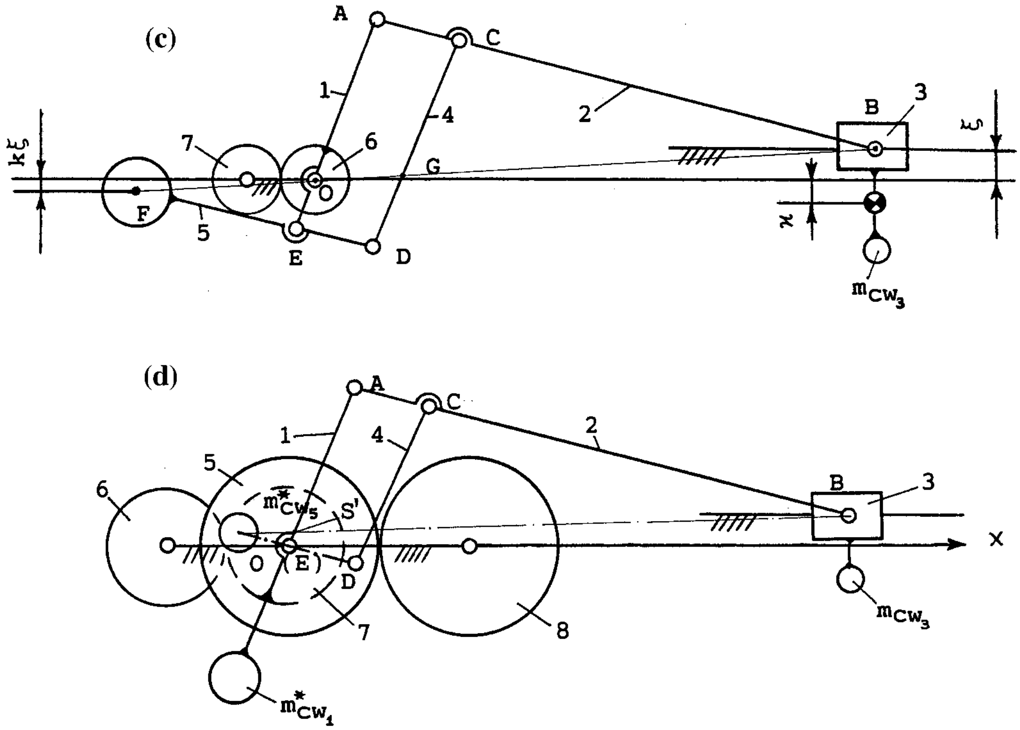


Fig. 3 (*continued*)

*2.3.2. Complete shaking force and shaking moment balancing of the mechanism based on the copying properties of the pantograph*

Fig. 3(c) shows an off-set crank–slider mechanism with an articulation dyad CDE connected to it which forms a pantograph, with the initial mechanism OAB. By selecting, for constructional reasons, the similarity factor of the formed pantograph, $k = l_{\mathrm{OB}}/l_{\mathrm{OA}}$, we determine the length of the articulation dyad: $l_{\mathrm{CD}} = l_{\mathrm{OA}} + l_{\mathrm{OE}}$, $l_{\mathrm{DF}} = kl_{\mathrm{AB}} + l_{\mathrm{AC}}$.

By substituting dynamically the mass $m_2$ of the coupler by point masses at the centres A, B and C and using the following condition

$$\begin{bmatrix} 1 & 1 & 1 \\ l_{\mathrm{AS}_2} & -l_{\mathrm{BS}_2} & l_{\mathrm{CS}_2} \\ l^2_{\mathrm{AS}_2} & l^2_{\mathrm{BS}_2} & l^2_{\mathrm{CS}_2} \end{bmatrix} \begin{bmatrix} m_{\mathrm{A}} \\ m_{\mathrm{B}} \\ m_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} m_2 \\ 0 \\ I_{\mathrm{S}_2} \end{bmatrix}, \tag{12}$$

where $l_{\mathrm{AS}_2}$, $l_{\mathrm{BS}_2}$ and $l_{\mathrm{CS}_2}$ are the distances of joint centres A, B and C from the centre of masses $S_2$ of the coupler; $I_{\mathrm{S}_2}$ is the axial moment of inertia of the coupler; we determine the value of the point masses

$$m_{\mathrm{A}} = D_{\mathrm{A}}/D_2, \quad m_{\mathrm{B}} = D_{\mathrm{B}}/D_2, \quad m_{\mathrm{C}} = D_{\mathrm{C}}/D_2, \tag{13}$$

where $D_A$, $D_B$, $D_C$ and $D_2$ are determinants of the third order obtained from the system of equations.

We now require link 1 to be balanced about point O, link 4 (masses $m_C$ , $m_D$ and $m_4$) about point G and finally masses $m_B$, $m_3$, $m_G$ and $m_F$ about point O. The necessary conditions are as follows:

$$\begin{cases} m_A l_{OA} + m_1 l_{OS_1} - m_E l_{OE} = 0; \\ m_D l_{DG} - m_4(l_{CG} - l_{CS_4}) - m_C l_{CG} = 0; \\ m_F = (m_B + m_3 + (m_C + m_D + m_4) l_{AC}/l_{AB})/k, \end{cases} \tag{14}$$

$$\begin{cases} m_D + m_E + m_F = m_5; \\ m_D l_{DS_5} + m_E(l_{DS_5} - l_{DE}) - m_F(l_{DF} - l_{DS_5}) = 0; \\ m_D l_{DS_5}^2 + m_E(l_{DS_5} - l_{DE})^2 - m_F(l_{DF} - l_{DS_5})^2 = I_{S_5}, \end{cases} \tag{15}$$

where $l_{OA}$, $l_{OE}$ and $l_{OS_1}$ are the distances of joint centres A, E and of the centre of mass $S_1$ of the crank from the pivot centre 0; $l_{DG}$, $l_{CG}$ are the distances of the centres of the joints D,C from the working point G of the pantograph; $l_{CS_4}$ is the distance of the centre of the joint C from the centre of mass $S_4$ of link 4; $l_{AB}$, $l_{AC}$ are the distances of the centres of the joints B, C from the centre of joint A; $l_{DE}$, $l_{DF}$ are the distances of the centres of the joints E, F from the centre of joint D; $m_4$ is the mass of link 4; $m_D$, $m_E$, $m_F$ are point masses obtained after dynamic substitution; $m_5$ is the mass of link 5; $l_{DS_5}$ is the distance of the centre of the joint D from the centre of mass $S_5$ of link 5; $I_{S_5}$ is the axial moment of inertia of link 5.

We now have the desired parameters

$$\begin{aligned} m_5 &= m_D + m_E + m_F; \\ l_{DS_5} &= (m_E l_{DE} + m_F l_{DF})/m_5; \\ I_{S_5} &= m_D l_{DS_5}^2 + m_E(l_{DS_5} - l_{DE})^2 - m_F(l_{DF} - l_{DS_5})^2, \end{aligned} \tag{16}$$

where

$$m_D = [m_C l_{CG} + m_4(l_{CG} - l_{CS_4})]/l_{DG}; \tag{17}$$

$$m_E = (m_A l_{OA} + m_1 l_{OS_1})/l_{OE}. \tag{18}$$

Thus, we obtain a dynamic model of the mechanism [see Fig. 3(c)] fully equivalent to the real mechanism involving the rotating links 1,4[2] and four point masses $m_3 + m_B$, $m_D$, $m_F$ and $m_G$, three of which perform a translational rectilinear motion in the horizontal sense. As may be seen from this equivalent model, a complete shaking force balancing of the movable links of

[2] The parameters of link 5 are selected so that the centre of mass of link 4, with the point masses $m_C$ and $m_D$ taken into account, coincides with the working point G of the pantograph, due to which the motion of this link is represented as a translational rectilinear motion of its centre of mass and a rotary motion relative to point G.

the mechanism has been achieved: $\overline{\mathrm{F}}_{\mathrm{F}}^{\mathrm{int}} = \overline{\mathrm{F}}_{\mathrm{B}}^{\mathrm{int}} + \overline{\mathrm{F}}_{3}^{\mathrm{int}} + \overline{\mathrm{F}}_{\mathrm{G}}^{\mathrm{int}}$, ($\overline{\mathrm{F}}_{\mathrm{G}}^{\mathrm{int}} = \overline{\mathrm{F}}_{\mathrm{C}}^{\mathrm{int}} + \overline{\mathrm{F}}_{\mathrm{D}}^{\mathrm{int}} + \overline{\mathrm{F}}_{4}^{\mathrm{int}}$, where $\overline{\mathrm{F}}_{i}^{\mathrm{int}}$ ($i$ = B, C, D, F, G, 3, 4) -inertia forces from corresponding masses).

The shaking moment of the mechanism is determined by the sum:

$$M^{\mathrm{int}} = M_1^{\mathrm{int}} + M_4^{\mathrm{int}} + M_{\mathrm{O}}(F_i^{\mathrm{int}}), \tag{19}$$

where $M_1^{\mathrm{int}}$ and $M_4^{\mathrm{int}}$ are the shaking moments of the rotating links 1 and 4 with the inertia of the replaced point masses taken into account:

$$\begin{aligned} M_1^{\mathrm{int}} &= (I_{\mathrm{S}_1} + m_1 l_{\mathrm{OS}_1}^2 + m_{\mathrm{A}} l_{\mathrm{OA}}^2 + m_{\mathrm{E}} l_{\mathrm{OE}}^2)\alpha; \\ M_4^{\mathrm{int}} &= (I_{\mathrm{S}_4} + m_4 l_{\mathrm{GS}_4}^2 + m_{\mathrm{C}} l_{\mathrm{CG}}^2 + m_{\mathrm{D}} l_{\mathrm{DG}}^2)\alpha, \end{aligned} \tag{20}$$

where $I_{\mathrm{S}_1}$ and $I_{\mathrm{S}_4}$ are the axial moments of inertia of links 1 and 4; $\alpha = \alpha_1 = \alpha_4$ is the angular acceleration of links 1 and 4; $M_{\mathrm{O}}(F_i^{\mathrm{int}})$ is the moment resulting from the force of inertia of the masses $m_3 + m_{\mathrm{B}}$, $m_{\mathrm{G}}$ and $m_{\mathrm{F}}$ performing a translational rectilinear motion relative to pivot 0.

The moments of the rotating links may be balanced by means of the gears [1,4–6] mounted on the base of the mechanism. The moment of inertia of such a gear is given by the following equation:

$$I_{\mathrm{gear}} = I_{\mathrm{S}_1} + I_{\mathrm{S}_4} + m_1 l_{\mathrm{OS}_1}^2 + m_4 l_{\mathrm{GS}_4}^2 + m_{\mathrm{A}} l_{\mathrm{OA}}^2 + m_{\mathrm{C}} l_{\mathrm{CG}}^2 + m_{\mathrm{E}} l_{\mathrm{OE}}^2 + m_{\mathrm{D}} l_{\mathrm{DG}}^2. \tag{21}$$

Regarding the moment $M_{\mathrm{O}}(F_i^{\mathrm{int}})$, it is necessary to redistribute the masses performing a translational motion, using counterweights mounted on slider 3 ($m_{\mathrm{CW}_3}$) and on link 5 ($m'_{\mathrm{F}}$).

The necessary conditions for balancing this moment are the following:

$$\begin{cases} m_{\mathrm{F}} + m'_{\mathrm{F}} = (m_{\mathrm{B}} + m_3 + m_{\mathrm{CW}_3} + (m_{\mathrm{C}} + m_{\mathrm{D}} + m_4) l_{\mathrm{AC}}/l_{\mathrm{AB}})k, \\ \xi k(m_{\mathrm{F}} + m'_{\mathrm{F}}) + \xi(m_{\mathrm{C}} + m_{\mathrm{D}} + m_4) l_{\mathrm{AC}}/l_{\mathrm{AB}} = (m_{\mathrm{B}} + m_3 + m_{\mathrm{CW}_3})\chi, \end{cases} \tag{22}$$

from which we determine $m_{\mathrm{CW}_3}$ and $m'_{\mathrm{F}}$.

It should be noted that in most constructions of such mechanisms, the eccentricity of slider guides is not significant and the moment $M_{\mathrm{O}}(F_i^{\mathrm{int}})$ is relatively small, so that in many mechanism balancing problems this moment may be neglected.

*2.3.3. Improvement on previous methods [4–6] by mounting the gear inertia counterweight on the mechanism frame.*

In Fig. 3(d) is illustrated an off-set mechanism OAB and an articulation dyad CDE connected to it. This dyad forms a parallelogram with the initial mechanism.

The conditions for balancing the system are determined from the following considerations. With the static substitution of mass $m_4$ of link 4 by the masses $m_{\mathrm{C}}$ and $m_{\mathrm{D}}$ situated in the centres of corresponding hinges and with the substitution of mass $m_2$ of the coupler (with the point mass $m_{\mathrm{C}}$ taken into account) by masses $m_{\mathrm{A}}$ and $m_{\mathrm{B}}$, we obtain a system of point masses performing either a rotational or a translational motion. By adding thereafter a counterweight with a mass $m_{\mathrm{CW}_3}$ on the slider, we transfer the slider mass centre with the point mass $m_{\mathrm{B}}$ into the line OX. However, since the slider performs a translational rectilinear motion, the mass $m_{\mathrm{B}} + m_3 + m_{\mathrm{CW}_3}$ may be considered as a point mass in the centre of joint B, since the balance of the inertia forces of the movable masses is not altered by this change. Mounting thereafter a counterweight of mass $m_5^*$ on link 5, we transfer the general centre of the masses $m_B + m_3 + m_{CW_3} + m_5$ onto point $S'$ [see Fig. 3(d)] which performs a rotational motion.

We now obtain a complete shaking force balance by the addition of a counterweight with mass $m^*_{CW_1}$ on the input crank, displacing the centre of mass of the movable links from point $S'$ to the centre of the pivot 0.

In this case, the static moments of the counterweights relative to the pivot will be given by

$$m_1^* r_1^* = m_1 l_{OS_1} + l_{OA}(m_2 + m_3 + m_4 l_{DS_4}/l_{CD}); \tag{23}$$

$$m_5^* r_5^* = m_2 l_{AS_2} + m_3 l_{AB} + m_4 l_{AC} \pm m_5 l_{OS_5}, \tag{24}$$

where $m_i$ is the mass of link $i$; $l_{OS_1}$, $l_{AS_2}$, $l_{CS_4}$, $l_{OS_5}$ are the distances of the centres of mass of links 1, 2, 4, 5 from the centres of joints O, A, C; $l_{OA}$, $l_{AB}$, $l_{AC}$, $l_{CD}$ are the distances between the centres of corresponding joints.

After such a redistribution of masses, the moment from inertia forces will be balanced by gear inertia counterweights 5–8 [1,4–6].

## 3. Application of the methods for complete shaking force and shaking moment balancing of multilink mechanisms

Let us consider the complete shaking force and shaking moment balancing of Stephenson (Fig. 4) and Watt (Figs. 5 and 6) linkages.

For the complete shaking force and shaking moment balancing of the Stephenson linkage we apply the following approach. First, we replace dynamically the mass of coupler 2 by three point masses located at the centres of the joints A, B and C. That permits us to solve the problem of complete shaking force and shaking moment balancing of the linkage as separate problems of the balancing of sub-linkages (case 2.1).

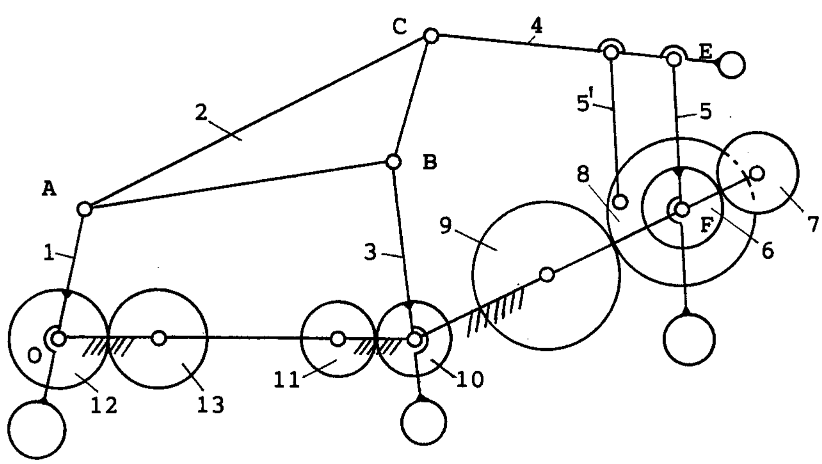


Fig. 4. Complete shaking force and shaking moment balancing of Stephenson linkage.

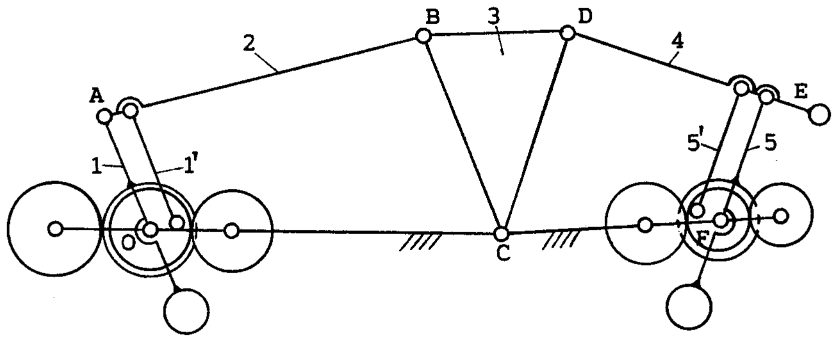


Fig. 5. Complete shaking force and shaking moment balancing of Watt linkage.

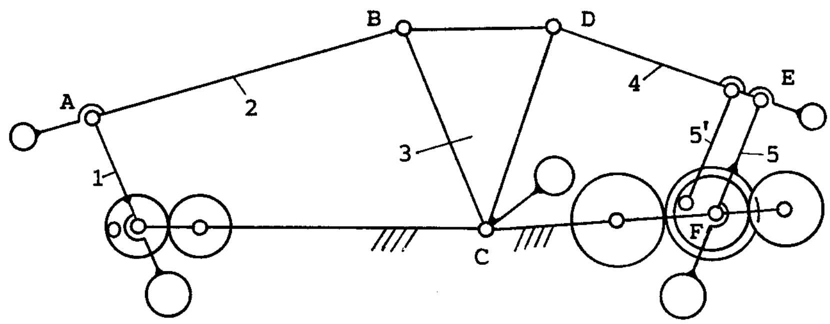


Fig. 6. Complete shaking force and shaking moment balancing of Watt linkage.

For the complete shaking force and shaking moment balancing of the Watt linkage, two methods have been developed. In the first method (Fig. 5), we replace dynamically the mass of link 3 by the point masses $m_B$ , $m_C$ and $m_D$ . Then, we consider the problem of sub-linkages OAB and DEF.

In the second method (Fig. 6), we replace dynamically the mass of link 2 by the point masses $m_A$ and $m_B$ . Then, taking into account the point mass $m_B$ we replace dynamically the mass of link 3 by the point masses $m_C$ and $m_D$ . That changes the problem of balancing the linkage into problems of balancing sub-linkages: crank OA and articulation dyad DEF.

## 4. Conclusions

This paper presents new balancing schemes relating to three types of sub-linkages and permitting complete shaking force and shaking moment balancing of mechanisms involving a smaller increase of link mass compared to earlier methods. An advantage of the schemes outlined here is the fact that all the gear inertia counterweights needed for balancing the shaking moment are mounted on the mechanism frame, which is constructively more efficient.

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