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Optimum Shaking Force Balancing of Planar 3-RRR Parallel Manipulators by means of an Adaptive Counterweight System

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Abstract

This paper deals with the problem of optimum balancing of planar 3-RRR parallel robots for fast manipulation. It is known that in fast robots shaking forces on the frame vary greatly during a cycle of operation. Such forces can cause vibrations having various negative impacts. Several balancing techniques have been developed to solve this problem in planar 3-RRR parallel robots. However, it is known that the manipulators after complete shaking force balancing become very heavy, which leads to the significant increase of input torques and dynamic loads in the joints. This is why in the present study an analytically tractable solution for optimum shaking force balancing of planar 3-RRR parallel manipulators is proposed. The proposed balancing has been carried out by only three counterweights mounted on the input links and having constant masses but adjustable locations. The efficiency of the suggested solution has been illustrated via a numerical simulations carried out by using ADAMS software.

Keywords: Optimum balancing, Shaking force, Parallel manipulator, Adaptive counterweight system.

Introduction

A fast robot with unbalance shaking force/moment transmits substantial vibration to the frame. Thus, a primary objective of the balancing is to cancel or reduce the variable dynamic loads transmitted to the frame and surrounding structures. Hence, the balancing problems are of continued interest to researchers and various design concepts for balancing of robot manipulators are available in the literature [1-6].

The review of methods devoted to the shaking force balancing of manipulators has shown that the following principal subgroups can be distinguished.

i) Shaking force balancing by adding counterweights in order to keep the total centre of mass of moving links stationary. In the case of open-chain manipulators, it is necessary to start from the outermost link and add a counterweight to it to bring the center of mass of this link on the immediately preceding joint axis. Such a balancing process must be repeated sequentially until the center of mass of the whole chain is fixed of the base pivot [4,7-9]. With regard to the parallel manipulators, the approach is the same: adding counterweights to keep the total centre of mass of moving links stationary [10,11].

ii) Shaking force balancing by adding auxiliary structures. Different approaches have developed in order to keep the total centre of mass of moving links stationary by adding auxiliary structures.

In Agrawal & Fattah [8,12,13], the parallelograms were used as auxiliary structures in order to create the balanced manipulators. As is shown in Fattah & Agrawal [12], the three scaled lengths are added to form parallelograms and are then used to identify the center of mass. For the 3-link mechanism, the system consists of parallelograms in two layers: the first layer has two parallelograms while the second layer has one. As is mentioned in the cited paper, this procedure can be extended to n links.

The pantograph has also been used in order to balance the shaking force. Different solutions were proposed for shaking force and shaking moment balancing of Delta robot by adding a pantograph to each leg or by adding a pantograph connected with the center of mass localized by using the parallelograms [14,15].

iii) Shaking force balancing by elastic components. These studies are focused on optimum force balancing of a five-bar parallel manipulator by a combination of a proper distribution of link masses with springs connected to the driving links [16,17]. The force balancing is formulated as a numerical optimization problem in such a way that the root-mean-square values of bearing and spring forces are minimized.

iv) Shaking force balancing by adjustment of kinematic parameters. These studies deal with the synthesis of the balanced five-bar mechanism via changing the geometric and kinematic parameters of the mechanical structure [18,19]. The shaking force balancing leads to the conditions which are traditionally satisfied by the redistribution of moving masses. In the mentioned studies, the mass of the link is considered unchanged and the length and the mass center of the links are determined in order to carry out the shaking force balancing. Thus, a new kinematic chain is obtained which is fully force balanced. This approach was also applied on the design of a spatial three-degree-of-freedom parallel manipulator [20].

v) Shaking force minimization via centre of mass acceleration control. In Briot & Arakelian [21,22] a resourceful solution was developed, which is based on the optimal control of the robot centre of masses. The aim of the suggested method consists in the fact that the manipulator is controlled not by applying end effector trajectories but by planning the displacements of the total mass centre of moving links. The trajectories of the total mass centre of moving links are defined as straight lines and are parameterized with "bang-bang" motion profiles. Such a control approach allows the reduction of the maximal value of the centre of mass acceleration and, consequently, leads to the reduction in the shaking force.

In the present paper an optimum shaking force balancing of planar 3-RRR parallel manipulators by means of an adaptive counterweight system is discussed. The proposed balancing is carried out by only three counterweights mounted on the input links and having constant masses but adjustable locations.

Optimum Shaking Force Balancing of Planar 3-RRR Parallel Manipulators

The moving platform of a planar 3-RRR parallel manipulator is connected to its legs by three revolute joints P_k $(k = 1,2,3)$ (Figure1). Each leg comprises two links connected by revolute joints A_k $(k = 1,2,3)$ and they are mounted on the frame by revolute joints O_k $(k = 1,2,3)$. The input parameters of such a manipulator are defined by the joint angles θ_k $(k = 1,2,3)$ of each leg and the output parameters by the pose of the moving platform, i.e. its orientation ϕ and position of one point of the moving platform, by example, the centre of mass of the moving platform (x, y).

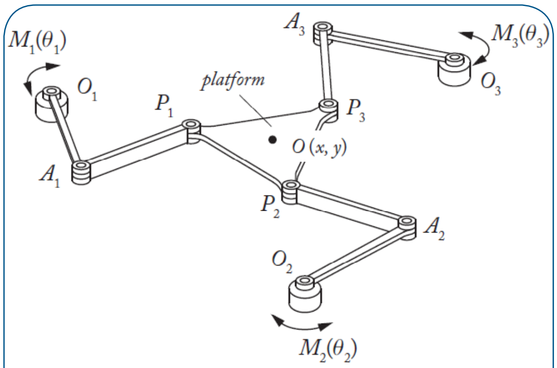

Figure 1: Planar 3-RRR parallel manipulator

Let us consider the complete shaking force balancing of the planar 3-RRR parallel manipulator. For this purpose it is necessary to add counterweights in order to change the mass redistribution. The traditional way to balance the shaking forces of the planar 3-RRR parallel manipulator is to add seven counterweights (Figure 2), which leads to the uniform redistribution of masses in the manipulator [4].

Please note that all axes of revolute joints are parallel, i.e. this is a mechanism in which all points of the links describe paths located in parallel planes.

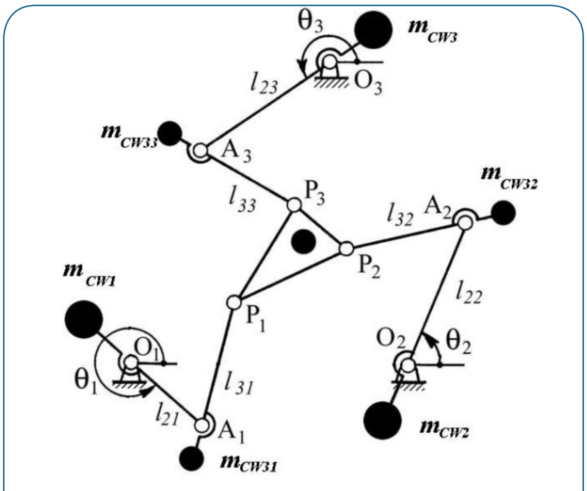

Figure 2: Fully force-balanced planar 3-RRR parallel manipulator

The complete shaking force balancing is based on the following consideration. The first counterweight added on the platform permits to substitute the mass of the moving platform by three equivalent point masses located at the points P_k $(k = 1,2,3)$ of legs. Then, each leg of the manipulator can be balanced independently by two counterweights. After such a redistribution of masses, all moving masses of the manipulator can be replaced by three fixed masses located at the axis of the fixed joints O_k $(k = 1,2,3)$. Thus, the centre of mass of the manipulator remains motionless for any motion of links and hence, the manipulator transmits no inertia loads to its base [23]. However, the added masses lead to significant incising of the moving masses and as a result, to incising of the input torques and the dynamic loads in the joints. Therefore, in the present study a partial balancing approach is proposed. It is carried out by only three counterweights having constant masses but adjustable locations.

Let us consider the proposed balancing technique.

Figure 3 shows the proposed adaptive counterweight system for optimum shaking force balancing of planar 3-RRR parallel manipulators.

The shaking force of this manipulator can be written as follows: $\mathbf{F} = \mathbf{F}^x + \mathbf{F}^y$, where $\mathbf{F}^x$ and $\mathbf{F}^y$ are the components of the shaking force relative to the fixed system of coordinates O_1xy

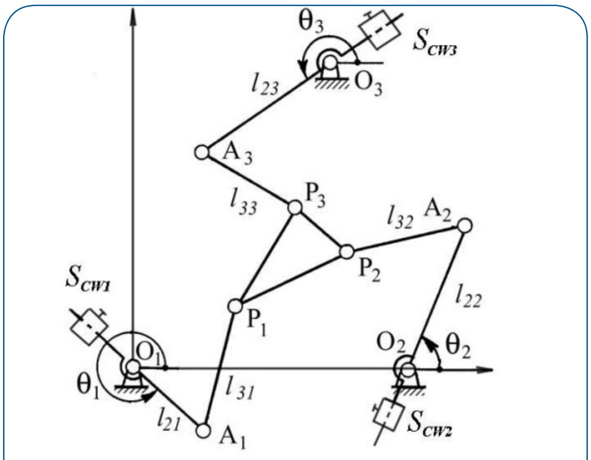

Figure 3: Partially force-balanced planar 3-RRR parallel manipulator with adaptive counterweight system

(Figure3). Let $S_{CWk} = m_{CWk} r_{CWk}$ $(k = 1,2,3)$ be static moment of the counterweight, where m_{CWk} is the mass of the counterweight mounted on the input link of the corresponding leg, and r_{CWk} is the distance of its center of mass from the fixed axes O_k.

The statement of the problem is the following: find such a selection of counterweights' parameters S_{CWi} for which the root-mean-square (RMS) values of the unbalanced force is the least, i.e.

$$\sqrt{\sum_{i=1}^{N} \left|\mathbf{F}_{CWki} + \mathbf{F}_i\right|^2 / N} \to \min_{S_{CWk}}, (k = 1,2,3) \tag{1}$$

where $\mathbf{F}_i \left(F_i^x, F_i^y\right)$ is the shaking force of the manipulator for the given trajectory of the gripper, $\mathbf{F}_{CWki}\left(F_{CWki}^x, F_{CWki}^y\right)$ is the force created by the counterweight and N is the number of calculated positions.

For the minimization of the RMS, it is necessary to minimize the sum:

$$\sum_{i=1}^{N}\left[\left(\sum_{k=1}^{3} F_{CWki}^x + F_i^x\right)^2 + \left(\sum_{k=1}^{3} F_{CWki}^y + F_i^y\right)^2\right] \to \min_{S_{CWk}} \tag{2}$$

with

$$\begin{bmatrix} F_{CWki}^x \\ F_{CWki}^y \end{bmatrix} = S_{CWk} \begin{bmatrix} a_{ki} \\ b_{ki} \end{bmatrix} \tag{3}$$

where $a_{ki} = \dot{\theta}_{ki}^2 \cos\theta_{ki} + \ddot{\theta}_{ki} \sin\theta_{ki}$, $b_{ki} = \dot{\theta}_{ki}^2 \sin\theta_{ki} - \ddot{\theta}_{ki} \cos\theta_{ki}$, θ_{ki}, $\dot{\theta}_{ki}$ and $\ddot{\theta}_{ki}$ are, respectively, the angular displacement, velocity and acceleration of input link k $(k = 1,2,3)$ for the given position i.

For minimization of the root-mean-square values of the unbalanced force, we shall achieve the conditions:

$$\frac{\partial\left(\sum_{i=1}^{N}\left[\left(\sum_{k=1}^{3} F_{CWki}^x + F_i^x\right)^2 + \left(\sum_{k=1}^{3} F_{CWki}^y + F_i^y\right)^2\right]\right)}{\partial S_{CWk}} = 0 \tag{4}$$

From which we obtain the following system of linear equations:

$$\begin{bmatrix} S_{CW1} \\ S_{CW2} \\ S_{CW3} \end{bmatrix} \begin{bmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & a_1a_3 + b_1b_3 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & a_2a_3 + b_2b_3 \\ a_1a_3 + b_1b_3 & a_2a_3 + b_2b_3 & a_3^2 + b_3^2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{5}$$

where

$$a_k = \sum_{i=1}^{N} a_{ki}, \ (k = 1,2,3) \tag{6}$$

$$b_k = \sum_{i=1}^{N} b_{ki}, \ (k = 1,2,3) \tag{7}$$

$$c_k = -\left[a_k \sum_{i=1}^{N} F_i^x + b_k \sum_{i=1}^{N} F_i^y\right], (k = 1,2,3) \tag{8}$$

Thus, from (5) we determine the static moment of counterweights:

$$S_{CW1} = \frac{\begin{vmatrix} 0.5c_1 & a_1a_2 + b_1b_2 & a_1a_3 + b_1b_3 \\ 0.5c_2 & a_2^2 + b_2^2 & a_2a_3 + b_2b_3 \\ 0.5c_3 & a_2a_3 + b_2b_3 & a_3^2 + b_3^2 \end{vmatrix}}{\begin{vmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & a_1a_3 + b_1b_3 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & a_2a_3 + b_2b_3 \\ a_1a_3 + b_1b_3 & a_2a_3 + b_2b_3 & a_3^2 + b_3^2 \end{vmatrix}} \tag{9}$$

$$S_{CW2} = \frac{\begin{vmatrix} a_1^2 + b_1^2 & 0.5c_1 & a_1a_3 + b_1b_3 \\ a_1a_2 + b_1b_2 & 0.5c_2 & a_2a_3 + b_2b_3 \\ a_1a_3 + b_1b_3 & 0.5c_3 & a_3^2 + b_3^2 \end{vmatrix}}{\begin{vmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & a_1a_3 + b_1b_3 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & a_2a_3 + b_2b_3 \\ a_1a_3 + b_1b_3 & a_2a_3 + b_2b_3 & a_3^2 + b_3^2 \end{vmatrix}} \tag{10}$$

$$S_{CW3} = \frac{\begin{vmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & 0.5c_1 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & 0.5c_2 \\ a_1a_3 + b_1b_3 & a_2a_3 + b_2b_3 & 0.5c_3 \end{vmatrix}}{\begin{vmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & a_1a_3 + b_1b_3 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & a_2a_3 + b_2b_3 \\ a_1a_3 + b_1b_3 & a_2a_3 + b_2b_3 & a_3^2 + b_3^2 \end{vmatrix}} \tag{11}$$

Then, taking into account that the masses of counterweights (m_{CWk}) are constant, we determine the locations of counterweights (r_{CWk}).

Let us consider an illustrative example in order to show the efficiency the suggested balancing approach.

Illustrative example and numerical simulations

The geometry and mass distribution parameters of the links are listed in Table 1, where X_{O_k} and Y_{O_k} are the coordinates of the fixed joints O_k $(k = 1,2,3)$; $l_{2k} = l_{O_kA_k}$ $(k = 1,2,3)$ are the lengths of the links jointed with the frame (see Figure 2); $l_{3i} = l_{A_iP_i}$

$(k=1,2,3)$ are the lengths of the links jointed with the platform (see Figure 2); x_{P_k} and y_{P_k} are the coordinates of the points P_k $(k=1,2,3)$ of the platform; m_{2k} are the masses of the links jointed with the frame; m_{3k} are the masses of the links jointed with the platform; $r_{S2k}=l_{O_kS_{2k}}$ is the distance of the centre of mass S_{2k} of the link $2k$ from the joint centre O_k, $r_{S3k}=l_{A_kS_{3k}}$ is the distance of the centre of mass S_{3k} of the link $3k$ from the joint centre A_k.

Table 1: Parameters of the manipulator modeled via ADAMS software

Parameter	Leg		
	1	2	3
X_{O_k} (m)	0	0.46	0.22
Y_{O_k} (m)	0	0	0.4
l_{2k} (m)	0.18	0.18	0.18
l_{3k} (m)	0.18	0.18	0.18
x_{P_k} (t=0) (m)	0.18	0.28	0.22
y_{P_k} (t=0) (m)	0	0	0.087
m_{2k} (kg)	2	2	2
r_{S2k} (m)	0.09	0.09	0.09
m_{3k} (kg)	1	1	1
r_{S3k} (m)	0.09	0.09	0.09

The platform of the examined manipulator is an equilateral triangle with a mass of 3kg.

The drivers are given by the expressions [24]:

$$\theta_k = a_k\pi + b_k\left(2\pi t/T - \sin\left(2\pi t/T\right)\right)$$

$(k=1,\ 2,\ 3)$, where $a_1=1/3$, $a_2=4/3$, $a_3=10/3$, $b_1=1/6$, $b_2=-1/6$, $b_3=1/12$, $T=0.3$ sec.

Thus, from equations (9) - (11), we determine the static moments of counterweights and assuming that $m_{CW1}=m_{CW2}=m_{CW3}=3kg$, we obtain $r_{CW1}=l_{O_1S_{CW1}}=0.1m$, $r_{CW2}=l_{O_2S_{CW2}}=0.09m$ and $r_{CW3}=l_{O_3S_{CW3}}=0.062m$.

The variations of shaking forces before and after balancing are given in Figures 4, 5 and 6.

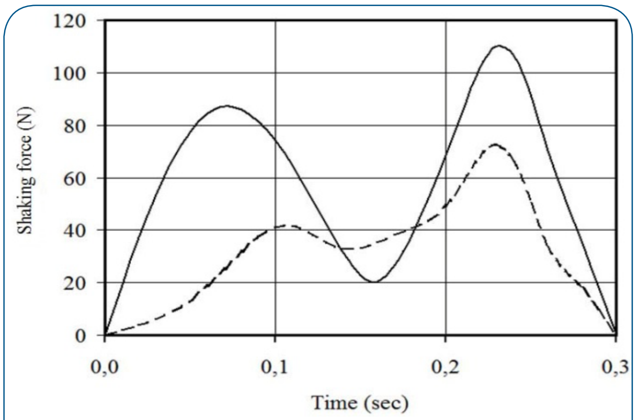

Figure 4: Shaking force of the manipulator before balancing (solid line) and after balancing (dashed line)

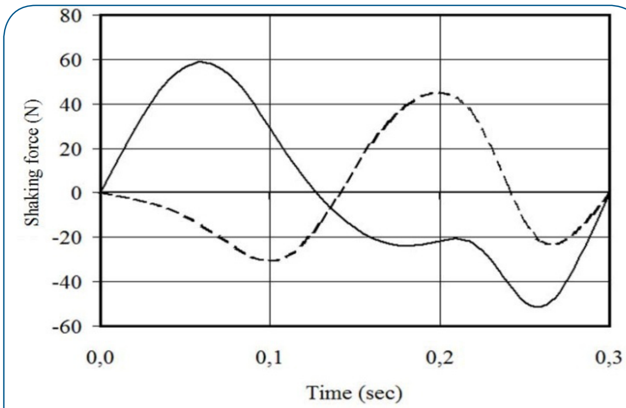

Figure 5: Shaking force along x-axis before balancing (solid line) and after balancing (dashed line)

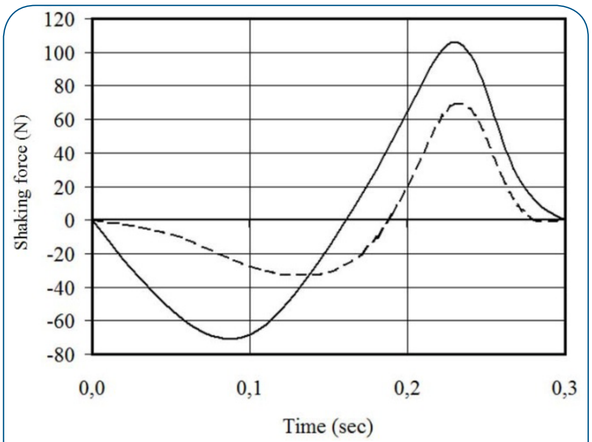

Figure 6: Shaking force along y-axis before balancing (solid line) and after balancing (dashed line)

The obtained results showed that a reduction of 35% of the shaking force has been achieved. It can seem that this is not significant since a complete shaking force balancing can provide a full cancellation of dynamic loads on the frame.

However, it is important to draw attention to the price that must be payed for complete shaking force balancing.

To balance the same manipulator via full cancellation of the shaking force, it is necessary to apply three counterweights with masses $m_{CW31}=m_{CW32}=m_{CW33}=3kg$ (when $r_{CW31}=r_{CW32}=r_{CW33}=0.09m$, see Fig. 2) and then three other counterweights with masses $m_{CW1}=m_{CW2}=m_{CW3}=12kg$

(when $r_{CW1} = r_{CW3} = r_{CW3} = 0.09m$).

Thus, we can note that the total mass of the unbalanced manipulator is 12kg and the total mass of a fully balanced manipulator is 57kg. The increase in total mass is significant which leads to the increase in input torques.

Let us carry out a comparative analysis between unbalanced, partially balanced and fully balanced manipulators (see Figures 7, 8 and 9).

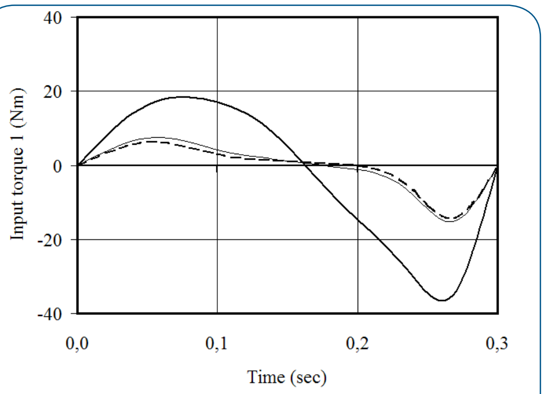

Figure 7: Torque variations of the first actuator for unbalanced (dashed line), partially balanced (slim line) and fully balanced (solid line) manipulators

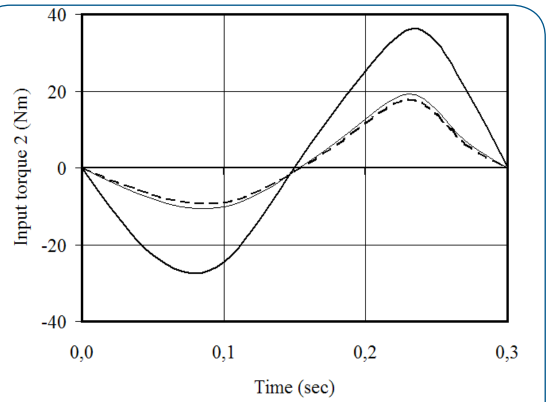

Figure 8: Torque variations of the second actuator for unbalanced (dashed line), partially balanced (slim line) and fully balanced (solid line) manipulators

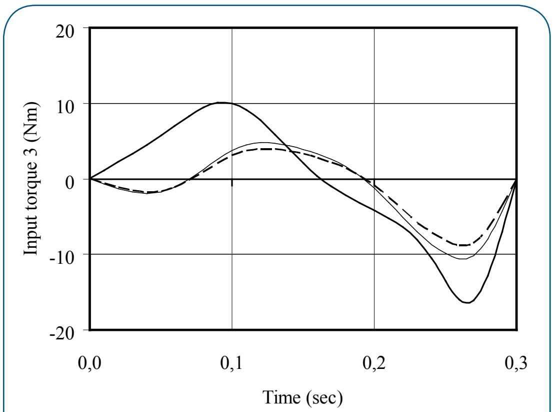

Figure 9: Torque variations of the third actuator for unbalanced (dashed line), partially balanced (slim line) and fully balanced (solid line) manipulators

The obtained results showed that in the fully balanced manipulator the input torques are almost doubled, while the increase of torques in the case of the partial balancing by means of the proposed solution is insignificant.

Conclusions

It is known that the manipulators after complete shaking force balancing become very heavy, which leads to the significant increase of input torques and dynamic loads in the joints. Therefore, in the present study, an analytically tractable solution for optimum shaking force balancing of planar 3-RRR parallel manipulators is proposed. It is carried out by only three counterweights mounted on the input links and having constant masses but adjustable locations, i.e. only the distances from fixed joints of the manipulator are adjustable. The means of the adjustment can be various. The technical solutions for variations of counterweights' locations are not examined. This can be a pneumatic system with linear displacements, a motorized lead screw or another linear driving mode. Obviously, the adding of the mechanism for adjustment of counterweights' locations will introduce some corrections taking into account the nature of the mechanical architecture. However, the masses of the added system can be easily reduced to the counterweights' masses. Then, the minimization of shaking force should be performed in the same way. The efficiency of the suggested solution has been illustrated via a numerical simulations carried out by using ADAMS software. For the examined manipulator the reduction of the shaking force was 35%, while the increase of the input toques compared to the unbalanced manipulator was insignificant. Such a solution can find a wide application in the design of fixed-sequence manipulators.

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