Generalized Lanchester balancer

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1. Introduction

The majority of linkage balancing works devoted to the crank–slider mechanisms in the past have been concentrated on axial mechanisms balancing (Arakelian et al., 2000; Arakelian and Smith, 2005a,b; Lowen et al., 1983; Dresig, 1999). They can be arranged in the following groups: (i) Balancing by counterweights attached to the links (Artobolevskii, 1968; Berkof and Lowen, 1969; Berkof, 1979; Campbell, 1979). The balancing based on the redistribution of mass of the mechanism by adding counterweights to moving links allows the immobility of the center of moving masses and the cancelling the shaking forces. It should be noted that such a balancing can only be attained by a considerable increase of the total moving mass of the mechanism; (ii) balancing using opposite movements (Arakelian, 1998; Arakelian and Smith, 1999; Artobolevskii, 1968; Davies, 1968; Dresig and Holzweißig, 2004; Filonov and Petrikovetz, 1987; Kamenski, 1968; Koropetz, 1979; Turbin et al., 1978). The addition of an axially symmetric duplicate mechanism to any given mechanism will allow the new combined center of mass to remain stationary and thus balances the shaking force. This approach involves building self-balanced mechanical systems, in which two identical mechanisms execute similar but opposite movements. For example, in the in-line 4-cylinder engine the first harmonic of the shaking forces is balanced and the mechanical system can be considered partially self-balanced. Different schemes of self-balanced crank–slider systems have been developed. However, almost all solutions are devoted to the axial mechanisms; (iii) balancing by means of added dyads (Arakelian, 1998; Arakelian and Smith, 1999; Doronin and Pospelov, 1991; Frolov, 1987). The parallelogram loop consisting of the initial links of the crank–slider mechanism and the added dyad transfer the motion of the coupler link to a shaft on the frame, where it is connected to a counterweight of considerably reduced mass (Arakelian, 1998). Partial shaking force balancing may be achieved by generating an approximate straight-line movement of a counterweight mounted on the added dyad (Doronin and Pospelov, 1991; Frolov, 1987). The crank–slider mechanisms can also be balanced by using the copying properties of the pantograph formed from the links of the initial mechanism and added links (Arakelian, 1998; Arakelian and Smith, 1999). The pantograph carries a counterweight that achieves the condition necessary for shaking force and shaking moment balancing of axial crank–slider mechanisms. In the case of off-set crank–slider mechanisms, this problem has been discussed in (Arakelian and Smith, 2005a,b).

However, one of the more efficient solution is the Lanchester balancer (Lanchester, 1914), in which the reduction of inertia effects is primarily accomplished by the balancing of certain harmonics of the shaking forces. Such an approach is used for balancing of divers linkages (Artobolevskii, 1968; Crossley, 1964; Dresig et al., 1994; Pantelic and Seculic, 1971; Shchepetilnikov, 1982; Tsai and Walter, 1984). As it is mentioned above, this solution has been
studied and successfully applied for balancing of axial slider–crank mechanisms.

In this paper the generalized Lanchester balancer is proposed. It allows the balancing of primary and secondary shaking forces of off-set slider–crank mechanisms.

2. Shaking force balancing of off-set crank-slider mechanism

Fig. 1 shows an off-set crank-slider mechanism. Let us firstly consider the kinematic analysis of off-set slider-crank mechanism and the inertia forces of the reciprocating motion.

The position of the slider can be determined by the following expression:

\[ d_B = \sqrt{(r + 1)^2 - e^2 - r \cos \varphi - l \cos \psi} \] (1)

where \( r = l_{OS} \) is the distance between the centers of joints A and O; \( l = l_{AB} \) is the distance between the centers of joints A and B; \( e = y_B \) is the eccentricity of slider guide; \( \varphi \) is the rotating angle of the input crank and \( \psi \) is the acute angle that the rod makes with the sliding axis (Fig. 1) determined from following expression:

\[ \cos \psi = (1 + p)^q \] (2)

where \( p = -(r \sin \varphi - e)/l^2 \) and \( q = 0.5 \).

This expression can be represented using Newton's binomial series as follows:

\[ (1 + p)^q = 1 + qp + \frac{q(q - 1)}{1 \cdot 2} p^2 + \frac{q(q - 1)(q - 2)}{1 \cdot 2 \cdot 3} p^3 + \ldots \] (3)

Substituting (3) into (2), which is then substituted into (1) and taking into account that it is sufficient to keep the two first terms of this series, we obtain the following expression for the position of the slider:

\[ d_B = \sqrt{(r + 1)^2 - e^2 - r \cos \varphi - l \left[ 1 - 0.5 \left( \frac{r \sin \varphi - e}{l} \right)^2 \right]} \] (4)

It should be noted that for off-set slider-crank mechanisms \( \psi \neq 90^\circ \), i.e. \( |p| < 1 \) (see Eq. (2)) and for the reasonable parameters of the mechanism’s links angle \( \psi \) is an acute angle with small value, i.e. \( |p| \ll 1 \). Therefore, we can state that

\[ 1 + qp \gg \frac{q(q - 1)}{1 \cdot 2} p^2 + \frac{q(q - 1)(q - 2)}{1 \cdot 2 \cdot 3} p^3 + \ldots \]

After differentiating this equation with respect to time, we obtain the following expressions for the velocity and acceleration of the slider:

\[ d_B = \dot{\psi}(r \sin \varphi + 0.5 \lambda r \sin 2 \varphi - \lambda e \cos \varphi) \] (5)

\[ d_B = \dot{\varphi}^2 (r \cos \varphi + \lambda r \cos 2 \varphi + \lambda e \sin \varphi) \] (6)

where \( \dot{\psi} \) is the constant velocity of the input crank and \( \lambda = r/l \).

Thus, the shaking force of the reciprocating motion can be expressed as:

\[ F_{sh} = -m d_B = F_{(1)} + F_{(1')} + F_{(2)} \] (7)

where \( m = m_3 + m_2 l_{c2}/l_{AB} \) is the reciprocating moving mass, \( m_3 \) is the slider’s mass, \( m_2 \) is the coupler link’s mass, \( l_{c2} \) is the distance between the center of the joint A and the center of the mass \( S_2 \) of the coupler link 2, \( l_{AB} \) is distance between the centers of the joints A and B,

\[ F_{(1)} = -m \dot{\psi}^2 r \cos \varphi \] (8)

\[ F_{(1')} = -m \dot{\psi}^2 \lambda e \sin \varphi \] (9)

\[ F_{(2)} = -m \dot{\psi}^2 \lambda r \cos 2\varphi \] (10)

Please note when \( e = 0 \), \( F_{(1')} = 0 \) and the terms \( F_{(1)}, F_{(2)} \) coincide with the classical solution of axial slider-crank mechanisms (Ulicker et al., 2003).

Eq. (7) can be rewritten as:

\[ F_{sh} = k_1 (\dot{\psi})^2 \cos(\varphi + \alpha) - k_2 (2\varphi)^2 \cos 2\varphi \] (11)

where

\[ \alpha = \arctan \left[ \frac{e}{l} \right] \] (12)

\[ k_1 = -\frac{m \lambda e}{\sin \alpha} \] (13)

\[ k_2 = 0.25 m \lambda r \] (14)

In the obtained expression the first term is the primary shaking force and the second is the secondary shaking force. The primary shaking force can be balanced by counterweights that rotate at the input speed but are out of phase with the input crank by angle \( \alpha \) (Fig. 2). The secondary shaking force can be balanced by counterweights that rotate at two times the input speed.

The parameters of the added counterweights are the following:

\[ 2m_{CW1} l_{CW1} = k_1 \] (15)

\[ 2m_{CW2} l_{CW2} = k_2 \] (16)

where \( m_{CW1} \) and \( m_{CW2} \) are the masses of the counterweights, \( l_{CW1} \) and \( l_{CW2} \) are the distances of the pivot centers from the centers of mass of the counterweights.

3. Numerical simulations

The off-set crank-slider mechanism \( OAB \) shown in Fig. 1 has the following parameters: \( r = 0.05 \) m, \( l = 0.2 \) m, \( e = 0.025 \) m, \( l_{OS} = 0.025 \) m (\( S_1 \) is the center of mass of the input crank), \( l_{c2} = l_{BG2} \) (\( S_2 \) is the center of mass of the coupler link 2), \( m_1 = 2 \) kg, \( m_2 = 2 \) kg, \( m_3 = 3 \) kg. Thus, for given parameters of the mechanism we obtain:

\( \alpha = 7.125^\circ, k_1 = 0.025 \) kg m, \( k_2 = 0.0125 \) kg m. Then, by selecting \( l_{CW1} = 0.0336 \) m and \( l_{CW2} = 0.0125 \) m, we determine \( m_{CW1} = 3 \) kg and \( m_{CW2} = 0.5 \) kg.

The simulation of this mechanism with obtained balancing parameters has been carried out using the software ADAMS. The shaking force variations of unbalanced and balanced mechanisms for input angular velocity \( \dot{\varphi} = 20 \pi \) s\(^{-1} \) are shown in Fig. 3.

Thus, the suggested balancing technique allows the reduction of the maximum value of the shaking force of the studied off-set slider-crank mechanism by 98%.

It should be noted, that the complete shaking force balancing of off-set crank-slider mechanisms can only be reached by a considerable increase of link masses of the mechanism.

The harmonic shaking force variations of unbalanced and balanced mechanisms.
balancing has not been applied to the off-set crank–slider mechanisms. As show the obtained results, the quasi-complete shaking force balancing has been achieved by a small increase in the total mass of mechanism.

4. Conclusion

This paper presents the generalized Lanchester balancer for shaking force balancing of off-set crank–slider mechanisms. The shaking force of the off-set slider–crank mechanism is provided by two terms: the first term is the primary shaking force and the second is the secondary shaking force. The primary shaking force is balanced by counterweights that rotate at the input speed but are out of phase with the input crank by an angle, which has been defined taking into account the eccentricity of slider guide. The secondary shaking force is balanced by counterweights that rotate at two times the input speed. The efficiency of the suggested solution is illustrated by the numerical simulations, which is carried out using the software ADAMS. The numerical example illustrates that a quasi perfect shaking force balancing (98%) has been achieved.

The author believe that the proposed study expand information about Lanchester balancer. It is based on the known constructive approaches consisting in the counter rotating shafts and various fields of industrial applications are possible.

References


