Vigen H. Arakelian
Professor,
INSA—Rennes, Department "GMA",
20 Avenue des Buttes de Coesmes, CS-14315,
F-35043 Rennes Cedex, France
e-mail: vigen.arakelyan@insa-rennes.fr
Mem. ASME

M. R. Smith[1]
Department of Mechanical, Materials, and
Manufacturing Engineering,
University of Newcastle,
Newcastle-upon-Tyne,
NE1 7RU United Kingdom

Shaking Force and Shaking Moment Balancing of Mechanisms: A Historical Review With New Examples

This paper deals with the solutions of the problem of the shaking force and shaking moment balancing of planar mechanisms by different methods based on the generation of the movements of counterweights. Some special cases are examined, such as balancing methods based on the copying properties of pantograph systems that carry the counterweights (formed by gears or by toothed-belt transmissions). The pantograph system executes a movement exactly opposite to the movement of the total center of the movable link masses. Such a solution provides the conditions for dynamic balancing with a relatively small increase of the total mass of the movable links. The methods are illustrated by many examples. [DOI: 10.1115/1.1829067]

Keywords: *Dynamics of Mechanisms, Balancing, Vibration, Shaking Force, Shaking Moment*

1 Introduction

In high-speed machines, mass balancing of the moving links brings about a reduction of the variable dynamic loads on the frame and, as a result, a reduction of vibrations. This considerably minimizes the noise, wear and fatigue characteristics of machines, and improves their performance. Many high-speed machines contain planar mechanisms and the problem of their mass balancing is of continuing interest to machine designers. Different approaches and solutions have been developed and documented [1] but, despite its long history, mechanism balancing theory continues to develop and new approaches and solutions are constantly being reported.

A review of the balancing methods based on the different movements of the counterweights is presented later. Previous work on this problem may be arranged in the following groups (Table 1).

Balancing by counterweights mounted on the movable links of the mechanism. The balancing methods based on the redistribution of mass of the mechanism by adding counterweights to links are well known[2]. Some schemes for such balancing are shown in Table 1. In the case of complete shaking force balancing this approach is generally limited to simple mechanisms having only revolute joints. In particular, it is difficult to apply to mechanisms with a slider because the conditions for complete shaking force balancing bring about a serious increase in the total mass of the balanced mechanism.

Harmonic balancing by two counter-rotating masses. These solutions are based on harmonic analysis. The reduction of inertia effects is primarily accomplished by the balancing of certain harmonics of the shaking forces and shaking moments. Unbalanced forces and moments are approximated by Fourier series (or Gaussian least-square formulation) and then each frequency component is studied. This solution has found wide application as it may be accomplished by attaching balancing elements to the crank (Table 1).

This approach has been used successfully for engine balancing [11,12]. For example the balancing shafts (Fig. 1) are used for balancing the second harmonic of the shaking force of an in-line four-cylinder engine.

Tsai and Walter [13] have suggested balancing the second harmonic of shaking force and shaking moment by using an Oldham-coupling balancer. The advantage of such an approach is that the balancer mechanism turns with the same velocity as the initial unbalanced system.

Balancing by opposite movements (Table 1). The addition of an axially symmetric duplicate mechanism to any given mechanism will make the new combined center of mass stationary (Fig. 2) and thus balances the shaking force. This approach involves building self-balanced mechanical systems, in which two identical mechanisms execute similar but opposite movements. In this case the shaking force is cancelled together with the shaking moment [15]. A partial balancing is also possible by this approach. For example, in the in-line four-cylinder engine only the first harmonic of the shaking forces is balanced and the second harmonic is eliminated by the previous approach.

Kamenski [18] first used the cam mechanism for balancing of linkages. In his work the reduction of inertia forces was performed by means of a cam carrying a counterweight and it was shown how cam-driven masses may be used to keep the total center of mass of a mechanism stationary.

Balancing by added dyads (Table 1). The parallelogram loop consisting of the initial links of the mechanism and the added dyad transfers the motion of the coupler link to a shaft on the frame, where it is connected to a counterweight of considerably reduced mass [15].

In Ref. [5], Shchepetilnikov suggested minimizing the imbalance of the inertia force moment by transferring the rotation axis of the counterweight mounted on the input crank. In his work the first harmonic of the shaking moment is eliminated by attaching the required input link counterweight, not to the input shaft itself, but to one suitably offset that rotates with the same angular velocity. This approach is original in that, while maintaining the static balance of the mechanism, it is possible to create an additional balancing moment, thereby reducing the shaking moment. This approach was developed in Refs. [20], [21], where the conditions for balancing are formulated by the minimization of the root-mean-square value of the shaking moment.

[1]Retired.
[2]More information about these methods can be found in Ref. [1].

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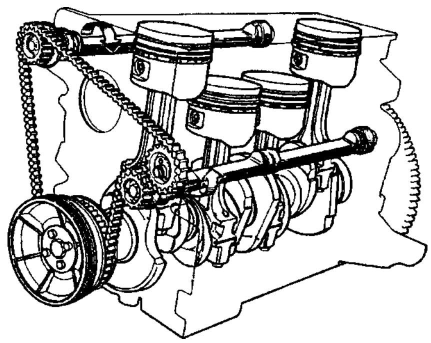

Fig. 1 Balancing shaft arrangement for an in-line four-cylinder engine

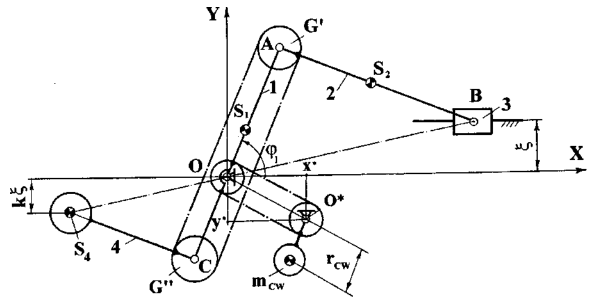

Fig. 3 Complete shaking force and partial shaking moment balancing

Partial balancing may be achieved by the generation of an approximate straight-line movement of a counterweight mounted on the added dyad [22,23].

Among several works may be distinguished also the study of Hilpert [25] in which a pantograph mechanism is used for the displacement of the counterweight. The symbiosis of this approach and balancing by added dyads allows the development of a new method. The aim of this approach is to balance the mechanism by using the copying properties of the pantograph formed from the links of the initial mechanism and added links. The pantograph carries a counterweight that achieves the condition necessary for shaking force and shaking moment balancing. This approach may be easily applied to the balancing of multibar mechanisms since many complex mechanical systems are based on the simple four bar linkage.

Some applications of such a balancing method are now examined.

2 Shaking Force and Shaking Moment Balancing of Slider-Crank Mechanisms Based on the Copying Properties of the Pantograph

2.1 Complete Shaking Force and Partial Shaking Moment Balancing of Off-Set Slider Crank Mechanisms

2.1.1 Shaking Force Balancing Using the Copying Properties of the Pantograph. Figure 3 shows an off-set slider-crank mechanism with a toothed-belt transmission. The two gears (G' and G'') are carried on the crank 1. The gear G' is connected to the coupler link 2 and gear G'' is connected to the counterweight 4. This system may be called a pantograph transmission because it has the same characteristics as a pantograph.

The principle of the shaking force balancing is as follows. By selecting, for constructional reasons, the magnification factor of the pantograph as $k=l_{OA}/l_{OC}$ and by static substitution of the mass m_2 of the coupler by point masses m_A and m_B, we determine the mass of the counterweight 4

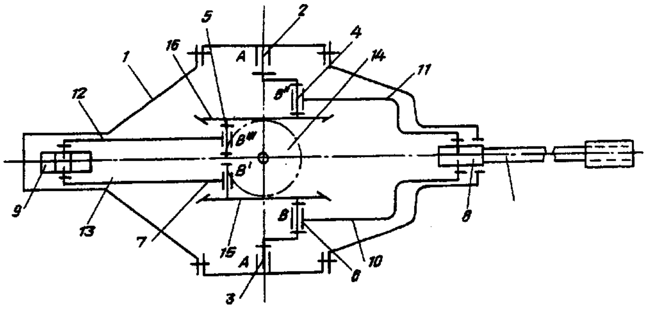

Fig. 2 Self-balanced slider-crank system [19]

$$m_4=k(m_2 l_{AS_2}/l_{AB}+m_3) \tag{1}$$

where m_2 and m_3 are the masses of the coupler link 2 and the slider 3, l_{AS_2} is the distance of the joint center A from the center of mass S_2 of the coupler link and l_{AB} is the distance between the centers of joints A and B. (A similar notation is used throughout this paper.)

Thus, the center of mass of counterweight 4 executes a motion that is similar but opposite to the motion of slider 3. This allows balancing of the slider mass by a counterweight with a similar translating motion. The advantage of this balancing scheme is that there is no additional sliding (prismatic) pair for realization of the translation motion and the increase in the total mass of movable links is relatively small.

After the cancellation of the inertia forces of slider 3, the balancing of crank 1, taking into account the mass $m_2(1-l_{AS_2}/l_{AB})$ of coupler link 2, presents no difficulty and can be carried out by a counterweight connected to the crank with a static moment given by

$$m_{CW_1}r_{CW_1}=(m_1 l_{OS_1}+m_2 l_{OA}(1-l_{AS_2}/l_{AB})) \tag{2}$$

where m_{CW_1} is the mass of the counterweight, r_{CW_1} is the distance of the joint center O from the center of mass S_{CW_1} of the counterweight, and m_1 is the mass of the crank 1.

Thus, the shaking force balancing of the slider-crank mechanism is achieved.

2.1.2 Partial Shaking Moment Balancing. After this redistribution of mass and assuming that the crank has a constant input angular velocity, the moment of the inertia forces will be represented by the following expression

$$M^{\text{int}}=-(I_{S_2}+I_{S_4}-m_2 l_{AS_2} l_{BS_2})(\ddot{\varphi}_2)^2 \pm 2\xi(\ddot{x}_B)(m_2 l_{AS_2}/l_{AB}+m_3) \tag{3}$$

where I_{S_2} is the moment of inertia of the coupler link 2, I_{S_4} is the moment of inertia of counterweight 4, $\ddot{\varphi}_2$ is the angular acceleration of the coupler link 2, ξ is the eccentricity of slider guides, and $\ddot{x}_B$ is the linear acceleration of the slider 3.

It should be noted that the product $m_2 l_{AS_2} l_{BS_2}$ must be added because of the static substitution of the coupler mass by point masses.

The shaking moment M^{int} may be balanced by a counterweight on crank 1. By a parallel displacement of the axis of rotation of this counterweight (see Fig. 3) from center O to center $O^*(x^*,y^*)$, the balancing of the shaking force of the slider-crank mechanism will be maintained but a supplementary moment is produced [5]

$$M^*=m_{CW}r_{CW}(\dot{\varphi}_1)^2(x^*\sin\varphi_1-y^*\cos\varphi_1) \tag{4}$$

where φ_1 and $\dot{\varphi}_1$ are the angle of rotation and the angular velocity of crank 1, respectively.

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The balancing of the shaking moment M^{int} by this supplementary moment may be achieved by using the approximation method [20,21] based on the minimization of the root-mean-square (rms) value

$$\text{rms} = \sqrt{\sum_{i=1}^{N} (M_i^{\text{int}} + M_i^*)^2 / N} \tag{5}$$

where N is the number of calculated positions of the slider-mechanism in one cycle.

For the minimization of the rms, it is necessary to minimize the sum

$$\Delta = \sum_{i=1}^{N} (M_i^{\text{int}} + M_i^*)^2 \rightarrow \min_{x^*, y^*} \tag{6}$$

For this purpose, we shall impose the conditions

$$\partial\Delta/\partial x^* = 0 \quad \text{and} \quad \partial\Delta/\partial y^* = 0 \tag{7}$$

from which we obtain

$$m_{CW} r_{CW} (\dot{\varphi}_1)^2 \left(y^* \sum_{i=1}^{N} \cos^2(\varphi_1)_i - x^* \sum_{i=1}^{N} \sin(\varphi_1)_i \cos(\varphi_1)_i \right) = \sum_{i=1}^{N} M_i^{\text{int}} \cos(\varphi_1)_i \tag{8}$$

$$m_{CW} r_{CW} (\dot{\varphi}_1)^2 \left(x^* \sum_{i=1}^{N} \sin^2(\varphi_1)_i - y^* \sum_{i=1}^{N} \sin(\varphi_1)_i \cos(\varphi_1)_i \right) = -\sum_{i=1}^{N} M_i^{\text{int}} \sin(\varphi_1)_i \tag{9}$$

From these expressions and taking into account the condition $\Sigma_{i=1}^{N} \sin(\varphi_1)_i \cos(\varphi_1)_i = 0$ for $\varphi \in [0; 2\pi]$, we determine the coordinates of the rotation axis O^*

$$x^* = -\sum_{i=1}^{N} M_i^{\text{int}} \sin(\varphi_1)_i \Big/ m_{CW} r_{CW} (\dot{\varphi}_1)^2 \sum_{i=1}^{N} \sin^2(\varphi_1)_i \tag{10}$$

$$y^* = \sum_{i=1}^{N} M_i^{\text{int}} \cos(\varphi_1)_i \Big/ m_{CW} r_{CW} (\dot{\varphi}_1)^2 \sum_{i=1}^{N} \cos^2(\varphi_1)_i \tag{11}$$

Numerical example. Table 2 shows the shaking force and shaking moment variations of a slider-crank mechanism with initial parameters: $\ddot{\varphi}_1 = 20.94\ \text{s}^{-1}$ (200 rev/min), $l_{OA} = 0.2$ m, $l_{AB} = 0.3$ m, $m_1 = 2$ kg, $m_2 = 2.2$ kg, $m_3 = 4.5$ kg, $I_{S_2} = 0.02\ \text{kg m}^2$.

The individual graphs are for the following states of slider-crank mechanisms: unbalanced, shaking force balanced, and shaking moment minimization of fully force balanced by the suggested method ($x^* = -0.01$ m, $y^* = -0.22$ m).

2.2 Complete Shaking Force and Shaking Moment Balancing of In-Line Slider-Crank Mechanisms

2.2.1 Shaking Force Balancing Using the Copying Properties of the Pantograph. Figure 4 shows an in-line slider-crank mechanism with gear transmissions. The three gears (G_2', G_4'', and G_O) are carried on crank 1. The gear G_2' is connected to the coupler link 2 and meshes with gear G_O mounted on the rotation axis of the crank 1. The gear G_4'' is connected to counterweight 4 and also meshes with gear G_O. The resulting mechanism is a pantograph system with well-known copying properties.

The principle of the shaking force balancing is as follows. By selecting, the magnification factor of the pantograph as $k = 1$ ($l_{OA} = l_{OC}$, $r_2' = r_4''$) and by static substitution of the mass m_2 of the coupler by point masses m_A and m_B, we determine the parameters of the counterweight 4

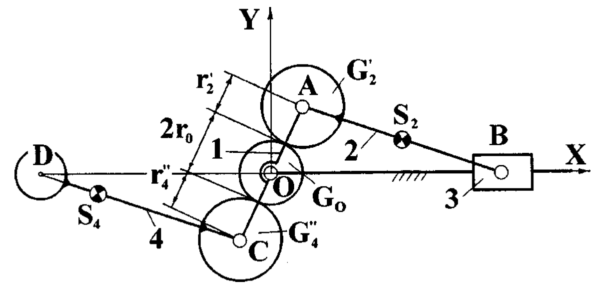

Fig. 4 Complete shaking force and shaking moment balancing

$$l_{CS_4} = (m_2 l_{AS_2} + m_3 l_{AB}) / (m_2 + m_3) \tag{12}$$

$$m_4 = m_2 l_{BS_2} / (l_{CD} - l_{CS_4}) \tag{13}$$

where l_{CS_4} is the distance of the joint center C from the center of mass S_4 of counterweight 4, m_4 is the mass of counterweight 4, and $l_{BS_2} = l_{AB} - l_{AS_2}$; $l_{CD} = l_{AB}$ (since $k = 1$). Thus, complete shaking force balancing is achieved by means of counterweight 4 and the resulting mechanism is a self-balanced force system.

2.2.2 Complete Shaking Moment Balancing. After this redistribution of mass and assuming that the crank has a constant input angular velocity, the shaking moment will be given by the following expression

$$M^{\text{int}} = -(I_{S_2} + I_{S_4} - m_2 l_{AS_2} l_{BS_2})(\ddot{\varphi}_2)^2 \tag{14}$$

which may be balanced by the moment of inertia of gear G_O taking into account the condition $\ddot{\varphi}_O = -u\ddot{\varphi}_2$, where u is the transmission ratio of the gear train [3].

3 Complete Shaking Force and Shaking Moment Balancing of the Four-Bar Linkage Based on the Copying Properties of the Pantograph

The four-bar linkage is a common element in high-speed machines. The methods for balancing [1] can be arranged into three classes: Complete shaking force balancing [6,18,25]; Complete shaking force and shaking moment balancing [26–31]; Optimum shaking force and shaking moment balancing [32,33].

The principal schemes for complete shaking force balancing of four-bar linkages are presented in Fig. 5[3]. In Fig. 5(*a*) shaking force balancing is achieved by two counterweights connected to crank 2 and to rocker 4 [6]. An alternative method that is less frequently applied is shown in Fig. 5(*b*) [6].

The parallelogram structure has been successfully applied for shaking force balancing of the four-bar linkage [Fig. 5(*c*)] [24]. The parallelogram loop ABEF transfers the motion of a coupler link to a shaft on the frame where two counterweights balance the masses of the moving links and the resultant force transmitted to the frame is zero.

Figure 5(*d*) shows the balancing of a four-bar linkage by means of a pantograph mechanism which copies the motion of the total mass center of the four-bar linkage and balances the shaking force [25].

Let us consider a solution to the problem of complete shaking force and shaking moment balancing of four-bar linkages based on the copying properties of the pantograph. This approach has

[3]In Fig. 5 the self-balanced scheme using two opposed four-bar linkages [18] is not included.

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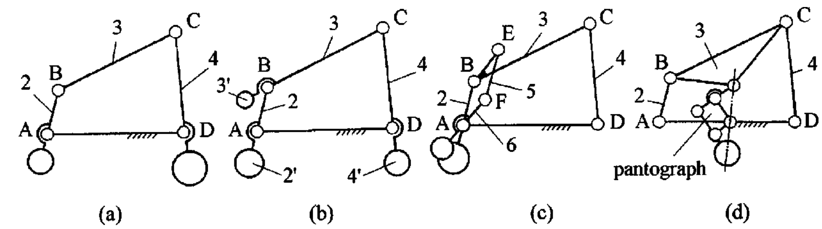

Fig. 5 Principal schemes for complete shaking force balancing of four-bar linkages

been used previously [15,27] and in the first part of the present paper for the balancing of slider-crank mechanisms. Here it is applied for the balancing of four-bar linkages.

The difference between the method developed by Hilpert [25] and the present suggested solution lies in the manner of the formation of the pantograph system. In Hilpert's method [Fig. 5(*d*)] the pantograph is separated from the four-bar linkage and forms an independent unit allowing only shaking forces to be balanced. In the present solution, however, the pantograph is composed of an additional dyad and the existing links of the four-bar linkage. This solution is simpler in design and easily applicable. Furthermore, it permits not only complete force balancing but also a complete shaking moment balancing. It will now be examined in detail.

3.1 Complete Shaking Force Balancing. Figure 6(*a*) shows a four-bar linkage ABCD and an articulation dyad EFG connected to it. This dyad forms a pantograph with the links of the initial mechanism.

The conditions for balancing this system are determined as follows. By static substitution of masses m_2, m_4, and m_5 by point masses at the centers A and B, C and D, E and F, we obtain a mechanical system model in which there are only two concentrated masses

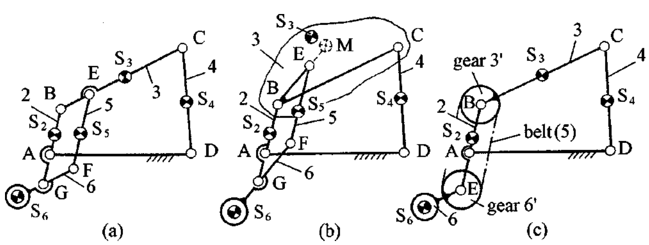

Fig. 6 Shaking force balancing based on the copying properties of the pantograph

$$M = m_2 l_{AS_2}/l_{AB} + m_3 + m_4 l_{DS_4}/l_{CD} + m_5 l_{FS_5}/l_{EF}$$

$$\text{and } M^* = m_5 l_{ES_5}/l_{EF} + m_6 \tag{15}$$

where m_i ($i=2,\ldots,6$) are the masses of the corresponding links.

Such a mechanical model will be balanced if the values of these masses are determined from the equation $M/M^*=k$, where $k = l_{AG}/l_{AB}$ is the magnification coefficient of the pantograph.

Thus, for prescribed parameters of links 2–5, the parameters of link 6 which ensure the conditions for complete shaking force balancing are the following

$$m_6 = M/k - m_5 l_{ES_5}/l_{EF}$$

$$\text{and } l_{FS_6} = (m_2 l_{AS_2} l_{BC}/l_{AB} + m_4 l_{DS_4} l_{BE}/l_{CD} + m_3 l_{BS_3} + M^* l_{FG})/m_6 \tag{16}$$

The balancing of a four-bar linkage with an asymmetrical coupler is shown in Fig. 6(*b*). In this case the disposition of the point E is changed because the center of mass of the coupler 3 is not on the line BC and as a consequence the concentrated mass M determined by the expression (15) is also not on the line BC.

The application of a pantograph with parallel links is complicated if in the mechanical system there are links that rotate completely about their axes. In such a case it is more reasonable to apply another type of pantograph system. Figure 6(*c*) shows a four-bar linkage with a toothed-belt transmission as a possible solution. The two gears (3′ and 6′) are located on the crank 2. The gear 3′ is connected to the coupler link 3 and gear 6′ is connected to counterweight 6. This system has the same properties as the pantograph mechanism shown in Fig. 6(*a*).

3.2 Complete Shaking Force and Shaking Moment Balancing. The principal schemes for complete shaking force and shaking moment balancing of four-bar linkages are presented in

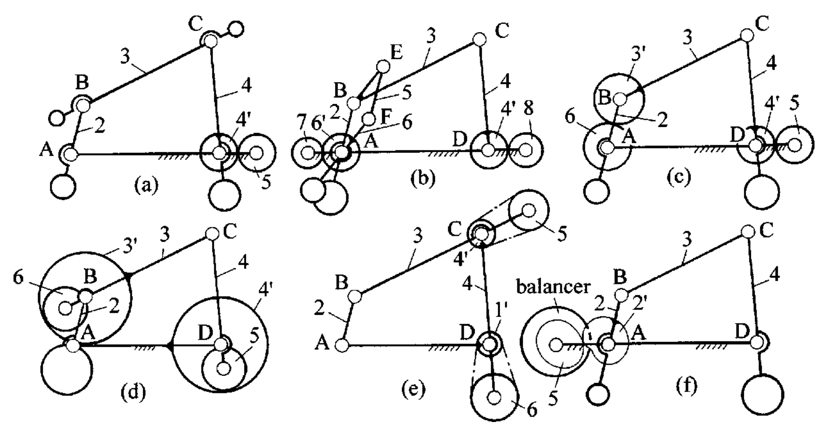

Fig. 7 Principal schemes for complete shaking force and shaking moment balancing of four-bar linkages

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Fig. 7[4]. In Berkof's approach [29] [Fig. 7(*a*)] the mass of the connecting coupler 3 is substituted dynamically by concentrated masses located at joints B and C. Thus, the dynamic model of the coupler represents a weightless link with two concentrated masses. This allows for the transformation of the problem of four-bar linkage dynamic balancing (shaking force and shaking moment) into a problem of balancing rotating links carrying concentrated masses.

The parallelogram structure [Fig. 7(*b*)] has also been applied for complete shaking force and shaking moment balancing of four-bar linkages [24].

Ye and Smith [26], Feng [30], and Berestov [31] [Figs. 7(*c*) and 7(*d*)] have proposed methods for complete shaking force and shaking moment balancing by counterweights with planetary gear trains. Esat and Bahai [34] [Fig. 7(*e*)] used a toothed-belt transmission to rotate counterweights 5 and 6 intended for shaking force balancing which also allowed shaking moment balancing.

Another approach applied by Kochev [28] [Fig. 7(*f*)] was to balance shaking moment (in the force balanced mechanism) by a prescribed input speed fluctuation achieved by noncircular gears or by a microprocessor speed-controlled motor.

In practice all the known methods for complete shaking force and shaking moment balancing of four-bar linkages face serious technical problems. The schemes presented in Figs. 7(*a*)–7(*d*) have a common disadvantage which is the connection of gears to the rocker. The resulting oscillations of the rocker create considerable noise unless expensive antibacklash gears are used. Thus, in high-speed systems it is inadvisable to use gears connected to oscillating links. In the solution presented in Fig. 7(*e*), this problem is solved partially by the use of toothed-belt transmission but the oscillations still cause serious technical problems. The method of active balancing [Fig. 7(*f*)] always presents great engineering difficulty requiring the development of a special type of driver-generator to achieve the prescribed input speed fluctuation.

Later is discussed the complete shaking force and shaking moment balancing of four-bar linkages based on the copying properties of the pantograph. Such a solution is applicable to high-speed machinery.

Figure 8(*a*) represents the suggested balancing scheme. The balancing conditions will be determined from the following considerations. With the dynamic substitution [27,29] of the mass m_4 of the rocker 4 by two concentrated masses m_C and m_D located at the centers of joints C and D, we obtain a dynamic model of the rocker that represents a weightless link with two point masses. For that it is sufficient to connect a counterweight to the rocker 4. Then, by using the copying properties of the pantograph, we balance the shaking force. In this case the pantograph system is composed in the following manner. The three gears (3′, 5, and 6′) are mounted on crank 2. The gear 3′ is connected to coupler link 3 and it meshes with gear 5 mounted on the rotation axis of crank 2. The gear 6′ is connected to the counterweight 6 and also meshes with gear 5. The resulting mechanical system is a pantograph with well-known copying properties. We shall not deal with the shaking force balancing of this system as it is similar to the preceding case (see balancing scheme of Fig. 8).

Using this redistribution of mass and assuming that crank 2 has a constant input angular velocity, the shaking moment of the force-balanced mechanism may be represented by the expression: $M^{\text{int}}=\Sigma_{i=3}^{6}K_i\ddot{\varphi}_i$ [29], where $K_i\ (i=3,\ldots,6)$ are the coefficients depending only on the mass distribution of the corresponding link and $\ddot{\varphi}_i\ (i=3,\ldots,6)$ are the angular accelerations of the corresponding links.

In this case, the rocker 4 represents a weightless link with two point masses m_C and m_D. The first concentrated mass m_D is stationary and the second concentrated mass m_C may be considered as a point mass located on the coupler 3. Thus, the shaking moment may be balanced by the moment of inertia of gear 5 taking into account the condition $\ddot{\varphi}_5=u\ddot{\varphi}_3\ (\ddot{\varphi}_3=\ddot{\varphi}_6)$, where u is the transmission ratio of the gear train [26].

[4]The schemes presented in Fig. 7 are simplified by assuming that the input speed is constant. Thus, the gears for shaking moment balancing of the input crank are eliminated.

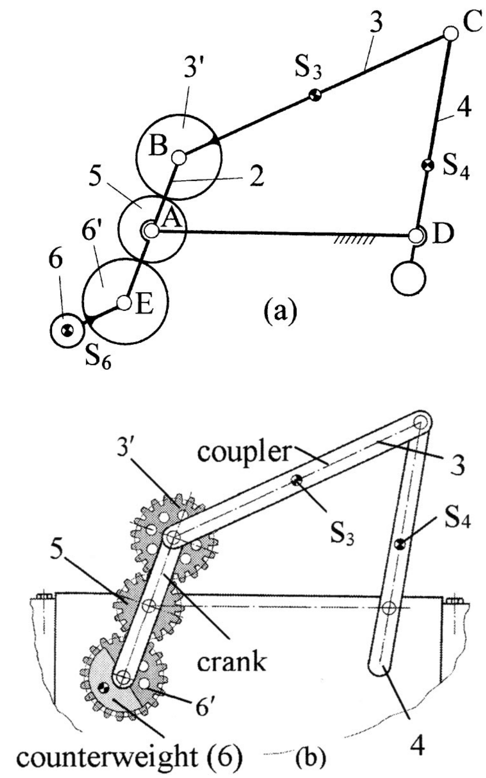

Fig. 8 Suggested balancing scheme

Figure 8(*b*) represents one version of a practical design for the suggested balancing scheme. It should be noted that for the rocker, which is in rod form, it is enough to extend it by a length $l_{CD}/2$. This extension allows the accurate dynamic substitution of the rocker mass by two concentrated masses located at the centres of joints C and D.

4 Conclusions

This paper presents a review of the shaking force and shaking moment balancing methods based on the generation of different movements of counterweights. The balancing of different mechanisms is demonstrated using the pantograph copying properties. The advantage of such a balancing method is to achieve the conditions of complete shaking force and shaking moment balancing of the mechanism with a relatively small increase of the total mass of the movable links. In the case of four-bar linkage balancing, the dynamic substitution of the rocker mass by two concentrated masses is used. The principal advantage of such a balancing method is the elimination of balancing gears connected to the rocker. Oscillating gears driven by the rocker can cause serious vibration of the mechanism and shock loading between the teeth. In the proposed solution, however, the gears driven by the coupler suffer no such sudden reversals so that this problem is almost eliminated and a favorable reduction in shock and vibration is achieved. This very important practical aspect makes possible the application of this balancing approach without serious technical problems. It should be noted that the proposed solution for complete shaking force and shaking moment balancing of four-bar linkage has been patented [35].

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