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Design of Scotch yoke mechanisms with improved driving dynamics

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Abstract

Input torque balancing through addition of an auxiliary mechanism is a well-known way to improve the dynamic behavior of mechanisms. One of the more efficient methods used to solve this problem is creating a cam-spring mechanism. However, the use of a cam mechanism is not always possible or desirable because of the wear effect due to the contact stresses and high friction between the roller and the cam. The Scotch yoke mechanism is most commonly used in control valve actuators in high-pressure oil and gas pipelines, as well as in various internal combustion engines, such as the Bourke engine, SyTech engine and many hot air engines and steam engines. This mechanism does not create lateral forces on the piston. Therefore, the main advantages of applications include reducing friction, vibration and piston wear, as well as smaller engine dimensions. However, the input torque of the Scotch yoke mechanism is variable and can be balanced. This paper proposes to balance the input torque of Scotch yoke mechanisms without any auxiliary linkage just by adding linear springs to the output slider. It is shown that after cancellation of inertial effects the input torque due to friction in joints becomes constant, which facilitates the control of the mechanism. An optimal control is considered to improve the operation of balanced Scotch yoke mechanisms. The efficiency of the suggested technique is illustrated via simulations carried out by using ADAMS software.

Keywords

Scotch yoke mechanism, input torque, balancing, optimal control, dynamics

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Introduction

It is obvious that whatever the power of control, even today, one cannot correctly operate a machine with poor mechanics. If the input torque, that is, the torque ensuring the constant speed, is highly variable, the resulting drive speed fluctuation will be substantial. Therefore, highly variable input torques might excite torsional vibration, while input torques with frequent sign changes present a very unfavorable loading case for the gears that are possibly present between the mechanism and its driving actuator.

This paper provides a simple and efficient input torque balancing method, which can be applied to Scotch yoke mechanisms. The Scotch yoke mechanism is subject to a wide range of applications and various publications have been devoted to its study.[1–5] This mechanism is most commonly used in control valve actuators in high-pressure oil and gas pipelines, as well as in various internal combustion engines, such as the Bourke engine, SyTech engine and many hot air engines and steam engines. It is also used in testing machines to simulate vibrations having simple harmonic motion.[6] The Scotch yoke mechanism does not create lateral forces on the piston. Therefore, the main advantages of applications include reducing friction, vibration and piston wear, as well as smaller engine dimensions.

The analysis of a Scotch yoke mechanism shows that its input torque is highly variable. The input torque may be reduced by optimal redistribution of moving masses.[7–11] or by using non-circular gears.[12] One of the more efficient methods used to solve the problem of input torque balancing is creating a cam-spring mechanism, in which the spring is used to absorb the energy from the system when the torque is low, and release energy to the system when the required torque is high. It allows reducing the fluctuation of the periodic torque in the high-speed mechanical systems.[13–21]

The input torque balancing technique proposed in this paper is achieved by adding linear springs.

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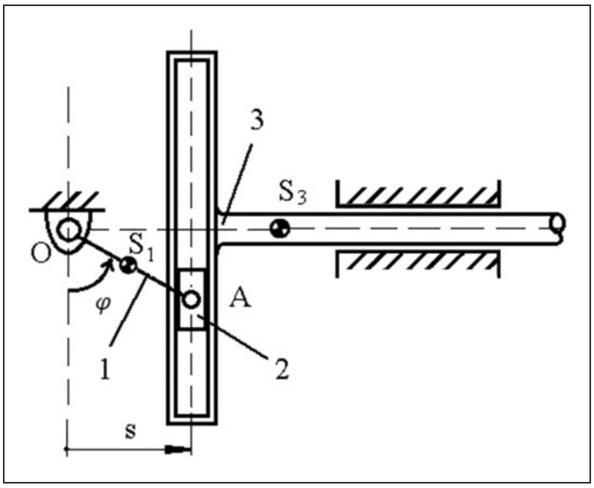

Figure 1. Scotch yoke mechanism.

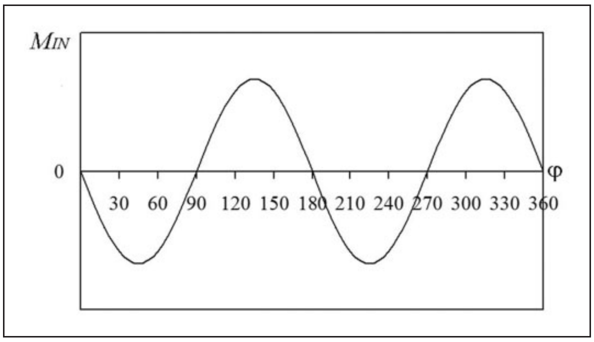

Figure 2. Input torque of a Scotch yoke mechanism.

Input torque of a Scotch yoke mechanism

The Scotch yoke mechanism is a reciprocating motion mechanism, converting the linear motion of a slider into rotational motion of a crank or vice versa (Figure 1). In the present study, it has been considered that the gravitational forces are perpendicular to the motion plane.

As is mentioned by Berkof,[7] the input torque of a single degree of freedom mechanism due to inertial effects can be found from equation

$$M_{IN} = \frac{1}{\dot{\varphi}}\frac{\mathrm{d}T}{dt} \tag{1}$$

where T is the total kinetic energy of the mechanism and $\dot{\varphi}$ is the input angular velocity.

The relationship between the rotation of link 1 and the translation of link 3 can be written as

$$s = l_{OA}\sin\varphi \tag{2}$$

where φ is the rotating angle of link 1; l_{OA} is the length of link 1, i.e. the distance between the joints O and A; s is the translational displacement of slier 3.

The slider velocity can be found by differentiating equation (2)

$$\dot{s} = l_{OA}\dot{\varphi}\cos\varphi \tag{3}$$

Considering that the input angular velocity is constant and differentiating equation (3), the slider acceleration can be written as

$$\ddot{s} = -l_{OA}(\dot{\varphi})^2\sin\varphi \tag{4}$$

The kinetic energy of the mechanism can be written as

$$T = 0.5(\dot{\varphi})^2\left(I_{S1} + m_1 r_{S1}^2 + m_2 l_{OA}^2 + m_3 l_{OA}^2\cos^2\varphi\right) \tag{5}$$

where I_{S1} is the axial inertia moment of link 1; m_i are the masses of the corresponding links ($i = 1, 2, 3$); r_{S1} is the distance between the centre of the joint O and the centre of mass S_1 of link 1.

Substituting equation (5) into equation (1), the input torque of the mechanism is

$$M_{IN} = -0.5 m_3 l_{OA}^2(\dot{\varphi})^2\sin 2\varphi \tag{6}$$

The obtained result shows that the input torque of a Scotch yoke mechanism varies according to $\sin 2\varphi$ (Figure 2).

It means that the average value of the input torque is equal to zero, and the correction moment created by the spring system should be similar to the input torque of the mechanism. Thus, in balancing the system for the periods $\varphi \in [0; \pi/2]$ and $\varphi \in [\pi; 3\pi/2]$, the spring must to absorb and accumulate the energy from the Scotch yoke mechanism because the input torque is low. With regard to the periods $\varphi \in [\pi/2; \pi]$ and $\varphi \in [3\pi/2; 2\pi]$, the spring should release energy to the Scotch yoke mechanism because the required torque is high.

It should be noted once again that in the present paper, the input torque due to inertial effects is considered. In the case of the presence of combustion forces, the input torque balancing will be different. For the case of engines please see combustion-induced torque variation in literature.[22]

Input torque balancing

The spring system should ensure the following condition

$$F_{sp}dx + M_{IN}\mathrm{d}\varphi = 0 \tag{7}$$

where $F_{sp} = kx$ is the elastic force of the spring; k is the stiffness coefficient of the spring, x is the displacement of the spring.

It should be noted that F_{sp} has a minus sign during the accumulation of energy and a plus sign during the restitution of energy.

For the period of the accumulation of potential energy

$$A_{accum} = -\int_0^{\varphi} M_{IN} \mathrm{d}\varphi = \int_0^{\varphi} 0.5 m_3 l_{OA}^2 (\dot{\varphi})^2 \sin 2\varphi \mathrm{d}\varphi = 0.5 m_3 l_{OA}^2 (\dot{\varphi})^2 \sin^2\varphi \quad (8)$$

The maximal value of the accumulate potential energy for both periods mentioned above is

$$A_{\max} = -\int_0^{\pi/2} M_{IN} \mathrm{d}\varphi = \int_0^{\pi/2} 0.5 m_3 l_{OA}^2 (\dot{\varphi})^2 \sin 2\varphi \mathrm{d}\varphi = 0.5 m_3 l_{OA}^2 (\dot{\varphi})^2 \quad (9)$$

Integrating equation (7) for the period of energy accumulation, the following relationship can be obtained

$$A_{accum} = -\int_0^{\varphi} M_{IN} \mathrm{d}\varphi = -\int_0^{x} kx\,\mathrm{d}x \quad (10)$$

and

$$A_{accum} = 0.5kx^2 \quad (11)$$

From which

$$x = \sqrt{\frac{2}{k} A_{accum}} \quad (12)$$

when $x = x_{\max}$, the accumulation of energy becomes maximum

$$A_{\max} = 0.5kx_{\max}^2 \quad (13)$$

Now, from equation (9) and equation (13) the stiffness coefficient of the spring can be determined

$$k = \frac{m_3 l_{OA}^2 (\dot{\varphi})^2}{x_{\max}^2} \quad (14)$$

To determine the displacement x of the spring let us introduce equation (8) and equation (14) into equation (12)

$$x = x_{\max} \sin\varphi \quad (15)$$

Let us now consider the period of energy restitution.

In this case, the following expression concerning input torque can be written as

$$A_{rest} = \int_{\pi/2}^{\varphi} m_3 l_{OA}^2 (\dot{\varphi})^2 \frac{\sin 2\varphi}{2} \mathrm{d}\varphi = 0.5 l_{OA}^2 (\dot{\varphi})^2 \cos^2\varphi \quad (16)$$

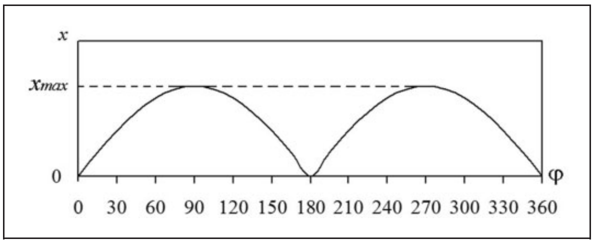

Figure 3. Displacements of the balancing spring.

With regard to the spring it can be written

$$A_{rest} = \int_x^{x_{\max}} kxdx = 0.5k(x_{\max}^2 - x^2) \quad (17)$$

or

$$x = \sqrt{x_{\max}^2 - \frac{2A_{rest}}{k}} \quad (18)$$

To determine the displacement x of the spring for the period of energy restitution, let us introduce equation (14) and equation (16) into equation (18)

$$x = x_{\max} \sin\varphi \quad (19)$$

So for two periods, accumulation and restitution, displacements of the spring are same, see Figure 3. The proposed traditional solution for input torque balancing in Scotch yoke mechanism is to add a cam to input crank in order to execute harmonic displacements of the spring. However, taking into account particularities of the Scotch yoke mechanism, it will be shown that a simple balancing technique of the input torque can be found.

Let us now turn our attention to the displacements of slider 3. The displacements of the slider vary with the sinusoidal law. Therefore, it is possible to balance the input torque of a Scotch yoke mechanism by adding linear springs between the frame and output slider 3. The added springs should ensure the condition: $x_{\max} = l_{OA}$.

Thus, by adding simple linear springs the input torque due to the inertial forces will be fully cancelled. Although the described solution is very simple, this is the first time it is proposed.

The input torque due to friction in joints

Let us now consider a Scotch yoke mechanism taking into account the friction in the mechanism's joints.

Several friction models have been proposed having different levels of accuracy, and wide variety of control solutions have been developed for its compensation.[23–30]

In Sawyer et al.,[32] a nearly ideal two-dimensional Scotch yoke mechanism was constructed to test a model of wear depth as a function cycle number. The model originally developed by Blanchet[33] was non dimensionalized and simplified under conditions of large numbers of cycles. Experiments, given in literature,[32] showed a linear progression of wear over two distinct regions, suggesting a sudden transition in wear modes just after 1.5 million cycles.

The review showed that friction must be considered in dynamic models in order to optimally control mechanisms.

In the present paper, the friction model developed by Wilson and Sadler[34] has been used. The choice of this model is due to the fact that it provides analytical results. It allows authors to keep the principal structure of the paper with only analytically tractable solutions.

After torque balancing described above, the reaction forces in prismatic joints are cancelled, i.e. $\mathbf{F_{23}} = \mathbf{F_{03}} = 0$, where $\mathbf{F_{23}}$ is the reaction force between links 2 and 3; $\mathbf{F_{03}}$ is the reaction force between link 3 and the frame (denoted as "0").

With regard to the reactions in revolute joints, they are constant due to the condition: $\mathbf{F_{21}} + \mathbf{F_{01}} + \mathbf{F_1^{int}} = \mathbf{0}$, where $\mathbf{F_{21}}$ is the reaction force between links 2 and 1; $\mathbf{F_{01}}$ is the reaction force between the frame and link 1; $\mathbf{F_1^{int}}$ is the resultant inertia force of link 1.

Thus, for determination of the input torque of the balanced mechanism, only the bearing friction in revolute joints O and A should be taken into consideration. It is known that the effect of the frictional contact at the bearing surfaces is always a torque which acts in a direction to oppose the relative rotation of the two links[34]

$$M_{ji}^{(fr)} = e_{ji}\mu_{ji}F_{ji}\cos\theta_{ji}\mathrm{sgn}(\dot{\varphi}_j - \dot{\varphi}_i) \tag{20}$$

where e_{ji} is the nominal radius of the bearing (in practical mechanisms the difference between the radii of the bearing and the shaft or pin is less than 0.2% and thus e_{ji} may be taken as the nominal size of the bearing); μ_{ji} is the coefficient of friction; F_{ji} is the bearing reaction force of link j on link i; θ_{ji} is the friction angle ($\theta_{ji} = \tan^{-1}\mu_{ji}$); $\dot{\varphi}_j$ is the angular velocity of link j; $\dot{\varphi}_i$ is the angular velocity of link i.

Therefore, the input torque of the balanced mechanism can be written as

$$\begin{aligned} M_{IN} = M_{01}^{(fr)} + M_{21}^{(fr)} &= e_{01}\mu_{01}F_{01}\cos\theta_{01}\mathrm{sgn}(\dot{\varphi}_0 - \dot{\varphi}_1) \\ &+ e_{21}\mu_{21}F_{21}\cos\theta_{21}\mathrm{sgn}(\dot{\varphi}_2 - \dot{\varphi}_1) \end{aligned} \tag{21}$$

Thus, after balancing of the inertia forces, the input torque of the Scotch yoke mechanism becomes constant and can be determined by the some of friction torques in joints O and A.

Let us now consider the optimal control of the Scotch yoke mechanism to ensure the constant input angular velocity and the given input torque due to friction in joints.

The input torque due to friction in joints

The differential equation describing the motion of the Scotch yoke mechanism without linear springs to the output slider is given by

$$\begin{aligned} \tau(t) = &\left[I_{S1} + m_1 r_{S1}^2 + m_2 l_{OA}^2 + m_3 l_{OA}^2\cos^2\varphi(t)\right]\ddot{\varphi}(t) \\ &- \frac{1}{2} m_3 l_{OA}^2 \dot{\varphi}^2(t)\sin 2\varphi(t) \end{aligned} \tag{22}$$

The joint variable is $\varphi(t)$ and the control torque is $\tau(t)$

The parameters of the Scotch yoke mechanism are

$$\begin{aligned} &I_{S1} = \frac{m_1 l_{OA}^2}{12}; \quad r_{S1} = \frac{l_{OA}}{2}; \quad l_{OA} = 0.1m; \\ &m_1 = 3kg; \quad m_2 = 0.5kg; \quad m_3 = 5kg \end{aligned} \tag{23}$$

In order to simplify the expression of $\tau(t)$, please note that

$$\begin{cases} \Delta[\varphi(t)] = \left[I_{S1} + m_1 r_{S1}^2 + m_2 l_{OA}^2 + m_3 l_{OA}^2\cos^2\varphi(t)\right] > 0 \\ \mathrm{B}[\varphi(t), \dot{\varphi}(t)] = \frac{1}{2} m_3 l_{OA}^2 \dot{\varphi}^2(t)\sin 2\varphi(t) \end{cases} \tag{24}$$

The dynamic model is shown below

$$\ddot{\varphi}(t) = \frac{\mathrm{B}[\varphi(t), \dot{\varphi}(t)]}{\Delta[\varphi(t)]} + \frac{\tau(t)}{\Delta[\varphi(t)]} \tag{25}$$

The steady-state for

$$\tau(t) = -\mathrm{B}[\varphi(t), \dot{\varphi}(t)] \text{ is} : \ddot{\varphi}(t) = 0 \tag{26}$$

By adding simple linear springs, the input torque due to the inertial forces is fully cancelled (null value for $\tau(t)$ in steady-state) but $\ddot{\varphi}(t) = 0$.

Now, for the steady-state, let us consider the Scotch yoke mechanism with friction in joints. It should be assumed that the input torque due to friction in joints can be represented through an additional constant disturbance $d(t)$ in the state equation as follows

$$\underbrace{\begin{bmatrix} \dot{\varphi}(t) \\ \ddot{\varphi}(t) \end{bmatrix}}_{\underline{x}(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \varphi(t) \\ \dot{\varphi}(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{E} d(t) \tag{27}$$

The observed variable is given by

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x(t) \tag{28}$$

This double integrator is unstable but completely controllable and observable. It is easily from equation

(27) that the zero-steady-state error optimal control law is given by

$$u(t) = -G\hat{x}(t) - \hat{d}(t) \quad (29)$$

The gain matrix G is an appropriate steady-state optimal feedback, $\hat{x}(t)$ is an estimate of the state-vector $x(t)$ and $\hat{d}(t)$ is an estimate of the disturbance $d(t)$.

The function of the gain matrix $G = \begin{bmatrix} g_1 & g_2 \end{bmatrix}$ is to stabilize the system by moving the closed-loop poles in the left-half complex plane.

For $\hat{d}(t) = d(t) = 0$ and $\hat{x}(t) = x(t)$, we seek $u(t)$ that minimizes the cost

$$J = \int_0^\infty \left[Ly^2(t) + u^2(t) \right] dt = \int_0^\infty \left[x^T(t) Q_C x(t) + u^2(t) \right] dt$$

The matrix L is based on the controllability transient gramian defined by

$$G_C(0, T_P) = \int_0^{T_P} \left[e^{At} B B^T e^{A^T t} \right] dt \quad (30)$$

For the matrix $L = \left[T_P C G_C(0, T_P) C^T \right]^{-1}$, the matrix $Q_C = C^T L C$ is symmetric and semi-definite positive. The parameter T_p assume that poles of closed-loop system may be placed, in the S plane, at the left or near of the vertical straight with the abscissa $-1/T_P$.

The output equation $u(t) = -Gx(t)$ of the controller is unique, optimal, full state feedback control law with $G = B^T \Sigma_C$ that minimizes the cost J.

The matrix Σ_C is the unique, symmetric, positive definite solution to the algebraic Riccati equation $A^T \Sigma_C + \Sigma_C A - \Sigma_C B B^T \Sigma_C + Q_C = 0$.

For the double integrator, the matrix G is

$$G = \left[g_1 = \frac{\sqrt{3}}{T_P^2} \quad g_2 = \frac{\sqrt{2\sqrt{3}}}{T_P} \right] \quad (31)$$

Then the closed-loop characteristic polynomial is given by

$$P_C(s) = s^2 + \frac{\sqrt{2\sqrt{3}}}{T_P} s + \frac{\sqrt{3}}{T_P^2}.$$

$$\text{If } P_C(s) = s^2 + 2\zeta\omega_n s + \omega_n^2,$$

$$\omega_n = \frac{\sqrt{\sqrt{3}}}{T_P} \text{ and } \zeta = \frac{\sqrt{2}}{2}$$

For obtain the observer, the constant disturbance is the following

$$\dot{d}(t) = 0 \quad (32)$$

The steady-state optimal observer which allows estimating $x(t)$ and $d(t)$ has the form

$$\underbrace{\begin{bmatrix} \dot{\hat{\varphi}}(t) \\ \ddot{\hat{\varphi}}(t) \end{bmatrix}}_{\dot{\hat{x}}(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\varphi}(t) \\ \dot{\hat{\varphi}}(t) \end{bmatrix}}_{\hat{x}(t)} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{E} \hat{d}(t) + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \left(y(t) - \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \hat{\varphi}(t) \\ \dot{\hat{\varphi}}(t) \end{bmatrix}}_{\hat{x}(t)} \right)$$

$$\dot{\hat{d}}(t) = k_3 \left(y(t) - \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \hat{\varphi}(t) \\ \dot{\hat{\varphi}}(t) \end{bmatrix}}_{\hat{x}(t)} \right) \quad (33)$$

The state-equations of the observer are

$$\underbrace{\begin{bmatrix} \dot{\hat{\varphi}}(t) \\ \ddot{\hat{\varphi}}(t) \\ \dot{\hat{d}}(t) \end{bmatrix}}_{\dot{x}_E(t)} = \underbrace{\begin{bmatrix} -k_1 & 1 & 0 \\ -k_2 & 0 & 1 \\ -k_3 & 0 & 0 \end{bmatrix}}_{A_E} \underbrace{\begin{bmatrix} \hat{\varphi}(t) \\ \dot{\hat{\varphi}}(t) \\ \hat{d}(t) \end{bmatrix}}_{x_E(t)} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{B_E} u(t) + \underbrace{\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}}_{K} y(t) \text{ with } y(t) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C_E} x_E(t) \quad (34)$$

The function of the gain matrix $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}^T$ is to stabilize asymptotically the observer. The duality between the optimal regulator and the optimal observer (Kalman filter) enables us to transfer from the regulator to the observer all important results.

The behavior of the Riccati equation can be rephrased as follows: $A_E \Sigma_O + \Sigma_O A_E^T - \Sigma_O C_E^T C_E \Sigma_O + Q_O = 0$

The matrix $Q_O = [T_R G_O(0, T_R)]^{-1}$ is based on the observability transient gramian defined by

$$G_O(0, T_R) = \int_0^{T_R} \left[e^{A_E^T t} C_E^T C_E e^{A_E t} \right] dt \quad (35)$$

The solution of the observer Riccati equation is

$$K = \Sigma_O C_E^T = \left[k_1 = \frac{c_1}{T_R} \quad k_2 = \frac{c_2}{T_R^2} \quad k_3 = \frac{c_3}{T_R^3} \right]^T$$

For $c_1^2 - 2c_2 = 9$ and $c_3 = 12\sqrt{5}$, the numerical values are: $c_1 = 7.198 \quad c_2 = 21.408 \quad c_3 = 26.83$

Then the characteristic polynomial is

$$P_O(s) = \left(s + \frac{3.0735}{T_R} \right) \left(s^2 + \frac{4.1248}{T_R} s + \frac{8.7303}{T_R^2} \right)$$

If $P_O(s) = (s + \omega_1)(s^2 + 2\zeta\omega_0 s + \omega_0^2)$
$\omega_0 = \frac{2.9547}{T_R}$ and $\zeta = 0.698$

Figure 4 shows the closed-loop control system which accumulates information about the plant during operation and allows a zero steady-state tracking error in spite of constant disturbance $d(t)$ defined by equation (32).

Let us now consider an illustrative example with simulations carried out by using ADAMS software.

Illustrative example and numerical simulations

Let us carry out the torque balancing of a Scotch yoke mechanism with parameters (Figure 4): $l_{OA} = 0.1$m; $m_1 = 3$kg $m_2 = 0.5$kg $m_3 = 5$kg; $\dot{\varphi} = 10\pi s^{-1}$; $e_{01} = e_{21} = 0.01$m; $\mu_{01} = \mu_{21} = 0.2$.

To balance the input torque of the Scotch yoke mechanism shown in Figure 5, two pairs of compression springs were used. The first pair balances the input torque for one haft of the crank rotation and the second pair for other haft of the crank rotation.

Figure 5(a) and (b) shows the Scotch yoke mechanism in dead point positions when $x = x_{\max}$.

The stiffness coefficient k of each pair of springs is 4934.8N/m, which are determined from equation (14) taking into account that $x_{\max} = l_{OA} = 0.1$m. The reaction forces in joints O and A are $|\mathbf{F_{01}}| = 197.39$N and $|\mathbf{F_{21}}| = 49.35$N respectively.

The variations of the reaction forces in revolute joints O and A for balanced and unbalanced mechanisms are given in Figure 6.

The input torque of the initial mechanism without balancing springs determined from equation (6) and with them determined from equation (21) is shown in Figure 7.

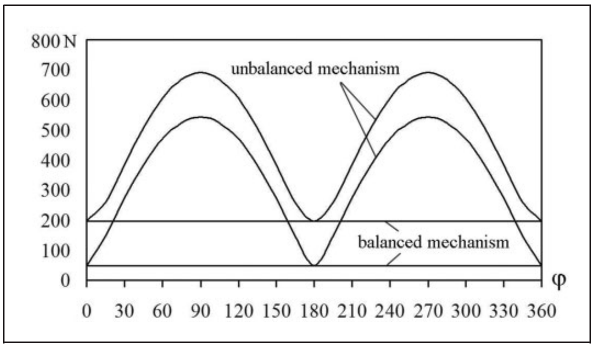

Figure 6. The reaction forces in the revolute joints of the Scotch yoke mechanism before and after balancing.

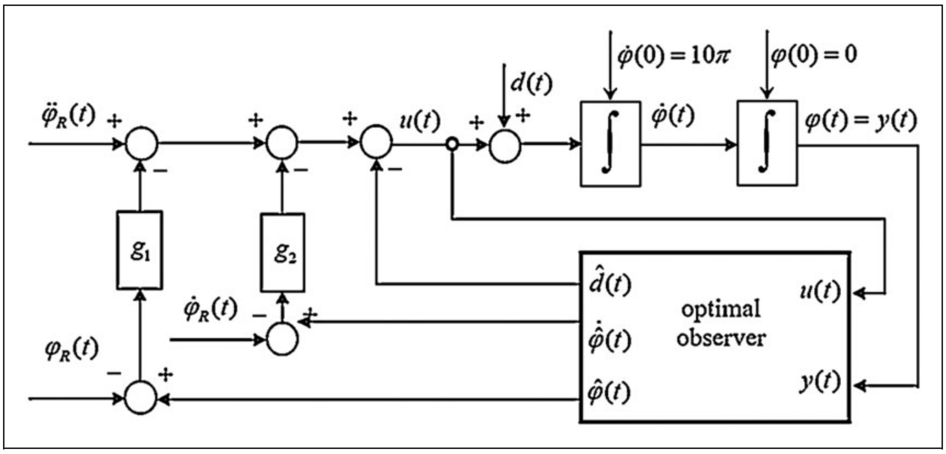

Figure 4. The closed-loop control system.

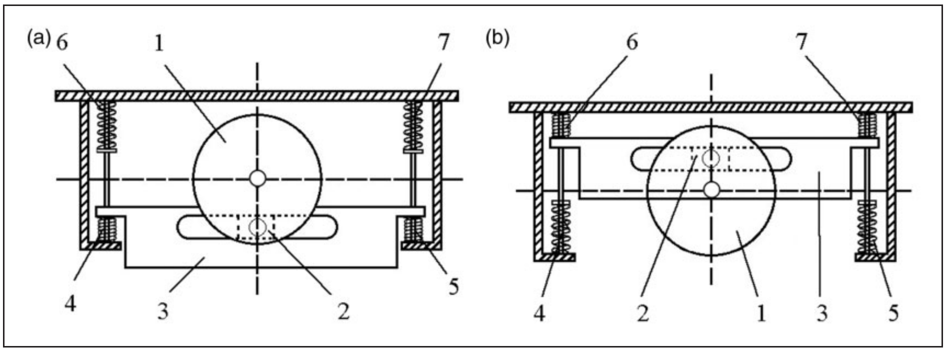

Figure 5. The Scotch yoke mechanism in dead point positions.

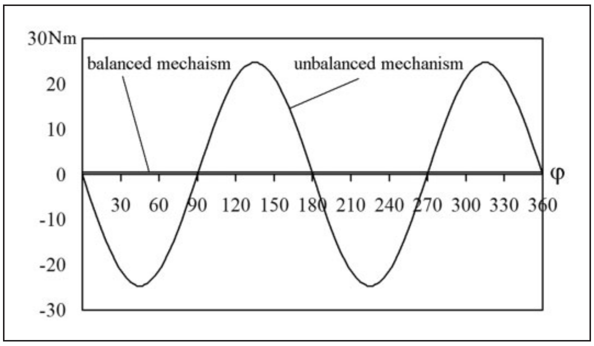

Figure 7. The input torque of the Scotch yoke mechanism before and after balancing.

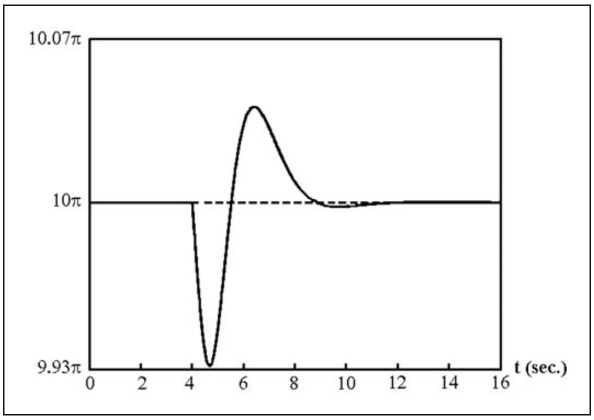

Figure 8. Angular velocity $\dot{\varphi}(t)$ (solid line) and $\dot{\varphi}_R(t) = 10\pi$ (dashed line).

The numerical simulations showed that in comparison with balanced mechanism, 98% reduction in input torque has been achieved (from 24.7 Nm to 0.48 Nm).

Let us now consider the optimal control of the mechanism to ensure the constant input angular velocity and the given input torque.

The closed-loop control law can be written as

$$u(t) = \ddot{\varphi}_R(t) - g_1[\hat{\varphi}(t) - \varphi_R(t)] - g_2\left[\dot{\hat{\varphi}}(t) - \dot{\varphi}_R(t)\right] - \hat{d}(t)$$

$\varphi_R(t)$, $\dot{\varphi}_R(t)$ and $\ddot{\varphi}_R(t)$ are given by the equations

$$\begin{cases} \varphi_R(t) = 10\pi t \\ \dot{\varphi}_R(t) = 10\pi \\ \ddot{\varphi}_R(t) = 0 \end{cases}$$

For $T_p = T_R = 1s$, the following results are obtained: $g_1 = 1.732$, $g_2 = 1.861$, $k_1 = 7.198$, $k_2 = 21.408$, $k_3 = 26.83$.

The responses, with Matlab software, to disturbance at $t = 4s$ are shown in Figures 8 and 9.

Figure 8 presents the angular velocity $\dot{\varphi}(t)$, which approaches at $t = 12s$, the constant reference $\varphi_R(t) = 10\pi$, independently of the disturbance $d(t) = -0.48Nm$.

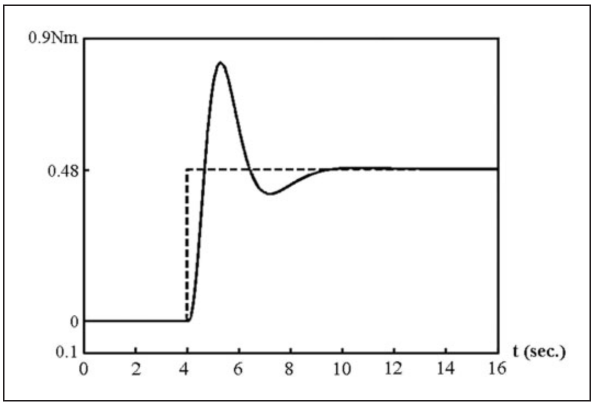

Figure 9. Control law $u(t) = -G\hat{x}(t) - \hat{d}(t)$ (solid line) and disturbance $-d(t) = 0.48\,Nm$.

Figure 9 shows the control law $u(t) = -G\hat{x}(t) - \hat{d}(t)$ with integral action which allows the disturbance rejection.

The control structure that results from combining observer with state feedback law has the property that the constant disturbance is always compensated so that a zero steady-state regulation or zero tracking error results.

Conclusion

This paper deals with the input torque balancing of Scotch yoke mechanisms due to inertia effects. The input torque balancing in linkages is usually carried out by adding cam-spring mechanisms. In this study it is disclosed that Scotch yoke mechanisms can be balanced without any auxiliary linkage by adding linear springs to the output slider. Although the described solution is very simple, this is the first time, it has been proposed. The analysis of the input torque showed that the variation of elastic balancing forces is a function of the slider displacement. Therefore, the balancing of the input torque of a Scotch yoke mechanism can be carried out by two pairs of springs connected with the output slider. The suggested balancing solution has been improved for the Scotch yoke mechanism taking into account the friction in the mechanism's joints. Numerical simulations showed that in comparison with balanced mechanism, 98% reduction in input torque has been achieved. It has been shown that after balancing the input torque becomes constant, which facilitate the control of the mechanism. It has also been shown that after balancing the reaction forces in two revolute joints become constant and far less than before balancing (for the considered mechanism about 91% in joint A and 72% in joint O).

However, the given reduction can be reached if the input link has a constant angular velocity. To ensure this condition an optimal control has been developed. It was shown that the given optimal control law ensures a constant input angular velocity taking into account the friction in joints, as well as the given input torque.

Declaration of Conflicting Interests

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