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Mechatronic design of adjustable serial manipulators with decoupled dynamics taking into account the changing payload

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ABSTRACT
This paper deals with the problem of dynamic decoupling of adjustable serial manipulators via a new mechatronic design approach, which is based on the opposite motion of manipulator links and the optimal command design. The goal is to simplify the control by reducing the effects of complicated manipulator dynamics. The opposite motion of links with optimal redistribution of masses allows the cancellation of the coefficients of nonlinear terms in the manipulator's kinetic and potential energy equations. Then, by using the optimal control design, the dynamic decoupling is achieved. The proposed approach, which is a symbiosis of mechanical and control solutions, improves the known design concepts permitting the dynamic decoupling of serial manipulators whilst taking into account the changing payload. It has two main advantages: at first, the dynamic decoupling of the manipulator is achieved without connection of gears to the links of the manipulator having oscillating motions and at second, the performance of the dynamic decoupling is improved by the changing payload. The suggested design methodology is illustrated by simulations carried out using ADAMS and MATLAB software, which have confirmed the efficiency of the developed approach.

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1. Introduction

It is known that manipulator dynamics can be highly coupled and nonlinear. The complicated dynamics results from varying inertia, interactions between the different joints, and nonlinear forces such as Coriolis and centrifugal forces. Nonlinear forces cause errors in position response at high speed, and have been shown to be significant even at slow speed (Brady 1982). Thus, the dynamic decoupling of robot mechanisms has attracted researchers' attention and different solutions have been proposed:

- The linearisation of the dynamic equations and their decoupling via actuator relocation, that is, by the kinematic decoupling of motion when the rotation of any link is due to only one actuator (Artobolevskii and Ovakimov 1976; Asada and Youcef-Toumi

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1987; Belyanin et al. 1981; Chung, Gang, and Lee 2002; Minotti 1991; Vukobratović and Stokić 1980; Youcef-Toumi 1985, 1992; Youcef-Toumi and Asada 1985a). In other terms, it should be assumed that the actuator displacements are a complete set of independent generalised coordinates that are able to locate the manipulator uniquely and completely. The design concept with remote actuation is not optimal from the point of view of the precise reproduction of the end-effector tasks because it accumulates all errors due to the clearances and elasticity of the belt transmission mainly used. Obviously, it is lot better to connect actuators directly with links than to use transmission mechanisms. The manufacturing and assembly errors of the added transmission mechanisms also have a negative impact to the robot precision.

- The linearisation of the dynamic equations and their decoupling via optimum inertia redistribution (Abdel-Rahman and Elbestawi 1991; Arakelian and Dahan 1995; Asada and Slotine 1986; Asada and Youcef-Toumi 1984; Filaretov and Vukobratović 1993; Minotti and Pracht 1992; Yang and Tzeng 1985, 1986; Youcef-Toumi and Asada 1985b; Youcef-Toumi and Asada 1986), which can be achieved when the inertia tensors are diagonal and independent of manipulator configuration. Such an approach is applied to serial manipulators in which the axes of joints are not parallel. In the case of parallel axes, such an approach allows linearisation of the dynamic equations but not their dynamic decoupling (Gompertz and Yang 1989). Thus, in the case of planar serial manipulators, it cannot be used.
- The linearisation of the dynamic equations and their decoupling via redesign of the manipulator by adding auxiliary links (Arakelian et al. 2011; Arakelian and Sargsyan 2012; Coelho, Yong, and Alves 2004; Moradi, Nikoobin, and Azadi 2010). Also the modification of the manipulator design to achieve high-quality dynamic performance is a promising new approach in the robotics. However, the design methodology proposed in, which claims that it is the first time the added links have been used for dynamic decoupling, leads to the unavoidable increase of the total mass of the manipulator. This is due to the disposition of the added elements in the end of each link. In (Arakelian and Sargsyan 2012) a solution has been proposed permitting the dynamic decoupling of the serial manipulators with a relatively small increase in the total mass of the moving links. Nevertheless, it should be noted that such a technique has a major disadvantage: the need for the connection of gears to the oscillating links. The gears added to the oscillating links of the manipulator are sources of shocks between teeth that will lead to the perturbation of the operation of the manipulator, and to noise and other negative effects.

The above-mentioned methods provide purely mechanical solutions but it should be noted that a number of procedures for the synthesis of control systems ensuring high-quality control of manipulators have been elaborated based on the general form of non-linear dynamic equations (Chen and Chang 2009; Chen and McInroy 2004; Davliakos and Papadopoulos 2008; Duchaine, Bouchard, and Gosselin 2007; Kim, Cho, and Lee 2005; Potkonjak 1982; Sciavicco and Siciliano 2000; Slotine et al. 1991; Ting, Chen, and Jar 2004; Vukobratović, Stokić, and Kirćanski 1985; Yang, Huang, and Han 2012). However, regardless of the permanent tendency to decrease the price of microcomputer systems, the cost of achieving complete dynamic control is still high for high-speed and precise dynamical tasks of industrial practice. Therefore, in many cases, the reduction (or cancellation) of coupling and of nonlinearity in the manipulators is necessary as shown in (Herman 2005, 2006, 2008; Pons et al. 1997).

It can therefore be concluded that all known mechanical solutions can only be reached by a considerably more complicated design of the initial structure via adding gears to the oscillating links leading to the above-mentioned drawbacks. On the other hand, dynamic decoupling via optimal control of a manipulator with a nonlinear system model and a changing payload is also rather complex task. That is why this paper proposes a new approach of dynamic decoupling, which is a symbiosis of mechanical and control solutions. It is carried out in two steps. In the first step, the dynamic decoupling of serial manipulator with adjustable lengths of links is achieved via the opposite rotation of links and their optimal redistribution of masses. Such a solution proposed in the first step eliminates the need for the connection of gears to the oscillating links. As was mentioned above, the gears added to the oscillating links of the manipulator are sources of shocks between teeth that lead to the perturbation of the operation of the manipulator, the noise and other negative effects. This is the first main advantage of the suggested decoupling technique. Thus, the proposed mechanical solution allows one to transform the original nonlinear system model into a fully linear system without using the feedback linearisation technique (Arakelian, Baron, and Mottu 2011). Then, it is shown that the changing payload leads to the perturbation of the dynamic decoupling of the manipulator. Therefore, in the second step, the dynamic decoupling of the equation of motion due to the changing payload is carried out using control techniques. The control of serial manipulators that are considered as controllable systems of rigid bodies is normally based on nonlinear regulation and nonlinear tracking. In nonlinear control, the concept of feedback plays a fundamental role in regulation design, as it does in linear systems. However, the concept of feed forward is always required to provide anticipative actions in tracking design. It is interesting to note that many tracking controllers use a feed forward part (to supply the necessary input for following the specified motion trajectory) and a feedback part (to stabilise the tracking errors dynamics). Sometimes, the feed forward part can be used to cancel effects of known disturbances. In the present case, the developed control technique is based on a feed forward part in the controller to take into account the payload.

Such an approach is promising because it combines the advantages of two different principles. It should be noted that the mechanical solutions, which can be used for dynamic decoupling of motion equations taking into account the changing payload, can only be reached with any undue complication of the design. Various actuated counterweights should be applied. Such an approach is not viable. However, the linearised dynamic of the manipulator via opposite rotation of manipulator's links, proposed in the present study, leads to relatively simple equations, which are easier to analyse for further dynamic decoupling, taking into account the changing payload. In other terms, the proposed mechanical solution leads to the linearised equations of the manipulator, which then facilitate the optimal control design for decoupling of dynamic equations, taking into account the changing payload. This is the second main advantage of the proposed mechatronic design.

2. Conditions of dynamic decoupling via opposite rotating links

Consider a serial planar manipulator with two degrees of freedom shown in Figure 1.

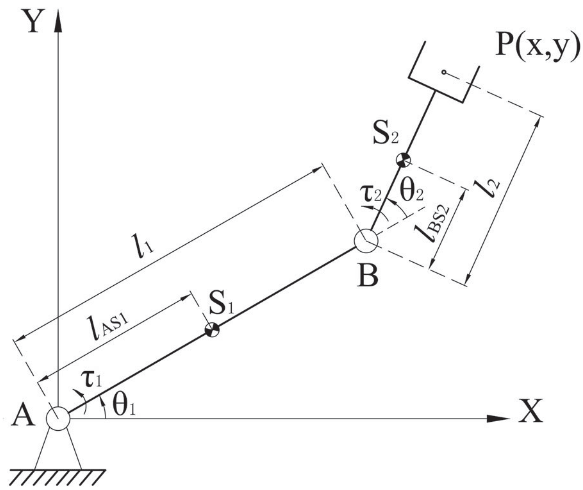

Figure 1. A 2-DOF planar serial manipulator.

According to Lagrangian dynamics, the equations of motion can be written as:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_{111} & D_{122} \\ D_{211} & D_{222} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} D_{112} & D_{121} \\ D_{212} & D_{221} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_1\dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \tag{1}$$

with

$$D_{11} = m_1 l_{AS1}^2 + m_2 l_1^2 + m_2 l_{BS2}^2 + 2m_2 l_1 l_{BS2} \cos\theta_2 + I_{S1} + I_{S2}, \tag{2}$$

$$D_{12} = D_{21} = m_2 l_{BS2}^2 + m_2 l_1 l_{BS2} \cos\theta_2 + I_{S2}, \tag{3}$$

$$D_{22} = m_2 l_{BS2}^2 + I_{S2}, \tag{4}$$

$$D_{111} = 0, \tag{5}$$

$$D_{122} = -m_2 l_1 l_{BS2} \sin\theta_2, \tag{6}$$

$$D_{211} = m_2 l_1 l_{BS2} \sin\theta_2, \tag{7}$$

$$D_{222} = 0, \tag{8}$$

$$D_{112} = D_{121} = -m_2 l_1 l_{BS2} \sin\theta_2, \tag{9}$$

$$D_{212} = D_{221} = 0, \tag{10}$$

$$D_1 = (m_1 l_{AS1} + m_2 l_1) g \cos\theta_1 + m_2 g l_{BS2} \cos(\theta_1 + \theta_2), \tag{11}$$

$$D_2 = m_2 g l_{BS2} \cos(\theta_1 + \theta_2), \tag{12}$$

where τ_1 and τ_2 are, respectively, the actuator torques in A and B; l_1, l_2 are the lengths of links 1 and 2; θ_1 is the angular displacement of link 1 relative to the base; θ_2 is the angular displacement of link 2 relative to link 1; $\dot{\theta}_1$ is the angular velocity of link 1 relative to the base; $\dot{\theta}_2$ is the angular velocity of link 2 relative to link 1; m_1, m_2 are the masses of links 1 and 2; l_{AS1} is the distance between the centre of mass S_1 of link 1 and joint centre A; l_{BS2} is the distance between the centre of mass S_2 of link 2 and joint centre B; I_{S1} is the axial moment of inertia of link 1 relative to the centre of mass S_1 of link 1; I_{S2} is the axial moment of inertia of link 2 relative to the centre of mass S_2 of link 2; g is the gravitational acceleration.

Now, let us consider that the second link is statically balanced, that is, $l_{BS2} = 0$ and the gravitational forces are perpendicular to the motion plane xOy, that is, $D_1 = D_2 = 0$.

In this case, Equation (1) can be rewritten as:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} I_{S2} + I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2 & I_{S2} \\ I_{S2} & I_{S2} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} a+b & a \\ a & a \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}, \tag{13}$$

where, $a = I_{S2}$; $b = I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2$.

$$\text{or} \begin{cases} \tau_1 = (I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2)\ddot{\theta}_1 + I_{S2}(\ddot{\theta}_1 + \ddot{\theta}_2) = b\ddot{\theta}_1 + a(\ddot{\theta}_1 + \ddot{\theta}_2), \\ \tau_2 = I_{S2}(\ddot{\theta}_1 + \ddot{\theta}_2) = a(\ddot{\theta}_1 + \ddot{\theta}_2). \end{cases} \tag{14}$$

From these equations, it follows that if $\ddot{\theta}_1 = -\ddot{\theta}_2$:

$$\begin{aligned} \tau_1 &= (I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2)\ddot{\theta}_1 = b\ddot{\theta}_1, \\ \tau_2 &= 0, \end{aligned} \tag{15}$$

that is, the dynamic equations are decoupled and the second actuator torque is cancelled.

Let us now consider the geometric synthesis for ensuring such a decoupling.

3. Adjustment lengths of links for ensuring opposite rotation of links

According to the inverse kinematics of the 2-DOF serial manipulator, the joint angles can be expressed as:

$$\theta_1 = \tan^{-1}\left[\frac{y(l_1 + l_2\cos\theta_2) - xl_2\sin\theta_2}{x(l_1 + l_2\cos\theta_2) + yl_2\sin\theta_2}\right], \tag{16}$$

$$\theta_2 = \pm\cos^{-1}\left[\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}\right], \tag{17}$$

where x and y are the coordinates of the end-effector with $l_2 = l_{BP}$ (see Figure 1).

The given expressions show that for the same end-effector position, there are two possible configurations of the manipulator called 'elbow down' (configuration noted (1)) and 'elbow up' (configuration noted (2)). The fact that a manipulator has multiple solutions would be used for ensuring the dynamic decoupling. Figure 2 shows two configurations of the manipulator corresponding to the initial end-effector position P_i and the final end-effector position P_f. As it has been mentioned above, the initial position of the end-effector can be found by the following solutions: $\theta^i_{1(1)}, \theta^i_{2(1)}$ 'elbow down' solution, $\theta^i_{1(2)}, \theta^i_{2(2)}$ 'elbow up' solution (not shown) and the final position of the end-effector by $\theta^f_{1(1)}, \theta^f_{2(1)}$ 'elbow

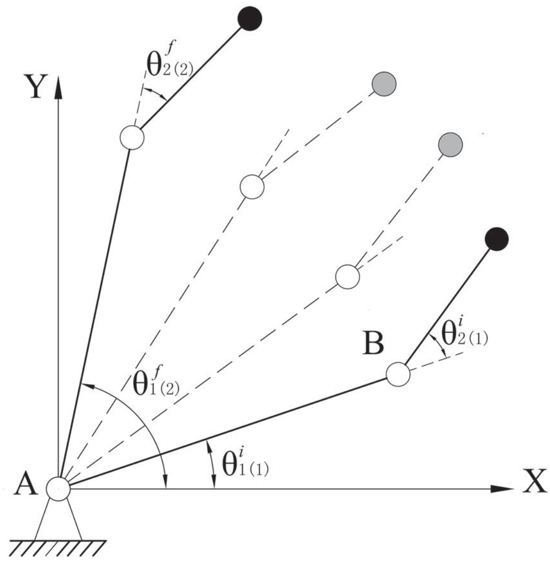

Figure 2. Two configurations of the 2-DOF planar serial manipulator corresponding to the initial and final end-effector positions.

down' solution (not shown), $\theta^f_{1(2)}, \theta^f_{2(2)}$ 'elbow up' solution. Thus, the links of the manipulator move in such a manner that in the initial end-effector position (P_i), where the configuration of the manipulator will correspond to the 'elbow down' solution; and, in the final end-effector position (P_f), where the configuration of the manipulator will correspond to the 'elbow up' solution.

This choice of initial and final end-effector configurations of the manipulator with an optimal selection of lengths l_1 and l_2 allows equal ($\Delta\theta_1 = \Delta\theta_2$) and opposite ($\dot{\theta}_1 = -\dot{\theta}_2$) rotations of links 1 and 2, that is, $|\theta^f_{1(2)} - \theta^i_{1(1)}| = -|\theta^f_{1(2)} + \theta^i_{1(1)}|$. These conditions lead to $\ddot{\theta}_1 = -\ddot{\theta}_2$ and consequently to Equations (15).

Figure 3 shows the proposed adjustable serial manipulator for ensuring the mentioned conditions. It is composed of link 1 with elements 1*a*, 1*b* and link 2 with elements 2*a*, 2*b*. The adjustable links of the manipulator allow an optimal selection of the lengths l_1 and l_2 of links 1 and 2, which ensures an identical and opposite rotation of links. It can also be seen that the proposed manipulator is provided with a double Scott–Russell mechanism, which ensures the static balancing of link 2 for any position of element 2*b* (Arakelian and Briot 2015; Briot et al. 2009).

Now consider the operation of the proposed manipulator. First, consider the selection of lengths l_1 and l_2 of links 1 and 2 for any given trajectory. To limit the variables in the specified conditions, suppose that the following parameters are given:

- the initial position P_i of the end-effector: x_i, y_i;
- the final position P_f of the end-effector: x_f, y_f;
- the initial angular position of the second link: $\theta^i_{2(1)}$;
- the rotating angle of the first link: $\Delta\theta_1 = \theta^f_{1(2)} - \theta^i_{1(1)}$.

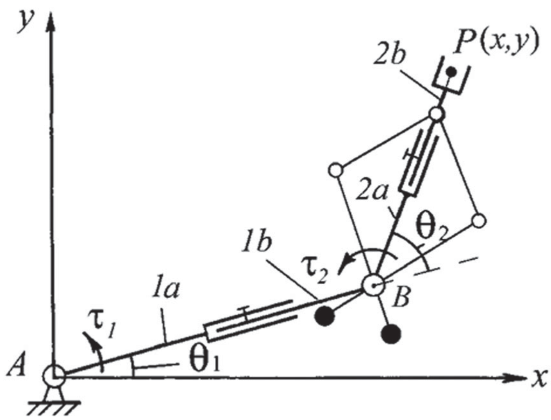

Figure 3. The 2-DOF adjustable serial manipulator with decoupled dynamics.

The geometrical equations of the manipulator with the specified conditions lead to the following expressions:

$$l_1 = \frac{x_f^2 + y_f^2 - x_i^2 - y_i^2}{2l_2(\cos\theta_{2(2)}^f - \cos\theta_{2(1)}^i)}, \tag{18}$$

$$l_2 = \left[\frac{-\xi - (\xi^2 - 4\chi)^{1/2}}{2}\right]^{1/2}, \tag{19}$$

where

$$\chi = \left[\frac{x_f^2 + y_f^2 - x_i^2 - y_i^2}{2(\cos\theta_{2(2)}^f - \cos\theta_{2(1)}^i)}\right]^2, \tag{20}$$

$$\xi = 2(\chi)^{1/2}\cos\theta_{2(1)}^i - x_i^2 - y_i^2, \tag{21}$$

$$\theta_{2(2)}^f = -(\Delta\theta_1 - \theta_{2(1)}^i). \tag{22}$$

Thus, the lengths l_1 and l_2 of links 1 and 2 determined from Equations (18) and (19) will ensure equal ($\Delta\theta_1 = \Delta\theta_2$) and opposite ($\dot{\theta}_1 = -\dot{\theta}_2$) rotations of links.

For illustration and validation of the suggested design concept, the simulations using ADAMS software have been carried out for a 2-DOF adjustable manipulator. The parameters of the manipulator are the following: $m_1 = 4$ kg, $m_2 = 1.2$ kg, $I_{S1} = 0.16$ kgm^2, $I_{S2} = 0.1$ kgm^2, $l_{BS2} = 0$and $l_{AS1} = 0.21$ m. The trajectory has been generated between the initial position P_i with coordinates $x_i = 0.5627$ m, $y_i = 0.3654$ m and the final position P_f with coordinates $x_f = 0.0657$ m, $y_f = 0.5236$ m. With $\theta_{2(1)}^i = 0°$ and$\Delta\theta_1 = \Delta\theta_2 = 77°$, from Equations (20) to (22), $\theta_{2(2)}^f, \chi, \xi$ are determined and then from Equations (18) to (19), the link lengths: $l_1 = 0.42$ m and $l_2 = 0.25$ m. The generation of motions between the initial and final positions of the links has been carried out using a fifth-order polynomial expression.

Figures 4 and 5 show actuator torques. It can be seen that the torque of second actuator is cancelled and the torque of the first actuator is linear to the actuator acceleration. Thus, the manipulator is dynamically decoupled.

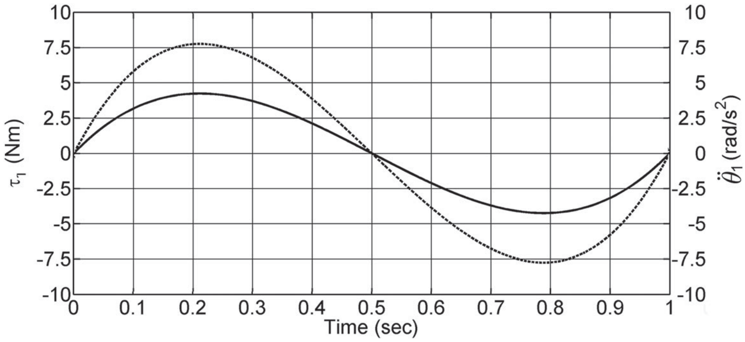

Figure 4. Torque (solid line) and angular acceleration (dashed line) of the first actuator.

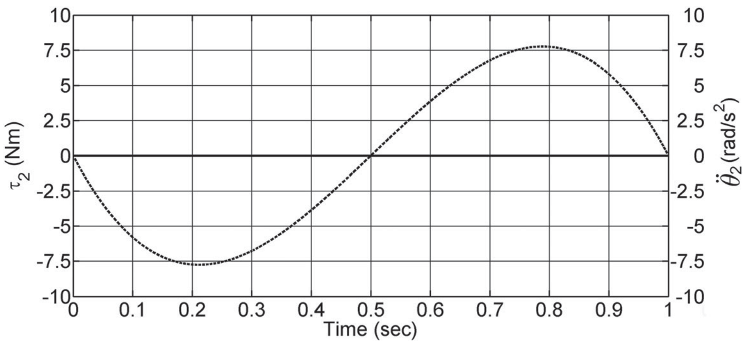

Figure 5. Torque (solid line) and angular acceleration (dashed line) of the second actuator.

In the end of this section, it should be noted that the links kinematically adjustable in its length can be designed either by adjustment of pivots or by the link length (Coppola et al. 2013; Peng 2010). However, the aim of this paper is to show the advantages of the mechatronic design combining both mechanical and control solutions. Therefore, the technological aspects including the design particularities of adjustment links will not be discussed.

4. Dynamic decoupling taking into account the payload

The previous sections have been devoted to the mechanical solution permitting the decoupling of the dynamic equations via optimal motions and special variations of mass redistribution. However, it is obvious that the changing payload creates the variable forces on the actuators, which are also nonlinear. According to the above results, the dynamic equations due to the mass Δ_m of the payload can be written as:

$$\tau_1 = [I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2]\ddot{\theta}_1 + \Delta_m l_1 [l_1 + l_2 \cos(\theta_2)]\ddot{\theta}_1 - [\Delta_m l_1 l_2 \sin(\theta_2)]\dot{\theta}_1 \dot{\theta}_2, \quad (23)$$

$$\tau_2 = \Delta_m l_1 l_2 [\cos(\theta_2)\ddot{\theta}_1 + \sin(\theta_2)\dot{\theta}_1^2]. \quad (24)$$

From these equations, it follows if $\Delta_m = 0$:

$$\tau_1 = [I_{S1} + m_1 l_{AS1}^2 + m_2 l_1^2]\ddot{\theta}_1 = b\ddot{\theta}_1, \tag{25}$$

$$\tau_2 = 0. \tag{26}$$

The payload compensation is given by:

$$\Delta\tau_1 = \Delta_m l_1 [l_1 + l_2 \cos(\theta_2)]\ddot{\theta}_1 - \Delta_m l_1 l_2 \sin(\theta_2)\dot{\theta}_1\dot{\theta}_2, \tag{27}$$

$$\Delta\tau_2 = \Delta_m l_1 l_2 [\cos(\theta_2)\ddot{\theta}_1 + \sin(\theta_2)\dot{\theta}_1^2]. \tag{28}$$

5. Illustrative example with simulation results

In this section, the performance of the proposed technique is examined through computer simulation for the study example presented in Figure 3, with payload compensation and without payload compensation. The introduction of the payload leads to an unbalance of the second link of the manipulator, and dynamic equations of the system with payload can be written as follows:

$$\tau_1 = \psi(\theta_2)\ddot{\theta}_1 + [\gamma_2 + \beta(\theta_2)]\ddot{\theta}_2 - 2\alpha(\theta_2)\dot{\theta}_1\dot{\theta}_2 - \alpha(\theta_2)\dot{\theta}_2^2, \tag{29}$$

$$\tau_2 = [\gamma_2 + \beta(\theta_2)]\ddot{\theta}_1 + \gamma_2\ddot{\theta}_2 + \alpha(\theta_2)\dot{\theta}_1^2, \tag{30}$$

where

$$\alpha(\theta_2) = M_2 l_1 l_{BS2} \sin(\theta_2); \quad \beta(\theta_2) = M_2 l_1 l_{BS2} \cos(\theta_2)$$

$$\psi(\theta_2) = \gamma_1 + \gamma_2 + M_2 l_1^2 + 2\beta(\theta_2); \quad \gamma_1 = [I_{S1} + m_1 l_{AS1}^2]; \quad \gamma_2 = [I_{S2} + M_2 l_{BS2}^2]$$

with $M_2 = m_2 + \Delta_m$ and $l_{BS2} = (\Delta_m/(m_2 + \Delta_m))l_2$

The inverse dynamic equations of the study system are:

$$\begin{aligned}\ddot{\theta}_1 = {} & \frac{\gamma_2}{\Delta(\theta_2)}\tau_1 - \frac{[\gamma_2 + \beta(\theta_2)]}{\Delta(\theta_2)}\tau_2 + \frac{\gamma_2\alpha(\theta_2)}{\Delta(\theta_2)}\dot{\theta}_1\dot{\theta}_2 \\ & + \frac{\alpha(\theta_2)[\gamma_2 + \beta(\theta_2)]}{\Delta(\theta_2)}\dot{\theta}_1^2 + \frac{\gamma_2\alpha(\theta_2)}{\Delta(\theta_2)}(\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2,\end{aligned} \tag{31}$$

$$\begin{aligned}\ddot{\theta}_2 = {} & -\frac{[\gamma_2 + \beta(\theta_2)]}{\Delta(\theta_2)}\tau_1 + \frac{\psi(\theta_2)}{\Delta(\theta_2)}\tau_2 - \frac{\alpha(\theta_2)[\gamma_2 + \beta(\theta_2)]}{\Delta(\theta_2)}\dot{\theta}_1\dot{\theta}_2 \\ & - \frac{\alpha(\theta_2)\psi(\theta_2)}{\Delta(\theta_2)}\dot{\theta}_1^2 - \frac{\alpha(\theta_2)[\gamma_2 + \beta(\theta_2)]}{\Delta(\theta_2)}(\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2,\end{aligned} \tag{32}$$

where $\Delta(\theta_2) = \gamma_1\gamma_2 + M_2 l_1^2 I_{S2} + [\alpha(\theta_2)]^2 > 0$

The manipulator, which is the system to be controlled, will be described by Equations (31) and (32). These equations are used for the simulation of the system with MATLAB software. The state of the system $x = [\theta_1\ \theta_2\ \dot{\theta}_1\ \dot{\theta}_2]^T$ has been integrated by Runge–Kutta method.

In the following section, two types of controllers for the simulation will be considered: open-loop controller and closed-loop controller. At first, to demonstrate the influence of the payload compensation, the open loop control system, which is a non-feedback system, will be used.

5.1. Open-loop control system

Figure 6 shows the open-loop control system. The open-loop controller obviously has no access to any information about the manipulator.

The open-loop control law can be written as:

$$\tau_1(t) = \tau_{1R}(t) + \Delta\tau_{1R}, \tag{33}$$

$$\tau_2(t) = \tau_{2R}(t) + \Delta\tau_{2R}, \tag{34}$$

where

$$\tau_{1R}(t) = (a+b)\ddot{\theta}_{1R}(t) + a\ddot{\theta}_{2R}(t), \tag{35}$$

$$\tau_{2R}(t) = a\ddot{\theta}_{1R}(t) + a\ddot{\theta}_{2R}(t). \tag{36}$$

The payload compensation is given by:

$$\Delta\tau_{1R} = \Delta_m l_1[l_1 + l_2\cos(\theta_{2R})]\ddot{\theta}_{1R} - \Delta_m l_1 l_2 \sin(\theta_{2R})\dot{\theta}_{1R}\dot{\theta}_{2R}, \tag{37}$$

$$\Delta\tau_{2R} = \Delta_m l_1 l_2[\cos(\theta_{2R})\ddot{\theta}_{1R} + \sin(\theta_{2R})\dot{\theta}_{1R}^2] \tag{38}$$

with

$$\left.\begin{aligned} \theta_{1R}(t) &= \theta_1^i + (\theta_1^f - \theta_1^i)\left(\frac{t}{T}\right)^3\left[10 - 15\left(\frac{t}{T}\right) + 6\left(\frac{t}{T}\right)^2\right] \\ \theta_{2R}(t) &= \theta_2^i + (\theta_2^f - \theta_2^i)\left(\frac{t}{T}\right)^3\left[10 - 15\left(\frac{t}{T}\right) + 6\left(\frac{t}{T}\right)^2\right] \end{aligned}\right\}, \quad 0 \le t \le T,$$

$$\left.\begin{aligned} \dot{\theta}_{1R}(t) &= \frac{30}{T}(\theta_1^f - \theta_1^i)\left(\frac{t}{T}\right)^2\left[1 - 2\left(\frac{t}{T}\right) + \left(\frac{t}{T}\right)^2\right] \\ \dot{\theta}_{2R}(t) &= \frac{30}{T}(\theta_2^f - \theta_2^i)\left(\frac{t}{T}\right)^2\left[1 - 2\left(\frac{t}{T}\right) + \left(\frac{t}{T}\right)^2\right] \end{aligned}\right\}, \quad 0 \le t \le T,$$

$$\left.\begin{aligned} \ddot{\theta}_{1R}(t) &= \frac{60}{T^2}(\theta_1^f - \theta_1^i)\left(\frac{t}{T}\right)\left[1 - 3\left(\frac{t}{T}\right) + 2\left(\frac{t}{T}\right)^2\right] \\ \ddot{\theta}_{2R}(t) &= \frac{60}{T^2}(\theta_2^f - \theta_2^i)\left(\frac{t}{T}\right)\left[1 - 3\left(\frac{t}{T}\right) + 2\left(\frac{t}{T}\right)^2\right] \end{aligned}\right\}, \quad 0 \le t \le T.$$

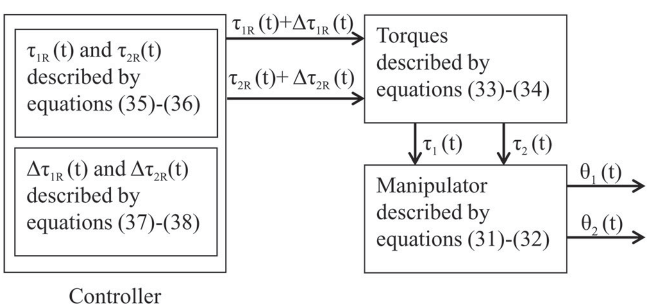

Figure 6. The open-loop control system.

The initial and final positions of the manipulator given as illustrative example in Section 3 are:

$\theta_1^i = \theta_{1(1)}^i = 33°; \theta_2^i = \theta_{2(1)}^i = 0°$ and $\theta_1^f = \theta_{1(2)}^f = 110°; \theta_2^f = \theta_{2(2)}^f = -77°$. The trajectories $\theta_{1R}(t)$ and $\theta_{2R}(t)$ are functions of time such that $[\theta_{1R}(0)\ \theta_{2R}(0)]^{\mathrm{T}} = [\theta_1^i\ \theta_2^i]^{\mathrm{T}}$ and $[\theta_{1R}(T)\ \theta_{2R}(T)]^{\mathrm{T}} = [\theta_1^f\ \theta_2^f]^{\mathrm{T}}$. In this case, T represents the amount of time taken to execute the trajectories. Since the trajectories are parameterised by time, velocities and accelerations can be obtained along the trajectories by differentiation.

Hence $\theta_2^f = \theta_1^i + \theta_2^i - \theta_1^f \Rightarrow \ddot{\theta}_{2R}(t) = -\ddot{\theta}_{1R}(t)$ and $\dot{\theta}_{2R}(t) = -\dot{\theta}_{1R}(t)$

For $T = 1s$ and $\Delta_m = 1$ kg (payload), the responses with MATLAB of the manipulator given for illustration in Section 3, are presented in Figures 7 and 8.

The dashed curves show the torques and the angular displacements of the manipulator without payload compensation and the solid curves show the same parameters with payload compensation. It can thus be seen that there are errors between these two cases. With

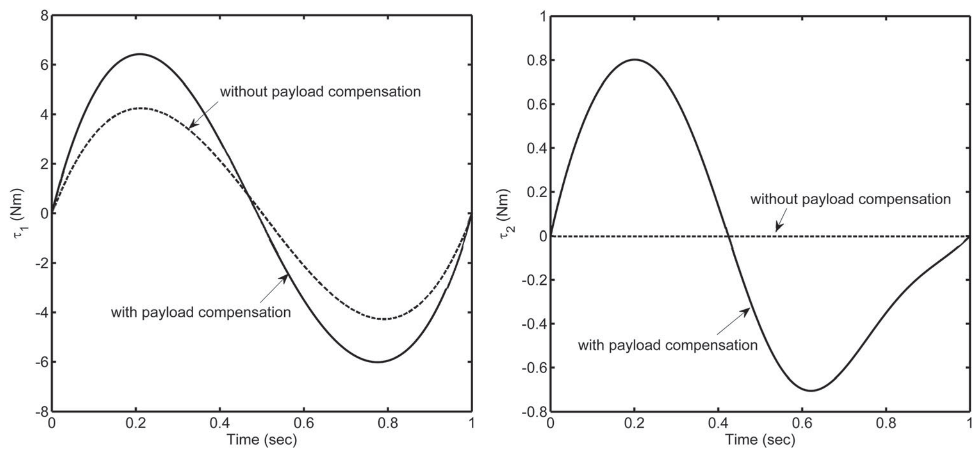

Figure 7. Torques with payload compensation (solid line) and without it (dashed line) for the open-loop system.

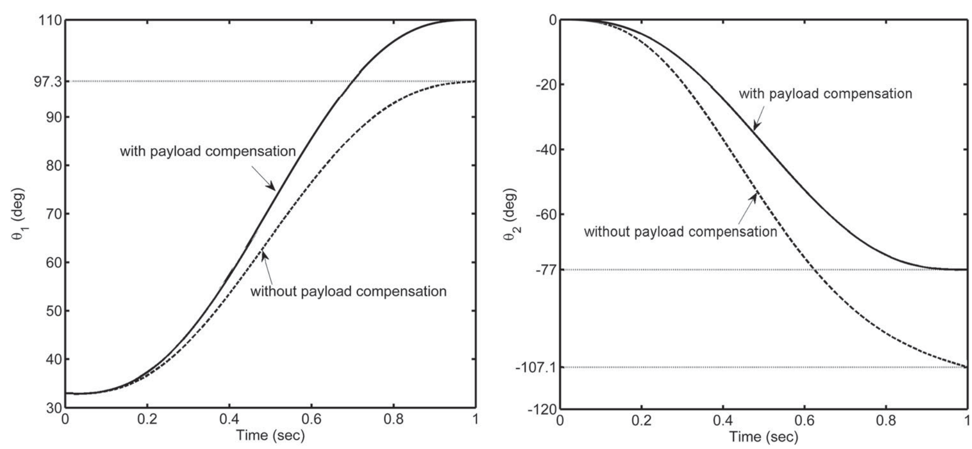

Figure 8. Angular displacements of links with payload compensation (solid line) and without it (dashed line) for the open-loop system.

payload compensation, both links of the manipulator can rotate exactly to the target angles. But, without payload compensation, the errors of angular displacements of link 1 and 2 are, respectively, 39% and 16.5%. The effect of feedforward control taking into account the payload is verified. Thus, a feedback control is needed to reduce the errors.

5.2. Closed-loop control system

Figure 9 shows the closed-loop control system, which accumulates information about the manipulator during operation and can reduce the effect of the payload.

The closed-loop control law can be written as:

$$\tau_1(t) = \tau_{1R}(t) + \Delta\tau_{1R}(t) - g_{11}[(a+b)[\dot{\theta}_1(t) - \dot{\theta}_{1R}(t)] + a[\dot{\theta}_2(t) - \dot{\theta}_{2R}(t)]] \\ - g_{12}[(a+b)[\theta_1(t) - \theta_{1R}(t)] + a[\theta_2(t) - \theta_{2R}(t)]], \quad (39)$$

$$\tau_2(t) = \tau_{2R}(t) + \Delta\tau_{2R}(t) - g_{21}[a[\dot{\theta}_1(t) - \dot{\theta}_{1R}(t)] + a[\dot{\theta}_2(t) - \dot{\theta}_{2R}(t)]] \\ - g_{22}[a[\theta_1(t) - \theta_{1R}(t)] + a[\theta_2(t) - \theta_{2R}(t)]], \quad (40)$$

where $\tau_{1R}(t)$, $\tau_{2R}(t)$, $\Delta\tau_{1R}(t)$ and $\Delta\tau_{2R}(t)$ are given by Equations (35)–(38).

The constant gain elements g_{11}, g_{12} and g_{21}, g_{22} are obtained by an optimal pole-placement design through state feedback. The state space representation of the 2-DOF adjustable serial manipulator, which is statically balanced, can be written as:

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{b} & -\frac{1}{b} \\ 0 & 0 \\ -\frac{1}{b} & \frac{1}{a} + \frac{1}{b} \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}.$$

The controllable canonical form is given by two independent subsystems as follows:

$$\begin{bmatrix} (a+b)\ddot{\theta}_1 + a\ddot{\theta}_2 \\ (a+b)\dot{\theta}_1 + a\dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} (a+b)\dot{\theta}_1 + a\dot{\theta}_2 \\ (a+b)\theta_1 + a\theta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tau_1$$

$$\begin{bmatrix} a\ddot{\theta}_1 + a\ddot{\theta}_2 \\ a\dot{\theta}_1 + a\dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a\dot{\theta}_1 + a\dot{\theta}_2 \\ a\theta_1 + a\theta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tau_2. \quad (41)$$

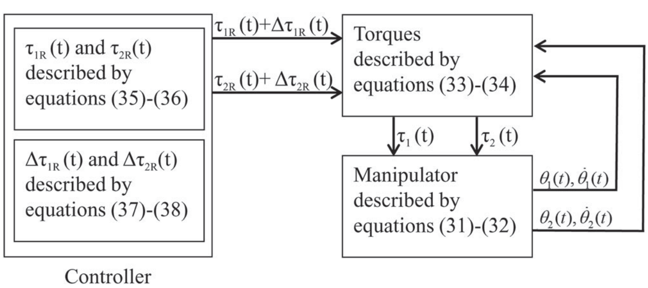

Figure 9. The closed-loop control system.

The subsystems (41) are represented by the normalised state space model of a double integrator (the two eigenvalues of the state matrix are equal to zero).

A double integrator

$$\underbrace{\begin{bmatrix}\ddot{\varphi}(t)\\ \dot{\varphi}(t)\end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}}_{A} \underbrace{\begin{bmatrix}\dot{\varphi}(t)\\ \varphi(t)\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{B} u(t)$$

is completely controllable.

Thus, $u(t)$ must be found that minimises the cost $J = \int_0^{\infty} [x^{\mathrm{T}}(t)Qx(t) + u^2(t)]\,\mathrm{d}t$ where the matrix Q is based on the controllability Gramian (Franklin, David Powell, and Emami-Naeini 1994) defined by:

$$G_c(0, T_p) = \int_0^{T_p} [e^{At}BB^{\mathrm{T}}e^{A^{\mathrm{T}}t}]\,dt.$$

The matrix $Q = [T_p G_c(0, T_p)]^{-1}$ is symmetric and positive definite. The parameter T_p assume that poles of closed-loop system may be placed in the S plane, at the left of the vertical straight with the abscissa $-1/T_p$.

The linear quadratic controller is unique, optimal $u(t) = -Gx(t)$, the full state feedback control law with, $G = B^{\mathrm{T}}\Sigma$, minimises the cost J.

The matrix Σ is the unique, symmetric, positive definite solution to the algebraic Riccati equation $A^{\mathrm{T}}\Sigma + \Sigma A - \Sigma BB^{\mathrm{T}}\Sigma + Q = 0$ (Lancaster and Rodman 1995).

For the double integrator, the matrix G gives:

$$G = \left[g_2 = \frac{2\sqrt{3}}{T_P^2} \quad g_1 = \frac{2\sqrt{1+\sqrt{3}}}{T_P} \right].$$

Then the closed-loop characteristic polynomial is:

$$P(s) = s^2 + \frac{2\sqrt{1+\sqrt{3}}}{T_P}.s + \frac{2\sqrt{3}}{T_P^2}.$$

If $P(s) = s^2 + 2\zeta\omega_n.s + \omega_n^2$, we obtain by identification:

$$\omega_n = \frac{\sqrt{2\sqrt{3}}}{T_P} \quad \zeta = \frac{\sqrt{1+\sqrt{3}}}{\sqrt{2\sqrt{3}}} = 0,9.$$

For $T = 1s, T_p = 0.2s$ and $\Delta_m = 1\,\text{kg}(\text{payload})$, the responses calculated with MATLAB are presented in Figures 10 and 11.

As in the previous case, the dashed curves show the torques and the angular displacements of the manipulator without payload compensation and the solid curves show the same parameters with payload compensation. With the feedback control, the payload compensation allows an exact reproduction of manipulator motions. However, with the feedback control, the errors of angular displacements of links 1 and 2 without payload compensation (see Figures 10 and 11) are not more than 2%. Thus, without payload compensation, the use of the feedback control reduces errors, but it does not eliminate them entirely.

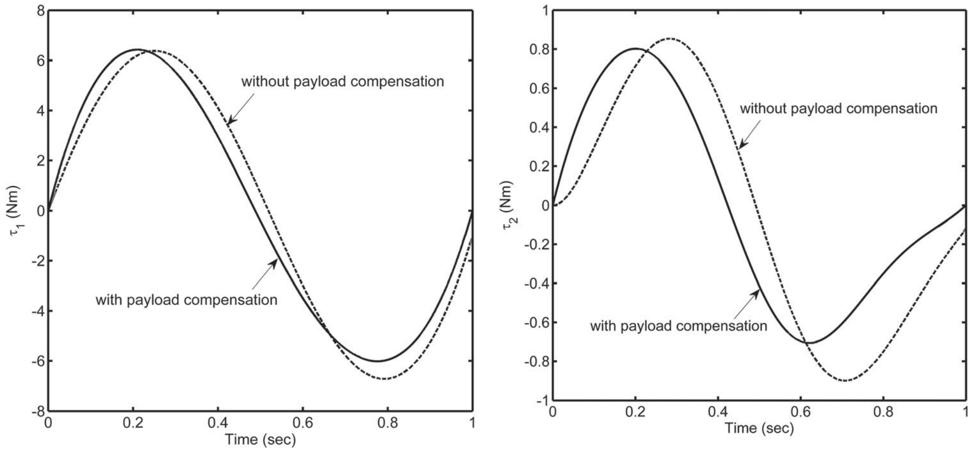

Figure 10. Torques with payload compensation (solid line) and without it (dashed line) for the closed-loop system.

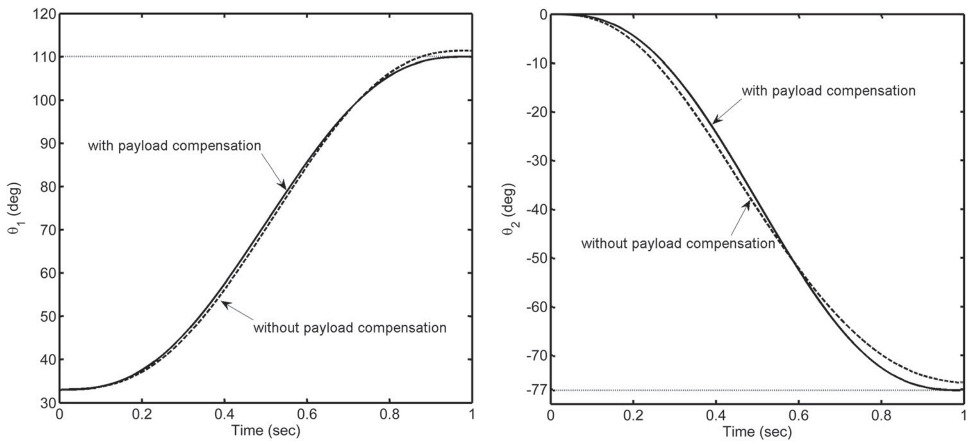

Figure 11. Angular displacements of links with payload compensation (solid line) and without it (dashed line) for the closed-loop system.

6. Discussion

The novelty of the developed method consist in the fact that the opposite rotation for dynamic decoupling is achieved not by including gears in the existing system but by opposite rotation of the links themselves. It is obvious that such a condition can only be satisfied by links with adjustable lengths. The design of links kinematically adjustable is known and the authors were limited just a few references. This is due also to the fact that each designer will solve this problem in his own way, based on the particularity of the developed manipulator (the types of motors and joints, the constraints on weight and size of links, etc.). With regard to possible examples of the application of this method, it should be noted that the frequent adjustment of the links' lengths is not optimal since it results in a loss of time and energy sources. Therefore, the authors see an eventual implementation of the developed method in fixed-sequence manipulators with predetermined initial and final positions of the manipulator gripper. This often occurs in pick-and-place systems with initial and final positions of the gripper predetermined for a certain number of similar cycles. It should be noted that the possible applications would result from these properties and cannot be identified more specifically. Any technological process which needs a fixed-sequence manipulation can be a potential field of application of this solution.

7. Conclusion

This paper deals with the design concept of adjustable serial manipulators with linearised and decoupled dynamics taking into account the changing payload. It is achieved by adding links of adjustable lengths to the initial architecture with a double Scott–Russell mechanism and by using an optimal control technique. Such a dynamic decoupling is a symbiosis of mechanical and control solutions. It is carried out in two steps. At first, the dynamic decoupling of the serial manipulator with adjustable lengths of links is accomplished via an opposite rotation of links and optimal redistribution of masses. Such a solution proposed for the first time allows one to carry out the dynamic decoupling without connection of gears to the oscillating links. The elimination of gears from design concept is a main advantage of the suggested solution. Thus, the proposed mechanical solution allows one to transform the original nonlinear system model into a fully linear system without using the feedback linearisation technique. However, it is obvious that the changing payload leads to the perturbation of the dynamic decoupling of the manipulator. To ensure linearised and decoupled dynamics of the manipulator for any payload, an optimal control technique is applied. It is shown that the dynamic decoupling of the manipulator simplifies the control solution ensuring the dynamic decoupling taking into account the changing payload. The perturbation of required motions of the manipulator with payload compensation and without it is shown via ADAMS and MATLAB simulations. Two kinds of simulations are carried out with open-loop control system which is a non-feedback system and closed-loop control system. The obtained results showed the efficiency of the proposed solution.

Disclosure statement

No potential conflict of interest was reported by the authors.

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