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SURVEY PAPER

Inertia forces and moments balancing in robot manipulators: a review

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ABSTRACT
The balancing of linkages is an integral part of the mechanism design. Despite its long history, mechanism balancing theory continues to be developed and new approaches and solutions are constantly being reported. Hence, the balancing problems are of continued interest to researchers. Several laboratories around the world are very active in this area and new results are published regularly. In recent decades, new challenges have presented themselves, particularly, the balancing of robots for fast manipulation. Various design concepts and methods for balancing of robot manipulators are available in the literature. The author believes that this is an appropriate moment to present the state of the art of the studies devoted to balancing of robot manipulators and to summarize their research results. Thus, the aim of this paper is to propose a review of shaking force and shaking moment balancing methods used in robotics, in particular, for serial and parallel architectures. The described methods are arranged into two principal parts: the resultant inertia force (shaking force) balancing and the resultant inertia moment (shaking moment) balancing. Then each part is divided into subgroups according to features of balancing methods and illustrated via kinematic schemes. At the end of the paper, the balanced robot manipulators having particular structures, the balancing taking into account the payload, the reactionless space robots and the optimization methods used in the balancing of robot manipulators are discussed.

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1. Introduction

It is known that fast-moving mechanical system with rotating and reciprocating masses is a significant source of vibration excitation. The high-speed manipulators can generate significant fluctuating forces with even small amounts of unbalance. In general, two types of forces must be considered: the externally applied forces and the inertial forces. Inertial forces arise when links of a mechanism are subjected to large accelerations. The inertial force system acting on a given link can be represented as an inertia force acting on a line through the center of mass and an inertia torque about the center of mass. The determination of the inertial forces and torques is well known and it has been disclosed in various hand books [1,2]. With regard to the external forces, for example input torques, in many cases, they constitute internal forces with respect to the manipulator as a whole. Thus, if all external active forces applied to the links of a manipulator are internal forces for the mechanism as whole, then the balance of the manipulator will be ensured under the fulfillment of inertia forces and inertia torque cancelation. Therefore, the balancing of shaking force and shaking moment due to the inertial forces of links acquires a specific importance. The quality of balancing of the moving masses has the influence not only on the level of vibrations but also on the resource, reliability, and accuracy of mechanisms. Besides the mentioned negative effects, vibrations bring to the environments pollution and the loss of energy, and can also provoke various health issues. Consequently, the quality improvement of the mass balancing has not only technical, technological, and economical aspects but also social.

Thus, a primary objective of the balancing of manipulators is to cancel or reduce the variable dynamic loads transmitted to the frame and surrounding structures. It is important to note that the reduction of vibrations leads to the increased accuracy of manipulators [3], which is one of the positive consequences of the balancing. As was mentioned in [4] can also be distinguished other advantages of balancing as the reduced cycle time [5], reduced noise, wear, and fatigue [6], as well as the improved ergonomics [7].

Different approaches and solutions devoted to the shaking force and shaking moment balancing have been developed and documented for one degree of freedom mechanisms [8–10]. A new field for their applications is the design of mechanical systems for fast manipulation, which is a typical problem in advanced robotics.

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The balancing of robot manipulators is generally carried out in two steps: (*i*) the cancelation (or reduction) of the shaking force and (*ii*) the cancelation (or reduction) of the shaking moment [11]. Therefore, in the present review, the balancing methods are arranged into two principal parts, which are then decomposed into various subgroups according to the particularities of balancing methods. The criteria for forming any subgroup can be various: structural particularity, nature of balancing force, control mode, etc. The given systematization is not the only way and can be modified according to the subjective preferences of each researcher. Nevertheless, the author believes that this review of state-of-the-art literature systematizes and generalizes the current research trends quite fine and provides a comprehensive view on the problem. However, there are some subgroups which are not included in the principal parts. They are given at the end of the paper: balanced robot manipulators having particular structures, balancing taking into account the payload, reactionless space robots, and optimization methods used in the balancing of robot manipulators. This paper summarizes an overview previously published in the proceedings of the 20th CISM-IFToMM Symposium on Theory and Practice of Robots and Manipulators [12] and presents a more extensive and detailed analysis of balancing methods including a more complete list of literature.

Thus, let us consider firstly the methods for the shaking force balancing of robot manipulators.

2. Inertia forces (shaking force) balancing in robot manipulators

The review of methods devoted to the shaking force balancing of manipulators has shown that following principal subgroups can be distinguished.

2.1. Shaking force balancing by adding counterweights in order to keep the total center of mass of moving links stationary

In the case of open-chain manipulators, we start from the outermost link and add a counterweight to it to bring the center of mass of this link on the immediately preceding joint axis. Such a balancing process must be repeated sequentially until the center of mass of the whole chain is fixed of the base pivot [13,14].

It is obvious that the adding of the supplementary mass due to the counterweights is not desirable because it leads to the increase of the total mass, of the overall size of the robot-manipulator and of the efforts in joints. That is why in many designs of industrial robots the masses of the motors are often used as counterweight (for example, KUKA R360 or PUMA 200, [15]).

With regard to the parallel manipulators, the approach is the same: adding counterweights to keep the total center of mass of moving links stationary. However, the approach is simpler to carry out in planar parallel manipulators (Figure 1) than in spatial parallel manipulators (Figure 2).

To achieve the balancing condition of inertia forces, different mathematical approaches are used, for example, the method of ‘principal vectors’ or the method of ‘static substitution of masses’ [9]. The aim of the ‘method of principal vectors’ is to study the balancing of the mechanism relative to each link and in the determination of those points on the links relative to which a static balance is reached. These points are called ’principal points’. Then, from the condition of similarity of the vector loop of the principal points and the structural loop of the mechanism, the necessary conditions of balancing are derived. It is of a particular importance as it serves to create several auxiliary devices intended for studying the motion of the

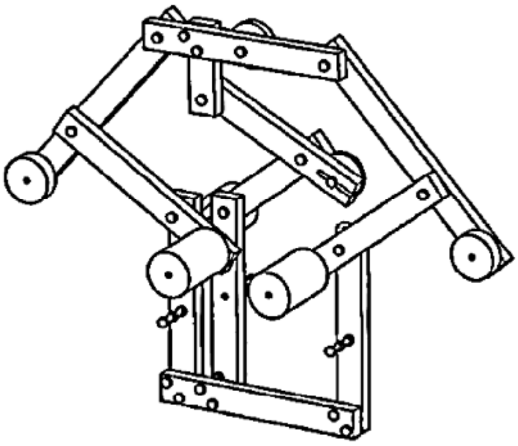

Figure 1. Counterweight balancing of a planar parallel manipulator [16].

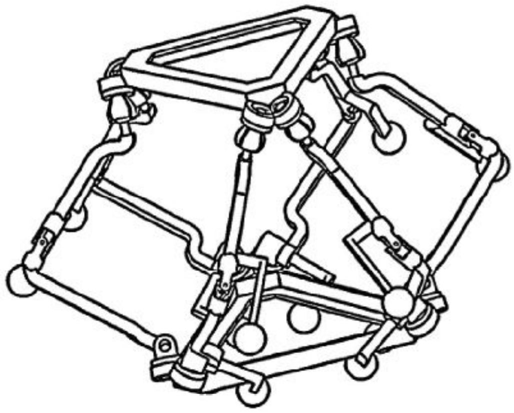

Figure 2. Counterweight balancing of a spatial parallel manipulator [17].

centers of mechanism masses, as well as to synthesize new types of balanced mechanisms [18]. The aim the method of 'static substitution of masses' is to statically substitute the mass of the link (or a platform if it is a parallel manipulator) by concentrated masses, which are balanced thereafter together with the rotating links. Such an approach permits changing the problem of manipulator balancing into a simpler problem of balancing rotating links.

2.2. Shaking force balancing by adding auxiliary structures

Different approaches have developed in order to keep the total center of mass of moving links stationary by adding auxiliary structures.

In [14,19,20], the parallelograms were used as auxiliary structures in order to create the balanced manipulators. As shown in Figure 3, the three scaled lengths are added to form parallelograms and are then used to identify the center of mass C. For the three-link mechanism, the system consists of parallelograms in two layers: the first layer has two parallelograms while the second layer has one. As is mentioned in the cited paper, this procedure can be extended to *n* links.

The pantograph has also been used in order to balance the shaking force. Different solutions were proposed for shaking force and shaking moment balancing of Delta robot [21,22]: by adding a pantograph to each leg or by adding a pantograph connected with the center of mass localized using the parallelograms.

2.3. Shaking force balancing by elastic components

These studies are focused on optimum force balancing of a five-bar parallel manipulator by a combination of a proper distribution of link masses with springs connected to the driving links [23,24]. The force balancing is formulated as an optimization problem in such a way that the root-mean-square values of bearing and spring forces are minimized. However, it should be noted that the springs connected to the driving links produce elastic forces which are internal forces and the added springs cannot have an influence on the shaking force minimization due to the inertia of the moving links. They influence the gravitational forces and the input torques which are also included in the objective function. In the mentioned studies, the authors overlook this fact.

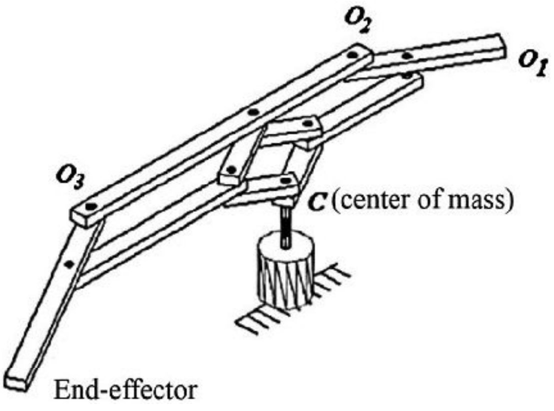

Figure 3. Manipulator with auxiliary parallelograms to locate the center of mass [14].

2.4. Shaking force balancing by adjustment of kinematic parameters

These studies deal with the synthesis of the balanced five-bar mechanism via changing the geometric and kinematic parameters of the mechanical structure [25,26]. The shaking force balancing leads to the conditions which are traditionally satisfied by the redistribution of moving masses. In the mentioned studies, the mass of the link is considered unchanged and the length and the mass center of the links are determined in order to carry out the shaking force balancing. Thus, a new kinematic chain is obtained which is fully force balanced. With regard to the trajectory planning, the authors propose to estimate the given positions of the end effector of the mechanism by the controllers of servomotors. As is rightly mentioned in these studies, the proposed design approach will change the workspace, so some regions of the original workspace may not be reachable. The drawback of this approach is that the project designers set the structural and kinematic tasks, and then the dynamic optimization, sequentially. Fixing the values of moving masses and then finding the kinematic parameters of the mechanism is quite unusual.

This approach was also applied on the design of a spatial three-degree-of-freedom parallel manipulator [27]. Theoretical results were obtained, but cannot be easily used for real application. Therefore, for the shaking force balancing of the proposed spatial three-degree-of-freedom parallel manipulator, another method was used [28].

It seems that the combined optimization including mass and geometric parameters will be more attractive for a wide range of applications of this technique.

2.5. Shaking force minimization via center of mass acceleration minimization

In [29,30], a resourceful solution was developed, which is based on the optimal control of the robot center of masses. The aim of the suggested method consists in the fact that the manipulator is controlled not by applying end-effector trajectories but by planning the displacements of the total mass center of moving links. The trajectories of the total mass center of moving links are defined as straight lines and are parameterized with "bang-bang" motion profiles. Such a control approach allows the reduction of the maximal value of the center of mass acceleration and, consequently, leads to the reduction in the shaking force. Such a balancing technique seems more promising since it is not based on a mass redistribution of moving links but their optimal motion. However, despite the obvious advantages, observations and attempts of practical implementations showed that it also has some drawbacks. Firstly, it is difficult to apply such a balancing on parallel manipulators due to the complexity of relationships between trajectories of the end-effector of the manipulator and its center of mass. Secondly, the control of a robot-manipulator based on the kinematic parameters of a virtual point as a center of masses leads to additional inconvenience. For example, measurements and refinements of the displacements of the total mass center of moving links becomes pretty complex. Another significant imperfection of the mentioned method is the fact that the end-effector trajectory becomes a derivative of the trajectory of the center of masses, i.e. using this balancing method it is possible to ensure only initial and final positions of the end-effector but not a straight line trajectory between them. By imposing a straight line trajectory for the center of masses, it will be obtained nonlinear characteristics for the end-effector trajectory.

3. Inertia moments (shaking moment) balancing in robot manipulators

With regard to the shaking moment balancing of manipulators, the developed approaches can be arranged into following subgroups.

3.1. Shaking moment balancing by counter-rotation

The concept of the shaking moment balancing by counter-rotation was studied for the first time in [31,32]. This approach was developed further in the various studies devoted to the balancing of 1-dof mechanisms and later in [33–39] to multi-dof mechanisms (Figure 4).

As is rightly pointed out in [40], this technique leads to the unavoidable increase in the initial mass and as a result, to the increase in input torques. Thus, the price paid for complete shaking moment balancing is extremely high.

In [41–46], a new design concept was proposed, studied and optimized for light-weight shaking moment balancing by gears. The aim of this concept is to assume both the functions of counter-rotation and counterweight simultaneously (Figure 5), which helps to reduce the mass of the resulting mechanism.

The major disadvantage of this technique is the need for the connection of gears to the oscillating links. The oscillations of the links of the manipulator will create noise unless expensive anti-backlash gears are used. Anti-backlash gears are devices that pre-load the gear always to favor one side of the tooth through spring action. Regardless of the direction of movement, they should always 'push' up against the same side of the tooth. They are basically comprised of two gears that are spring loaded in opposite directions. One gear is attached to the mechanism being moved, while the other simply 'floats' to provide the pre-loading.

In the recent study [47], a new counter-mechanism is proposed in order to dynamically balance a force balanced two degrees of freedom mechanism with variable inertia, which is expected to reduce the drawbacks of the counter-rotation technique.

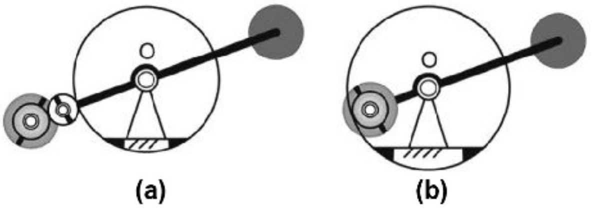

Figure 5. Counter-rotary counter-mass. (a) External gears [4] and (b) Internal gears [4].

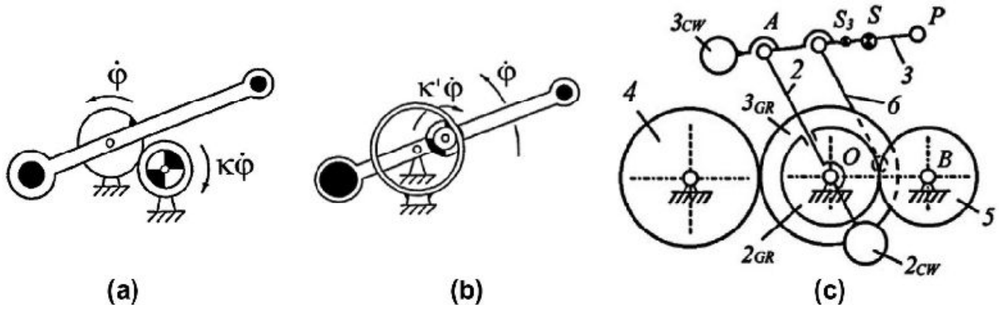

Figure 4. Shaking moment balancing by counter-rotation. (a) Gears with external teeth [41], (b) Gears with internal teeth [41], and (c) Balancing an articulated dyad by gears [36].

3.2. Shaking moment balancing with modules based on dynamically balanced four-bar linkages

In this case, the complete shaking force and shaking moment balancing is carried out without any separate counter-rotation [48–51]. It becomes possible thanks to the synthesis of fully balanced four-bar linkages. It was shown that a four-bar linkage having specific geometric parameters and assuming some report between the lengths of links can be fully balanced only by optimal choice of mass and inertia parameters of moving links. This principle is also practicable when the input angular velocity of the four-bar linkage is variable. Thus, the various structures of manipulators are designed by special legs constructed with modules based on dynamically balanced four-bar linkages (Figure 6).

3.3. Shaking moment balancing by generating optimal trajectories of moving links

In [52], a redundant 3-dof manipulator is designed in which the system center of mass is fixed by an optimal redistribution of masses. Moreover, the dynamics of the system is decoupled. The latter feature simplified the planning of optimal motions in order to balance the shaking moment of the manipulator. A similar study is carried out in [53].

In [54] was proposed a method for obtaining globally optimal motions with minimal base reactions for a redundant mechanical manipulator. The forces transmitted to the supporting base of a manipulator are desired to be small so as to reduce the stresses and the magnitude of vibration in the supporting structure. The proposed method was illustrated via a planar 3R manipulator.

Shaking moment balancing by prescribed rotation of the end-effector was proposed in [55–57]. The shaking moment of 3-dof planar parallel manipulator [55] was canceled using two approaches: through a proper choice of inertia and geometric parameters and by using appropriate motion planning. The shaking moment on the frame of the SCARA-type robots with 4-dof has been eliminated by a prescribed velocity of the end-effector [56]. Taking into account that the two angles of the linear positioning do not depend on the orientation angle, it was proposed to rotate the end-effector during the linear displacements of the end-effector and to balance in such a manner the shaking moment of the robot. The advantage of such a balancing is its simplicity because the complete balancing of the shaking moment is achieved without significant design modifications. The major drawback is the increase of the inertia moment of the end-effector in order to compensate the inertia moment of the other rotating links. A similar approach has been applied on the PAMINSA manipulator in [57].

3.4. Shaking moment balancing by adding an inertia flywheel rotating with a prescribed angular velocity

It is well known that after shaking force balancing, the shaking moment applied on the base is constant relative to any point [58], i.e. for a given position of the mechanism it has the same value for any point of the base. Taking into account this property, the shaking moment of any planar manipulator can be balanced adding an inertia flywheel rotating with a prescribed angular velocity (Figure 7) [36]. This balancing technique has been also applied on the 3-dof planar parallel manipulator with unlimited rotation capability [59]. A similar approach based on the active balancing of the shaking moment of the Delta robot by three additional rotating inertia was discussed in [21,60]. Active balancing of the Hummingbird minipositioner with three axis servo mechanisms was discussed in [61].

The redundant servomotors was also used in order to improve the dynamic performance of robots and to carry out shaking moment, driving torque, and ground joint force minimization [62,63].

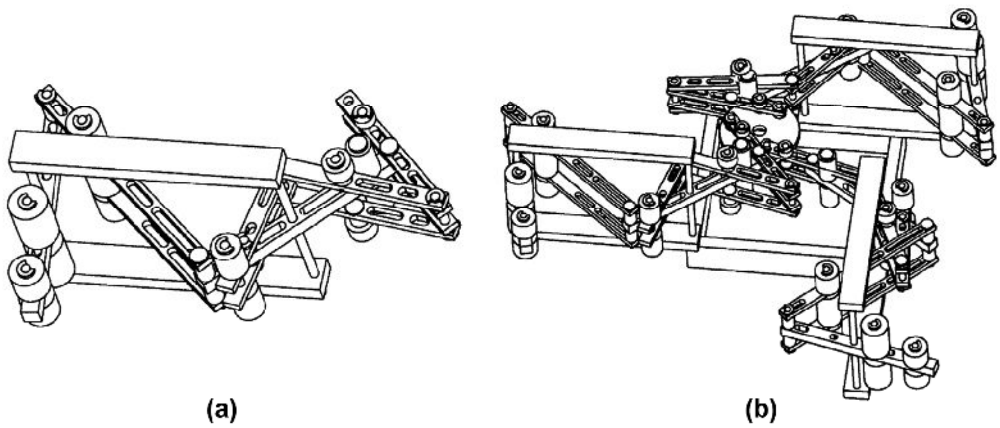

Figure 6. Balancing by adding four-bar linkages. (a) Planar 2-DOF mechanism [49] and (b) Planar 3-DOF mechanism [49].

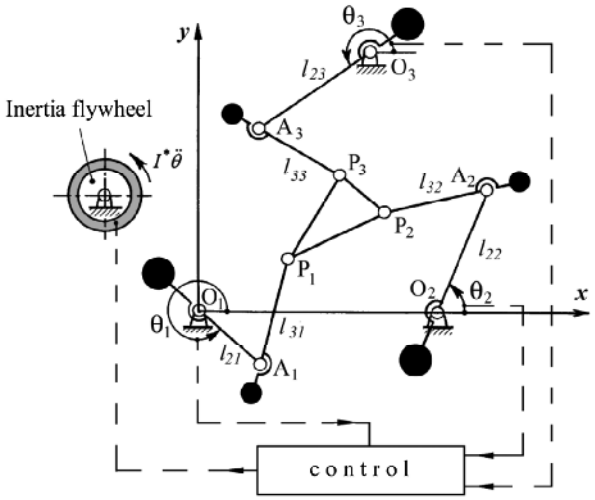

Figure 7. Shaking moment balancing of fully force-balanced 3-DOF 3-RRR parallel manipulator by an inertia flywheel [36].

However, it should be noted that the practical validity of this balancing solution depends on the technical potentials to generate the desirable input motion with great precision. In reality, several factors affect unfavorably, such as the elasticity of the links, joint clearance, and superposition of natural vibrations over the prescribed motion. Unfortunately, the mentioned factors act simultaneously. Therefore, only extensive experimental studies would assess the real effect of the balancing procedure.

It is difficult to provide an overall systematization of all results in a single system. That is why let us now consider some studies which are not included in the review structure of shaking force and shaking moment balancing methods given below.

At first, it should be noted that the new balanced structures were also developed. In [64], a dynamically balanced 3-DoF planar parallel manipulator has been presented and tested. The manipulator is composed of two independently force-balanced five-bar linkages pivoted to the base and coupled with an end-effector link. In this manipulator, each leg was balanced separately, which has been made possible by distributing the inertia of the platform on each of its attachment points [36,65].

In [66], a novel 3-DOF parallel mechanism referred to as the parallelepiped mechanism has been developed. Counterweights and counter-rotations were used to dynamically balance the proposed mechanism.

Design of a fully force-balanced redundant planar 4-RRR parallel manipulator has also been developed [67,68]. In this case, some properties of the balanced parallelogram system are used. The counterweights added only in the links mounted on the frame allows a complete cancelation of inertia forces in the manipulator.

The complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs [69] and with variable payload [33,51,60,70,71] have also been studied. In this case, the adjustable parameters are introduced into the balancing system to ensure the shaking force and shaking moment balancing taking into account the payload. However, it should be noted that such solutions are quite complex.

In the field of free-floating space robots the design of reactionless robots was also studied. The formalism called “Reaction Null Space” was initially introduced in [72] (see also [73]). Later, it has been applied to reactionless motion generation and vibration control with flexible base robots [74–78]. The dynamic balancing of a novel 2-dof reactionless pointing mechanism for satellite antennas was studied in [79]. In regard to the shaking moment, its elimination is achieved by the use of the ball joint as a mounting mechanism of the antenna. As mentioned in this study, the torques are produced by the actuators which are counteracted internally in the counterweight due to the special actuation principle.

The study [80] deals with a novel scheme for the motion planning of a dual-arm free-floating planar manipulator where one arm is commanded to perform desired tasks while the other provides compensating motions to keep the base inertially fixed.

The use of kinematic redundancy for robot base reaction reduction was explored in [81,82]. The given numerical examples demonstrate that the developed approach is effective for reducing base reactions for planar and spatial robots.

The study [83] demonstrated that three orthogonally mounted wheels in the attitude-control system can compensate the total moment of the system. They further show that induced translational motion of the base can be counteracted using a set of augmented inverse-kinematic relations when calculating the commanded joint variables.

In [84], the control-moment gyroscopes are proposed as actuators for a spacecraft-mounted robotic arm to reduce reaction forces and torques on the spacecraft base.

The balancing has been also used to pass through a type II singularity of parallel robots [85]. The balancing conditions were derived based on the previously developed studies [86,87].

In the context of dynamic decoupling of serial manipulators new balancing solutions have been developed in [88,89]. The developed balancing solutions accomplish the dynamic decoupling via opposite rotation of adjustable links [88] or using a Scott Russell mechanism [89].

Finally, it should be noted that the various optimization methods were also applied in order to reduce the shaking force and shaking moment of robot manipulators [90–95]. In [90], it is proposed to carry out the shaking force and shaking moment balancing via minimization of global dynamic characteristics combining generalized inertia ellipsoid or isotropy. The sensitivities of the shaking moment to the position, velocity and acceleration of the manipulator are also used as the objective function to minimize the shaking moment [91,92]. In studies [93] and [94], the derivation of the shaking force and moment balancing conditions for a five-bar planar manipulator is obtained. These conditions are expressed as a system of seven equations and three inequalities with twelve parameters. Then, the parameters are solved as an optimum design problem, under nonlinear equality constraints. A similar study has been discussed in [95]. An approach to optimally locate a given trajectory profile and path, permitting to minimize the shaking force of parallel kinematics machines, has been proposed in [96]. The proposed methodology has been applied to the Orthoglide.

However, without reducing of usefulness and efficiency of all mentioned optimization methods, it should be noticed that they are nevertheless based on numerical approaches and don't reveal details of general properties of balancing solutions.

4. Conclusions

The survey of investigations into the shaking force and shaking moment balancing of robot manipulators showed that not only were the known methods of linkage balancing found further development, but proposed new approaches. The counter-rotation balancing, which seemed to be an essential technique in linkage balancing, was not so indispensable in robotics. Other solutions can be emphasized, such as the design of reactionless manipulators with dynamically balanced modules. For the moment, there is not a large choice of these modules, but the development of new balanced modules seems to be a promising technique, with genuinely potential. An interesting technique that should also be noted is the balancing by generation optimal motions of moving links of robot manipulators. Indeed, the input control laws generating optimized motions of moving masses can be a useful source to minimize the shaking force and shaking moment on the frame of a robot manipulator. It is important to note that often the balancing of robot manipulators leads to the increase of moving masses and as a result, to the increase of input torques. On one hand, the added masses help to reduce the variable dynamic loads on the frame. On the other hand, they increase inertia of moving masses and as a result input torques. From this point of view, the balancing via optimal motion generation of moving links of robot manipulators is more efficient because it is carried out without modification of the initial structure or masses of the robot manipulator.

With regard to the various optimization techniques, it is obvious that they can be useful tools for designers. However, it is important to pay attention to the physical interpretation of the obtained results in order to avoid possible errors.

It is to be hoped that this overview will be useful for the readers and provide a global view on the problem of shaking force and shaking moment balancing of robot manipulators.

Disclosure statement

No potential conflict of interest was reported by the author.

Notes on contributor

Vigen Arakelian is a full professor in Robotics and Mechatronics at the INSA (Institut National des Sciences Appliquées) in Rennes, as well as researcher in LS2N (Le Laboratoire des Sciences du Numérique de Nantes). His research interests span mechanics and control of robots, design of assistive rehabilitative devices, balancing and synthesis of mechanisms, numerical simulation and optimization of mechanical systems by using ADAMS software. He is author of two books devoted to the study of linkages and robots, published by HERMES in 1997 and SPRINGER in 2015, more than 200 scientific publications and 23 inventions. He has both industrial and academic experience. He developed many innovative engineering projects which were sources of several patents applied in the fields of machine design. He was a project leader of international and national grants, as well as many industrial research programs.

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