Degeneracy Conditions of the Dynamic Model of Parallel Robots

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Séminaire
Université Laval
Québec, QC, Canada

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Outline

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A few data about IRCCyN

Institut de Recherche en Communications et Cybernétique de Nantes

- 270 people, among which ∼110 researchers / professors, ∼130 PhD std., ∼30 staff.
- 11 research teams in different areas (Robotics, Control, Process, Signal Processing, etc.)
- The biggest lab. in the mentioned fields for the North-West of France

One lab, different locations

- Ecole Centrale Nantes
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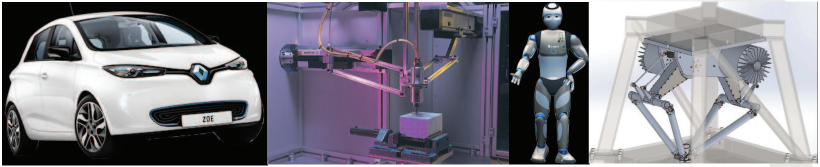
A large panel of benchmarks



But also : Orthgolide 5 dof, Kuka LWR, Staubli TX40, Baxter, Nao, NaVARo, Eel-like robot, Electro-sensing benchmark, helicopters, etc.

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Introduction

Parallel robots :

- Advantages : large payload-to-weight ratio, high accelerations, etc
- Drawbacks : small workspace, with singularities in it

Various types of singularities, but two categories of phenomena :

- Loss of motion capabilities : Type 1 sing. (belong to the serial sing.)
- Gain of uncontrollable motion : Type 2 sing., constraint sing., but also a less known type of serial sing. due to the degeneracy of the passive twist system in the leg (LPJTS sing. – ex : Tripteron)

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Solutions to enlarge the operational workspace size :

- Optimal design (decoupled robots, redundancy, etc.)
- Changing working modes
  - Passing a Type 1 singularity (ex : DexTAR of Bonev/Bigras)
  - Passing a LPJTS singularity (never done)
- Changing assembly modes
  - Non singular changing : motion around a cusp point
  - Singular changing : crossing a Type 2 singularity / Necessity of optimal traj. planning using a criterion based on the robot dynamics
- The two last solutions are promising !

Aim of the presentation :

- to use a straightforward formalism for the dynamic modeling of pkm
- to analyze all degeneracy conditions for the pkm dynamics
- to show that passing a LPJTS singularity require to respect another criterion based on the robot dynamics

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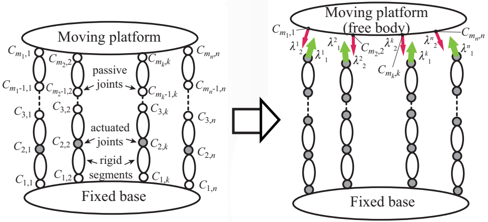
General Methodology

- To virtually open the robot loop and compute the dynamic models of :
  - A virtual tree structure (all joints actuated)

$$\tau_t = \mathcal{F}_t \left(\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t, \chi_{st_t} \right)$$

  - A free moving platform

$$\tau_p = \mathcal{F}_p \left(\mathbf{x}, \mathbf{t}, \dot{\mathbf{t}}, \chi_p \right)$$



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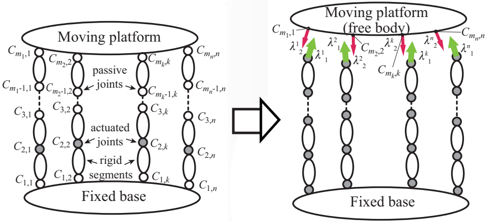

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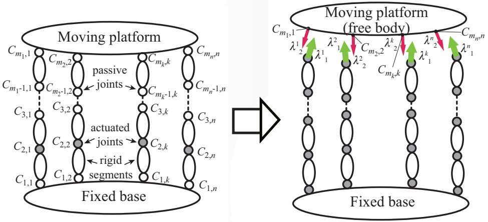

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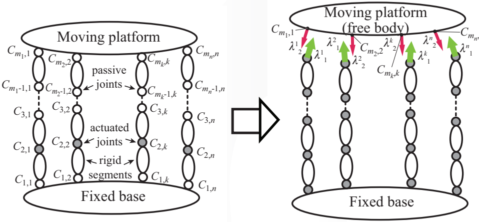

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Kinematic constraints

- Dynamic model of the virtual structure expressed as a function of :
  - $\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t$
  - $\mathbf{x}, \mathbf{t}, \dot{\mathbf{t}}$
  - not of $\mathbf{q}_a, \dot{\mathbf{q}}_a, \ddot{\mathbf{q}}_a$
- Necessity to express the kinematic relations between $\dot{\mathbf{q}}_t$, $\dot{\mathbf{q}}_a$ and $\mathbf{t}$
- Skipping the mathematical derivations,

$$\mathbf{A}_p\mathbf{v} + \mathbf{B}_p\dot{\mathbf{q}}_a = \mathbf{0}$$

$$\mathbf{J}_{tk}\mathbf{v} - \mathbf{J}_{k_a}\dot{\mathbf{q}}_a - \mathbf{J}_{k_d}\dot{\mathbf{q}}_d = \mathbf{0}$$

with

- $\mathbf{v}$ independent components of $\mathbf{t}$ ($\dim(\mathbf{v}) = \dim(\dot{\mathbf{q}}_a) \leq \dim(\mathbf{t}) = 6$)
- $\mathbf{A}_p$ square matrix : $\det(\mathbf{A}_p) = 0$ iff Type 2 sing.
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General Dynamic Model

- Use of Lagrange multipliers

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{t_a} - \mathbf{J}_{k_a}^T \boldsymbol{\lambda}_1 - \mathbf{B}_p^T \boldsymbol{\lambda}_2$$

$$\mathbf{J}_{k_d}^T \boldsymbol{\lambda}_1 = \boldsymbol{\tau}_{t_d}$$

$$-\mathbf{J}_{tk}^T \boldsymbol{\lambda}_1 + \mathbf{A}_p^T \boldsymbol{\lambda}_2 = \boldsymbol{\tau}_{pr}$$

- Skipping all mathematical derivations, it comes that

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{t_a} - (\mathbf{J}_{k_a}^T + \mathbf{B}_p^T \mathbf{A}_p^{-T} \mathbf{J}_{tk}^T)\mathbf{J}_{k_d}^{-T} \boldsymbol{\tau}_{t_d} - \mathbf{B}_p^T \mathbf{A}_p^{-T} \boldsymbol{\tau}_{pr}$$

- Thus, the dynamic model degenerates
  - if $\det(\mathbf{A}_p) = 0$ (i.e. Type 2 sing.)
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Degeneracy due to the matrix $\mathbf{A}_p$

- If $\mathbf{A}_p$ is rank-deficient, a twist $\mathbf{t}_s$ exists such that

$$\mathbf{A}_p \, \mathbf{t}_s = 0 \Leftrightarrow \mathbf{t}_s^T \, \mathbf{A}_p^T = 0$$

- $\mathbf{t}_s$ defines the gained motion inside the Type 2 singularity
- Let us rewrite the last Lagrange equation :

$$-\mathbf{J}_{tk}^T \lambda_1 + \mathbf{A}_p^T \lambda_2 = \boldsymbol{\tau}_{pr}$$

$$\Rightarrow \mathbf{A}_p^T \lambda_2 = \boldsymbol{\tau}_{pr} - \mathbf{J}_{tk}^T \lambda_1 = \boldsymbol{\tau}_{pr} - \mathbf{J}_{tk}^T \mathbf{J}_{k_d}^{-T} \boldsymbol{\tau}_{t_d} = \mathbf{w}_p$$

- Left-multiplying by $\mathbf{t}_s^T$

$$\mathbf{t}_s^T \, \mathbf{A}_p^T \lambda_2 = 0 \Rightarrow \mathbf{t}_s^T \, \mathbf{w}_p = 0$$

- $\mathbf{t}_s^T \, \mathbf{w}_p = 0$: in order to avoid infinite input efforts while crossing a Type 2 singularity, the sum of the wrenches applied on the platform by the legs, inertia/gravitational effects and external environment must be reciprocal to the uncontrollable motion of the platform inside the singularity (in other words, the power of these wrenches along the platform uncontrollable motion must be null).

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- $\dot{\mathbf{q}}_d^s$ defines the gained motion inside the LPJTS singularity
- Let us rewrite the second Lagrange equation :

$$\mathbf{J}_{k_d}^T \boldsymbol{\lambda}_1 = \boldsymbol{\tau}_{t_d}$$

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- $\dot{\mathbf{q}}_d^s$ defines the gained motion inside the LPJTS singularity
- Let us rewrite the second Lagrange equation :

$$\mathbf{J}_{k_d}^T \boldsymbol{\lambda}_1 = \boldsymbol{\tau}_{t_d}$$

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- Left-multiplying by $\dot{\mathbf{q}}_d^{s\,T}$

$$\dot{\mathbf{q}}_d^{s\,T}\mathbf{J}_{k_d}^T \boldsymbol{\lambda}_1 = 0 \Rightarrow \dot{\mathbf{q}}_d^{s\,T}\boldsymbol{\tau}_{t_d} = 0$$

- $\dot{\mathbf{q}}_d^{s\,T}\boldsymbol{\tau}_{t_d} = 0$: the input efforts of the virtual system in the joints that correspond to the passive joints of the real robot must be reciprocal to the uncontrollable motion of the passive legs inside the singularity (in other words, the power of these efforts along the leg uncontrollable motion must be null).

Degeneracy due to the matrix $\mathbf{J}_{k_d}$

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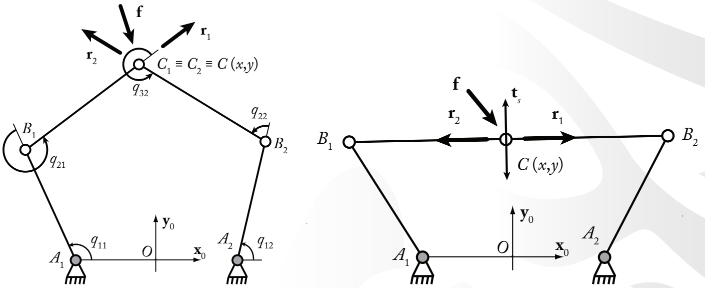
Physical interpretation

Impact of the degeneracy of the matrix $\mathbf{A}_p$

- Analysis of a five-bar mechanism / A single loading **f**



(a) out of Type 2 sing. (b) in a Type 2 sing.

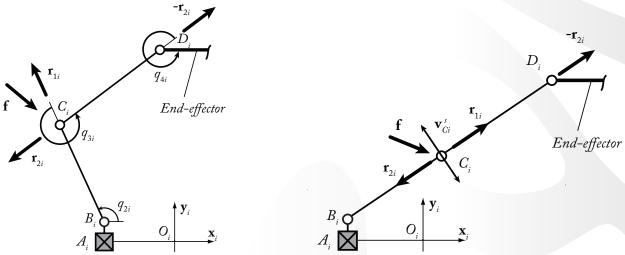
FIGURE: The five-bar mechansim

- If $\mathbf{t}_s^T\mathbf{f} \neq 0$: infinite joint reactions
- If $\mathbf{t}_s^T\mathbf{f} = 0$: finite joint reactions

Physical interpretation

Impact of the degeneracy of the matrix $\mathbf{J}_{k_d}$

- Analysis of a Tripteron leg / A single loading **f**



(a) out of LPJTS sing. (b) in a LPJTS sing.

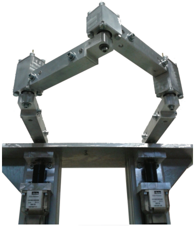
FIGURE: A leg of the Tripteron

- If $\mathbf{t}_s^T \mathbf{f} = \dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d \neq 0$: infinite joint reactions
- If ${\mathbf{v}_C^s}^T \mathbf{f} = \dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$: finite joint reactions

Outline

Benchmark

- Prototype of five-bar mechanism (IFMA, Clermont-Ferrand)



- Dynamic model (identified parameters) :

$$\tau = \mathbf{w}_b - \mathbf{B}_p^T \lambda_2, \mathbf{A}_p^T \lambda_2 = \mathbf{w}_p$$

with

$$\mathbf{w}_p = m_3 \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}, \mathbf{w}_b = \begin{bmatrix} zz_{11R} \ \ddot{q}_{11} \\ zz_{12R} \ \ddot{q}_{12} \end{bmatrix} + \begin{bmatrix} f_{v11} \dot{q}_{11} \\ f_{v21} \dot{q}_{12} \end{bmatrix} + \begin{bmatrix} f_{s11} sign(\dot{q}_{11}) \\ f_{s12} sign(\dot{q}_{12}) \end{bmatrix}$$

- Criterion for crossing the Type 2 singularities : $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{2i})$

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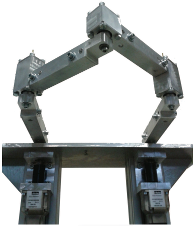

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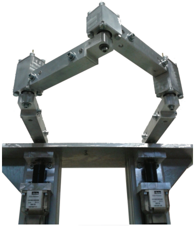

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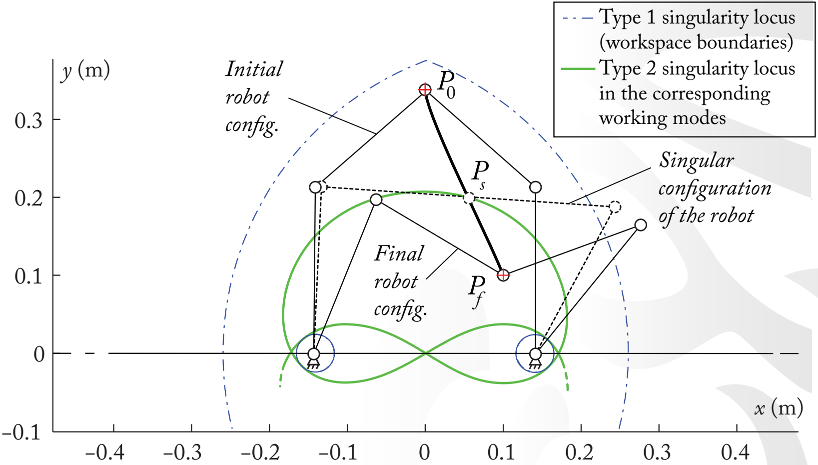
Crossing Type 2 singularities



- Two types of trajectory
  - Case 1 : without respecting the criterion $\ddot{y} = \ddot{x}\tan(q_{1i} + q_{2i})$
  - Case 2 : respecting the criterion $\ddot{y} = \ddot{x}\tan(q_{1i} + q_{2i})$

Trajectory planning

TABLE: Boundary conditions for the two trajectories used on the five-bar mechanism.

Trajectory for Case 1		Trajectory for Case 2		
$t = 0$ sec	$t = t_f = 1.5$ sec	$t = 0$ sec	$t = t_f = 1.5$ sec	$t = t_s = 0.75$ sec
$x(t=0) = x_{p_0}$	$x(t=t_f) = x_{p_f}$	$x(t=0) = x_{p_0}$	$x(t=t_f) = x_{p_f}$	$x(t=t_s) = x_{p_s} = 0.0543$ m
$\dot{x}(t=0) = 0$	$\dot{x}(t=t_f) = 0$	$\dot{x}(t=0) = 0$	$\dot{x}(t=t_f) = 0$	$\dot{x}(t=t_s) = 0.1671$ m/s
$\ddot{x}(t=0) = 0$	$\ddot{x}(t=t_f) = 0$	$\ddot{x}(t=0) = 0$	$\ddot{x}(t=t_f) = 0$	$\ddot{x}(t=t_s) = 6.8e^{-4}$ m/s^2
$y(t=0) = y_{p_0}$	$y(t=t_f) = y_{p_f}$	$y(t=0) = y_{p_0}$	$y(t=t_f) = y_{p_f}$	$y(t=t_s) = y_{p_s} = 0.2$ m
$\dot{y}(t=0) = 0$	$\dot{y}(t=t_f) = 0$	$\dot{y}(t=0) = 0$	$\dot{y}(t=t_f) = 0$	$\dot{y}(t=t_s) = -0.4812$ m/s
$\ddot{y}(t=0) = 0$	$\ddot{y}(t=t_f) = 0$	$\ddot{y}(t=0) = 0$	$\ddot{y}(t=t_f) = 0$	$\ddot{y}(t=t_s) = -0.01$ m/s^2

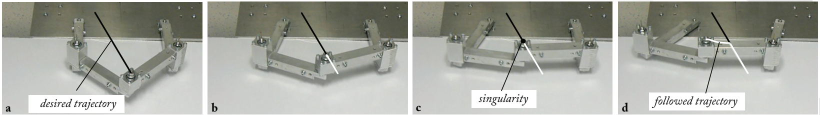
Trajectory planning

TABLE: Coefficients of the polynomials for each trajectory used on the five-bar mechanism.

	Polynomials for Case 1		Polynomials for Case 2	
	$x(t)=\sum_{i=0}^{5} a_i t^i$	$y(t)=\sum_{i=0}^{5} a_i t^i$	$x(t)=\sum_{i=0}^{8} a_i t^i$	$y(t)=\sum_{i=0}^{8} a_i t^i$
a_0	0	0.338175237168	0	0.338175237168
a_1	0	0	0	0
a_2	0	0	0	0
a_3	0.296296296296	−0.705704406423	0.030616142296	0.403081224309
a_4	−0.296296296296	0.705704406423	0.364976100965	−1.953915773554
a_5	0.079012345679	−0.188187841713	−0.089638089174	0.149034398769
a_6	−−	−−	−0.638891733361	3.132585241250
a_7	−−	−−	0.555392794445	−2.569364459356
a_8	−−	−−	−0.131974503408	0.600075981487

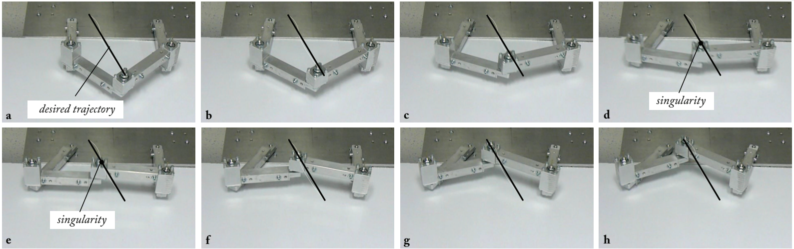
Results

Case 1 : without respecting the criterion $\ddot{y} = \ddot{x}\tan(q_{1i} + q_{2i})$



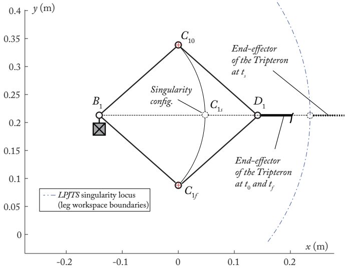
Results

Case 2 : respecting the criterion $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{2i})$



Crossing LPJTS singularities

- Simulate the Tripteron behavior with the five-bar mechanism



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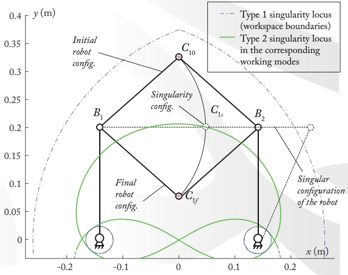

- Two types of trajectory
- Case A : without respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$
- Case B : respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$

Trajectory planning

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$\dot{y}(t=0) = 0$	$\dot{y}(t=t_f) = 0$	$\dot{y}(t=0) = 0$	$\dot{y}(t=t_f) = 0$	$\dot{y}(t=t_s) = 0.147$ m/s
$\ddot{y}(t=0) = 0$	$\ddot{y}(t=t_f) = 0$	$\ddot{y}(t=0) = 0$	$\ddot{y}(t=t_f) = 0$	$\ddot{y}(t=t_s) = -0.693$ m/s^2

$$x(t) = x_{B_1} + \sqrt{d_{31}^2 + (y(t) - y_{B_1})^2}$$

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a_0	0.338174999999996	0.338175000000007
a_1	0	0
a_2	0	0
a_3	−2.503500000000006	3.051722491923807
a_4	3.755249999999982	−23.590518369448382
a_5	−1.502099999999988	43.558974062571224
a_6	−−	−26.660836890258111
a_7	−−	−0.254587389021722
a_8	−−	3.644896094233372

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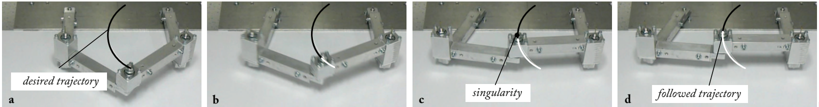
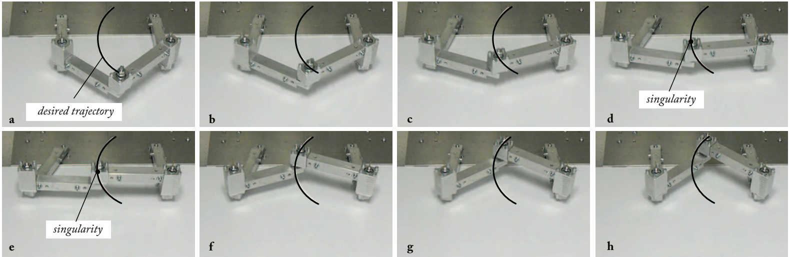
Case A : without respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$



Results

Case B : respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$



Outline

Conclusion

Aim of the present work

- To fulfil the lacks of the previous studies and to analyze all degeneracy conditions of the full parallel robot dynamic model which takes into account all link dynamic parameters,
- To demonstrate that the LPJTS singularities impact the robot effort transmission, as this point is usually bypassed in the literature, and
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