Degeneracy Conditions of the Dynamic Model of Parallel Robots

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> > 22 juillet 2014



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Institut de Recherche en Communications et Cybernétique de Nantes

- $\bullet~270$ people, among which ${\sim}110$ researchers / professors, ${\sim}130$ PhD std., ${\sim}30$ staff.
- 11 research teams in different areas (Robotics, Control, Process, Signal Processing, etc.)
- The biggest lab. in the mentioned fields for the North-West of France

One lab, different locations

- Ecole Centrale Nantes
- Ecole des Mines Nantes
- Polytech Nantes
- **Université Nantes**
- IUT Nantes

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The Robotics	team a	t IRCCyN	i R	Cyn

The biggest team of the IRCCyN

- \bullet 55 people, among which ${\sim}19$ researchers / professors, ${\sim}23$ PhD std., ${\sim}8$ staff.
- 4 main research themes (Industrial robots, Humanoid Robots, Bio-inspired Robots, Mobile Robots)

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A large panel of benchmarks



But also : Orthgolide 5 dof, Kuka LWR, Staubli TX40, Baxter, Nao, NaVARo, Eel-like robot, Electro-sensing benchmark, helicopters, etc.

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Introduction			ĨR	C:n

Parallel robots :

- Advantages : large payload-to-weight ratio, high accelerations, etc
- Drawbacks : small workspace, with singularities in it

Various types of singularities, but two categories of phenomena :

- Loss of motion capabilities : Type 1 sing. (belong to the serial s
- Gain of uncontrollable motion : Type 2 sing., constraint sing., but also a less known type of serial sing. due to the degeneracy of the passive twist system in the leg (LPJTS sing. – ex : Tripteron)

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- Optimal design (decoupled robots, redundancy, etc.)
- Changing working modes
 - Passing a Type 1 singularity (ex : DexTA
 - Passing a LPJTS singularity (never done
- Changing assembly modes
 - Non singular changing : motion around a cusp point
 - Singular changing a crossing of operating aligned and consistivation of the essity of a constraint of the constraintof the constraint of the constraint of the constraint of the co
- The two last solutions are promising !

Aim of the presentation :

- to use a straightforward formalism for the dynamic modeling of pkm
- to analyze all degeneracy conditions for the pkm dynamics
- to show that passing a LPJTS singularity require to respect another criterion based on the robot dynamics

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General Met	nodology		IR	CYN

- To virtually open the robot loop and compute the dynamic models of :
 - A virtual tree structure (all joints actuated)

$$\boldsymbol{\tau}_{t} = \mathcal{F}_{t}\left(\mathbf{q}_{t}, \dot{\mathbf{q}}_{t}, \ddot{\mathbf{q}}_{t}, \boldsymbol{\chi}_{st_{t}}\right)$$

$$\boldsymbol{ au}_{p}=\mathcal{F}_{p}\left(\mathbf{x},\mathbf{t},\dot{\mathbf{t}},\boldsymbol{\chi}_{p}
ight)$$



To close the loop by using the Lagrange multipliers

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• To close the loop by using the Lagrange multipliers

^{oo} Kinematic o	constraints	00000	0000000	°

• Dynamic model of the virtual structure expressed as a function of :

- $\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t$
- x, t, ṫ
- not of q_a, q_a, q_a
- Necessity to express the kinematic relations between $\dot{\mathbf{q}}_{b}, \dot{\mathbf{q}}_{a}$ and \mathbf{r}
- Skipping the mathematical derivations,

$$\mathbf{A}_{p}\mathbf{v}+\mathbf{B}_{p}\dot{\mathbf{q}}_{a}=\mathbf{0}$$
 $\mathbf{J}_{tk}\mathbf{v}-\mathbf{J}_{k_{a}}\dot{\mathbf{q}}_{a}-\mathbf{J}_{k_{d}}\dot{\mathbf{q}}_{d}=\mathbf{0}$

with

- **v** independent components of **t** $(\dim(\mathbf{v}) = \dim(\dot{\mathbf{q}}_a) \leq \dim(\mathbf{t}) =$
- A_p square matrix : det(A_p) = 0 iff Type 2 sing.
- \mathbf{B}_p square matrix : det $(\mathbf{B}_p) = 0$ iff Type 1 sing.
- \mathbf{J}_{k_d} square matrix : det $(\mathbf{J}_{k_d}) = 0$ iff LPJTS sing.

Kinematic co	onstraints		ÎR	<u>C</u> yn
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• Necessity to express the kinematic relations between $\dot{\mathbf{q}}_t,\,\dot{\mathbf{q}}_a$ and t

Skipping the mathematical derivations,

$$\mathbf{A}_{p}\mathbf{v} + \mathbf{B}_{p}\dot{\mathbf{q}}_{a} = \mathbf{0}$$

 $\mathbf{J}_{tk}\mathbf{v} - \mathbf{J}_{ks}\dot{\mathbf{q}}_{a} - \mathbf{J}_{ks}\dot{\mathbf{q}}_{d} = \mathbf{0}$

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• Use of Lagrange multipliers

$$egin{aligned} & m{ au} = m{ au}_{t_a} - m{J}_{k_a}^T m{\lambda}_1 - m{B}_p^T m{\lambda}_2 \ & m{J}_{k_d}^T m{\lambda}_1 = m{ au}_{t_d} \ & -m{J}_{tk}^T m{\lambda}_1 + m{A}_p^T m{\lambda}_2 = m{ au}_{pr} \end{aligned}$$

Skipping all mathematical derivations, it comes that

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{t_a} - (\mathbf{J}_{k_a}^{\mathsf{T}} + \mathbf{B}_{\rho}^{\mathsf{T}} \mathbf{A}_{\rho}^{-\mathsf{T}} \mathbf{J}_{t_k}^{\mathsf{T}}) \mathbf{J}_{k_d}^{-\mathsf{T}} \boldsymbol{\tau}_{t_d} - \mathbf{B}_{\rho}^{\mathsf{T}} \mathbf{A}_{\rho}^{-\mathsf{T}} \boldsymbol{\tau}_{\rho_d}$$

Thus, the dynamic model degenerates

• if det(\mathbf{A}_{p}) = 0 (i.e. Type 2 sing.)

• but also if $det(\mathbf{J}_{k_d}) = 0$ (i.e. LPJTS sing.) \Rightarrow never observed

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• Use of Lagrange multipliers

$$oldsymbol{ au} = oldsymbol{ au}_{t_a} - oldsymbol{\mathsf{J}}_{k_a}^T oldsymbol{\lambda}_1 - oldsymbol{\mathsf{B}}_p^T oldsymbol{\lambda}_2 \ oldsymbol{\mathsf{J}}_{k_d}^T oldsymbol{\lambda}_1 = oldsymbol{ au}_{t_d} \ -oldsymbol{\mathsf{J}}_{tk}^T oldsymbol{\lambda}_1 + oldsymbol{\mathsf{A}}_p^T oldsymbol{\lambda}_2 = oldsymbol{ au}_{pr}$$

• Skipping all mathematical derivations, it comes that

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{t_a} - (\mathbf{J}_{k_a}^{\mathsf{T}} + \mathbf{B}_{p}^{\mathsf{T}} \mathbf{A}_{p}^{-\mathsf{T}} \mathbf{J}_{tk}^{\mathsf{T}}) \mathbf{J}_{k_d}^{-\mathsf{T}} \boldsymbol{\tau}_{t_d} - \mathbf{B}_{p}^{\mathsf{T}} \mathbf{A}_{p}^{-\mathsf{T}} \boldsymbol{\tau}_{pr}$$

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• Use of Lagrange multipliers

$$egin{aligned} & \mathbf{ au} = oldsymbol{ au}_{t_a} - \mathbf{J}_{k_a}^{ op} oldsymbol{\lambda}_1 - \mathbf{B}_p^{ op} oldsymbol{\lambda}_2 \ & & \mathbf{J}_{k_d}^{ op} oldsymbol{\lambda}_1 = oldsymbol{ au}_{t_d} \ & & - \mathbf{J}_{tk}^{ op} oldsymbol{\lambda}_1 + \mathbf{A}_p^{ op} oldsymbol{\lambda}_2 = oldsymbol{ au}_{pr} \end{aligned}$$

• Skipping all mathematical derivations, it comes that

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{t_a} - (\boldsymbol{\mathsf{J}}_{k_a}^{\mathsf{T}} + \boldsymbol{\mathsf{B}}_{\rho}^{\mathsf{T}} \boldsymbol{\mathsf{A}}_{\rho}^{-\mathsf{T}} \boldsymbol{\mathsf{J}}_{tk}^{\mathsf{T}}) \boldsymbol{\mathsf{J}}_{k_d}^{-\mathsf{T}} \boldsymbol{\tau}_{t_d} - \boldsymbol{\mathsf{B}}_{\rho}^{\mathsf{T}} \boldsymbol{\mathsf{A}}_{\rho}^{-\mathsf{T}} \boldsymbol{\tau}_{\rho r}$$

- Thus, the dynamic model degenerates
 - if det(\mathbf{A}_p) = 0 (i.e. Type 2 sing.)
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Degeneracy	due to the	e matrix \mathbf{A}_{p}	i R	Cyn

$$\mathbf{A}_{p} \, \mathbf{t}_{s} = \mathbf{0} \Leftrightarrow \mathbf{t}_{s}^{T} \, \mathbf{A}_{p}^{T} = \mathbf{0}$$

• t_s defines the gained motion inside the Type 2 singularity

Let us rewrite the last Lagrange equation :

$$-\mathbf{J}_{tk}^{T}\boldsymbol{\lambda}_{1}+\mathbf{A}_{p}^{T}\boldsymbol{\lambda}_{2}=\boldsymbol{\tau}_{p}$$

$$\Rightarrow \mathbf{A}_{\rho}^{T} \boldsymbol{\lambda}_{2} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{T} \boldsymbol{\lambda}_{1} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{T} \mathbf{J}_{k_{d}}^{-T} \boldsymbol{\tau}_{t_{d}} = \mathbf{w}$$

Left-multiplying by t¹_s

$$\mathbf{t}_s^T \mathbf{A}_p^T \boldsymbol{\lambda}_2 = \mathbf{0} \Rightarrow \mathbf{t}_s^T \mathbf{w}_p = \mathbf{0}$$

 t_s¹ w_p = 0 : in order to avoid induce input efforts while crossing a Type 2 singularity, the sum of the wrenches uplied on the platform by the legs, inertia/gravitational effects and external environment must be reciprocal to the uncontrollable motion of the platform inside the singularity (in other words, the power of these wrenches along the platform uncontrollable motion must be null).

Degeneracy	due to the	matrix A _	, R	EVN
A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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$$\mathbf{A}_{p} \, \mathbf{t}_{s} = \mathbf{0} \Leftrightarrow \mathbf{t}_{s}^{T} \, \mathbf{A}_{p}^{T} = \mathbf{0}$$

• \mathbf{t}_s defines the gained motion inside the Type 2 singularity

$$-\mathbf{J}_{tk}^T \boldsymbol{\lambda}_1 + \mathbf{A}_p^T \boldsymbol{\lambda}_2 = \boldsymbol{\tau}_{pl}$$

$$\Rightarrow \mathbf{A}_{\rho}^{T} \boldsymbol{\lambda}_{2} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{T} \boldsymbol{\lambda}_{1} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{T} \mathbf{J}_{k_{d}}^{-T} \boldsymbol{\tau}_{t_{d}} = \mathbf{I}_{k}^{T} \mathbf{J}_{k_{d}}^{-T} \boldsymbol{\tau}_{t_{d}} = \mathbf{I}_{k}^{T} \mathbf{J}_{k_{d}}^{-T} \mathbf{J}_{k_{d}}^{T} \mathbf{J}_{k_{d}}^{-T} \mathbf{J}_{k_{d}}^{T} \mathbf{J}_{k_{d}}^{-T} \mathbf{J}_{k_{d}}^{T} \mathbf{J}_$$

Left-multiplying by t⁷_s

$$\mathbf{t}_{s}^{T} \mathbf{A}_{p}^{T} \boldsymbol{\lambda}_{2} = \mathbf{0} \Rightarrow \mathbf{t}_{s}^{T} \mathbf{w}_{p} = \mathbf{0}$$

• $\mathbf{t}_s^T \mathbf{w}_p = 0$: in order to avoid induce input efforts while crossing a Type 2 singularity, the sum of the wrenches uplied on the platform by the legs, inertia/gravitational effects and external environment must be reciprocal to the uncontrollable motion of the platform inside the singularity (in other words, the power of these wrenches along the platform uncontrollable motion must be null).

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$$\mathbf{A}_{p}\,\mathbf{t}_{s}=0\Leftrightarrow\mathbf{t}_{s}^{T}\,\mathbf{A}_{p}^{T}=0$$

- t_s defines the gained motion inside the Type 2 singularity
- Let us rewrite the last Lagrange equation :

$$-\mathbf{J}_{tk}^{ op}oldsymbol{\lambda}_1+\mathbf{A}_{
ho}^{ op}oldsymbol{\lambda}_2=oldsymbol{ au}_{
hor}$$

$$\Rightarrow \mathbf{A}_{\rho}^{\mathsf{T}} \boldsymbol{\lambda}_{2} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{\mathsf{T}} \boldsymbol{\lambda}_{1} = \boldsymbol{\tau}_{\rho r} - \mathbf{J}_{tk}^{\mathsf{T}} \mathbf{J}_{k_{d}}^{-\mathsf{T}} \boldsymbol{\tau}_{t_{d}} = \mathbf{w}_{\rho}$$

• Left-multiplying by t

$$\mathbf{t}_{s}^{T} \mathbf{A}_{p}^{T} \boldsymbol{\lambda}_{2} = 0 \Rightarrow \mathbf{t}_{s}^{T} \mathbf{w}_{p} = 0$$

• $\mathbf{t}'_s \mathbf{w}_p = \mathbf{0}$: in order to avoid infinite input efforts while crossing a Type 2 singularity, the sum of the wrenches applied on the platform by the legs, inertia/gravitational effects and external environment must be reciprocal to the uncontrollable motion of the platform inside the singularity (in other words, the power of these wrenches along the platform uncontrollable motion must be null).

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Degeneracy	due to th	e matrix $\mathbf{\Delta}$	-	Curr

$$\mathbf{A}_{p}\,\mathbf{t}_{s}=0\Leftrightarrow\mathbf{t}_{s}^{T}\,\mathbf{A}_{p}^{T}=0$$

- t_s defines the gained motion inside the Type 2 singularity
- Let us rewrite the last Lagrange equation :

$$- \mathsf{J}_{\textit{tk}}^{ op} oldsymbol{\lambda}_1 + \mathbf{A}_{
ho}^{ op} oldsymbol{\lambda}_2 = oldsymbol{ au}_{\textit{pr}}$$

$$\Rightarrow \mathbf{A}_{p}^{\mathsf{T}} \boldsymbol{\lambda}_{2} = \boldsymbol{\tau}_{pr} - \mathbf{J}_{tk}^{\mathsf{T}} \boldsymbol{\lambda}_{1} = \boldsymbol{\tau}_{pr} - \mathbf{J}_{tk}^{\mathsf{T}} \mathbf{J}_{k_{d}}^{-\mathsf{T}} \boldsymbol{\tau}_{t_{d}} = \mathbf{w}_{p}$$

• Left-multiplying by \mathbf{t}_s^T

$$\mathbf{t}_s^T \, \mathbf{A}_p^T \boldsymbol{\lambda}_2 = \mathbf{0} \Rightarrow \mathbf{t}_s^T \, \mathbf{w}_p = \mathbf{0}$$

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• Left-multiplying by \mathbf{t}_s^T

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• $\mathbf{t}_s^T \mathbf{w}_p = 0$: in order to avoid infinite input efforts while crossing a Type 2 singularity, the sum of the wrenches applied on the platform by the legs, inertia/gravitational effects and external environment must be reciprocal to the uncontrollable motion of the platform inside the singularity (in other words, the power of these wrenches along the platform uncontrollable motion must be null).

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Degeneracy	due to the	matrix $\mathbf{J}_{k_{d}}$, ir	Cyn

$$\mathbf{J}_{k_d} \, \dot{\mathbf{q}}_d^s = \mathbf{0} \Leftrightarrow \dot{\mathbf{q}}_d^{s \, T} \, \mathbf{J}_{k_d}^T = \mathbf{0}$$

- $\dot{\mathbf{q}}_d^s$ defines the gained motion inside the LPJTS singularity
- Let us rewrite the second Lagrange equation :

$$\mathsf{J}_{k_d}^{\mathsf{T}} \boldsymbol{\lambda}_1 = \boldsymbol{\tau}_{t_d}$$

$$\Rightarrow \mathsf{J}_{k_d}^{\,\prime} \lambda_1 = \tau_t$$

• Left-multiplying by $\dot{\mathbf{q}}_d^{s T}$

$$\dot{\mathbf{q}}_{d}^{s\,\mathsf{T}}\mathbf{J}_{k_{d}}^{\mathsf{T}}\boldsymbol{\lambda}_{1}=0\Rightarrow\dot{\mathbf{q}}_{d}^{s\,\mathsf{T}}\boldsymbol{ au}_{t_{d}}=0$$

• $\dot{\mathbf{q}}_{d}^{s\,T} \boldsymbol{\tau}_{t_{d}} = 0$: the input efforts of the virtual estem in the joints that correspond to the passive joints of the real robot must be reciprocal to the uncontrollable motion of the passive legs inside the singularity (in other words, the power of these efforts along the leg uncontrollable motion must be mult).

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Degeneracy	due to the	matrix \mathbf{J}_{k_d}	ir	<u>C</u> w

$$\mathbf{J}_{k_d} \, \dot{\mathbf{q}}_d^s = \mathbf{0} \Leftrightarrow \dot{\mathbf{q}}_d^{s \, T} \, \mathbf{J}_{k_d}^T = \mathbf{0}$$

- $\dot{\mathbf{q}}_d^s$ defines the gained motion inside the LPJTS singularity
- Let us rewrite the second Lagrange equation :

$$egin{aligned} & oldsymbol{\mathsf{J}}_{k_d}^{\mathsf{T}} oldsymbol{\mathsf{\lambda}}_1^{\mathsf{T}} & oldsymbol{\mathsf{L}}_{t_d}^{\mathsf{T}} \end{aligned} \ \Rightarrow oldsymbol{\mathsf{J}}_{k_d}^{\mathsf{T}} oldsymbol{\lambda}_1 = oldsymbol{ au}_{t_d} \end{aligned}$$

• Left-multiplying by $\dot{\mathbf{q}}_d^{s\,T}$

$$\dot{\mathbf{q}}_d^{s\,T} \mathbf{J}_{k_d}^T \boldsymbol{\lambda}_1 = 0 \Rightarrow \dot{\mathbf{q}}_d^{s\,T} \boldsymbol{\tau}_{t_d} = 0$$

• $\dot{\mathbf{q}}_{d}^{s\,T} \boldsymbol{\tau}_{t_{d}} = 0$: the input efforts of the virtual stem in the joints that correspond to the passive joints of the real robot must be reciprocal to the uncontrollable motion of the passive legs inside the singularity (in other words, the power of these efforts along the leg uncontrollable motion must be mult).

Degeneracy	due to th	e matrix \mathbf{J}_{k_d}	ir	Cyn
A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Degeneracy	due to th	e matrix \mathbf{J}_{k_d}	ir	Cyn
A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Impact of the degeneracy of the matrix \mathbf{A}_{n}

• Analysis of a five-bar mechanism / A single loading f





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Introduction

Computation of the dynamic model of pkm

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Benchmark	00	000000	ĩ	Cen 1
A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions

• Prototype of five-bar mechanism (IFMA, Clermont-Ferrand)



Dynamic model (identified parameters) :

$$oldsymbol{ au} = oldsymbol{w}_b - oldsymbol{\mathsf{B}}_p^T oldsymbol{\lambda}_2, oldsymbol{\mathsf{A}}_p^T oldsymbol{\lambda}_2 = oldsymbol{\mathsf{w}}_p$$

with

$$\mathbf{w}_{p} = m_{3} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}, \ \mathbf{w}_{b} = \begin{bmatrix} zz_{11R} & \ddot{q}_{11} \\ zz_{12R} & \ddot{q}_{12} \end{bmatrix} + \begin{bmatrix} f_{v11}\dot{q}_{11} \\ f_{v21}\dot{q}_{12} \end{bmatrix} + \begin{bmatrix} f_{s11}sign(\dot{q}_{11}) \\ f_{s12}sign(\dot{q}_{12}) \end{bmatrix}$$

• Criterion for crossing the Type 2 singularities : $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{1i})$

Benchmark	00	000000	ĩ	Cen 1
A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions

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• Criterion for crossing the Type 2 singularities : $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{1i})$

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A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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• Criterion for crossing the Type 2 singularities : $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{2i})$



A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Trajectory pla	anning		ir.	Cyn

$\ensuremath{\mathsf{TABLE}}$: Boundary conditions for the two trajectories used on the five-bar mechanism.

Trajector	y for Case 1	Trajectory for Case 2		Case 2
$t = 0 \sec$	$t = t_f = 1.5 \text{ sec}$	t = 0 sec	$t = t_f = 1.5 \text{ sec}$	$t = t_{s} = 0.75 \text{ sec}$
$x(t = 0) = x_{p_0}$	$x(t = t_f) = x_{p_f}$	$x(t = 0) = x_{p_0}$	$x(t = t_f) = x_{p_f}$	$x(t = t_s) = x_{p_s} = 0.0543 \text{ m}$
$\dot{x}(t=0)=0$	$\dot{x}(t=t_f)=0$	$\dot{x}(t=0)=0$	$\dot{x}(t=t_f)=0$	$\dot{x}(t = t_s) = 0.1671 \text{ m/s}$
$\ddot{x}(t = 0) = 0$	$\ddot{x}(t = t_f) = 0$	$\ddot{x}(t=0) = 0$	$\ddot{x}(t = t_f) = 0$	$\ddot{x}(t = t_s) = 6.8e^{-4} \text{ m/s}^2$
$y(t = 0) = y_{p_0}$	$y(t = t_f) = y_{p_f}$	$y(t = 0) = y_{p_0}$	$y(t = t_f) = y_{p_f}$	$y(t = t_s) = y_{p_s} = 0.2 \text{ m}$
$\dot{y}(t=0)=0$	$\dot{y}(t = t_f) = 0$	$\dot{y}(t=0)=0$	$\dot{y}(t=t_f)=0$	$\dot{y}(t = t_s) = -0.4812 \text{ m/s}$
$\ddot{y}(t = 0) = 0$	$\ddot{y}(t = t_f) = 0$	$\ddot{y}(t=0)=0$	$\ddot{y}(t = t_f) = 0$	$\ddot{y}(t = t_s) = -0.01 \text{ m/s}^2$

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Trajectory pl	anning		i rs	Cyn

 $\ensuremath{\mathrm{TABLE}}$: Coefficients of the polynomials for each trajectory used on the five-bar mechanism.

	Polynomial	s for Case 1	Polynomials for Case 2		
	$x(t) = \sum_{i=0}^{5} a_i t^i$	$y(t) = \sum_{i=0}^{5} a_i t^i$	$x(t) = \sum_{i=0}^{8} a_i t^i$	$y(t) = \sum_{i=0}^{8} a_i t^i$	
<i>a</i> 0	0	0.338175237168	0	0.338175237168	
a_1	0	0	0	0	
a ₂	0	0	0	0	
a ₃	0.296296296296	-0.705704406423	0.030616142296	0.403081224309	
a4	-0.296296296296	0.705704406423	0.364976100965	-1.953915773554	
a_5	0.079012345679	-0.188187841713	-0.089638089174	0.149034398769	
a ₆			-0.638891733361	3.132585241250	
a ₇			0.555392794445	-2.569364459356	
a ₈			-0.131974503408	0.600075981487	

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Results			ir	<u>C</u> w

Case 1 : without respecting the criterion $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{2i})$



A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Results			ÎR.	C w

Case 2 : respecting the criterion $\ddot{y} = \ddot{x} \tan(q_{1i} + q_{2i})$



A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Crossing LP.	JTS singi	ularities	ir	Cyn

• Simulate the Tripteron behavior with the five-bar mechanism



- Two types of trajectory
- Case A : without respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$
- Case B : respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Trajectory pl	anning		173	Cyn

TABLE: Boundary conditions for the two trajectories used on the Tripteron.

Trajectory	y for Case A	Trajectory for Case B		Case B	
$t = 0 \sec t$	$t = t_f = 1.5 \text{ sec}$	$t = 0 \sec$	$t = t_f = 1.5 \text{ sec}$	$t = t_{s} = 0.75 \text{ sec}$	
$y(t = 0) = y_{c_0}$	$y(t = t_f) = y_{Cf}$	$y(t = 0) = y_{c_0}$	$y(t = t_f) = y_{Cf}$	$y(t = t_s) = y_{c_s} = 0.2021 \text{ m}$	
$\dot{y}(t=0)=0$	$\dot{y}(t = t_f) = 0$	$\dot{y}(t=0)=0$	$\dot{y}(t = t_f) = 0$	$\dot{y}(t = t_s) = 0.147 \text{ m/s}$	
$\ddot{y}(t = 0) = 0$	$\ddot{y}(t = t_f) = 0$	$\ddot{y}(t=0)=0$	$\ddot{y}(t = t_f) = 0$	$\ddot{y}(t = t_{s}) = -0.693 \text{ m/s}^2$	
$x(t) = x_{B_1} + \sqrt{a_{31}^2 + (y(t) - y_{B_1})^2}$					

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Trajectory p	lanning		ÎR.	Cenv

TABLE: Coefficients of the polynomials for each trajectory used on the Tripteron.

	Polynomial for Case A	Polynomial for Case B
<i>a</i> 0	0.338174999999996	0.338175000000007
a_1	0	0
<i>a</i> ₂	0	0
a ₃	-2.503500000000006	3.051722491923807
a ₄	3.755249999999982	-23.590518369448382
a_5	-1.502099999999988	43.558974062571224
<i>a</i> ₆		-26.660836890258111
a7		-0.254587389021722
<i>a</i> 8		3.644896094233372

 $x(t) = x_{B_1} + \sqrt{d_{31}^2 + (y(t) - y_{B_1})^2}$

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Results			Ĩĸ	Crw

Case A : without respecting the criterion $\dot{\mathbf{q}}_s^T \boldsymbol{\tau}_d = 0$



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Results			Ĩĸ	Cynv

Case B : respecting the criterion $\dot{\mathbf{q}}_s^{\mathsf{T}} \boldsymbol{\tau}_d = 0$



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A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Conclusion			ĨR	·Cyw

- To fulfil the lacks of the previous studies and to analyze all degeneracy conditions of the full parallel robot dynamic model which takes into account all link dynamic parameters,
- To demonstrate that the LPJTS singularities impact the robot effort transmission, as this point is usually bypassed in the literature, and
- To provide all physical criteria that make it possible to defin trajectories allowing the passing through Type 2 and LPJTS singularities

Miss some important questions, among which

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Conclusion			ĨR	·Cyw

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- The trajectory is considered projectly tracted \Rightarrow impossible in reality
- Robustness to model errors

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Conclusion			ĨR	Cyny

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 - Robustness to model errors

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Conclusion			ĨR	Cyny

- To fulfil the lacks of the previous studies and to analyze all degeneracy conditions of the full parallel robot dynamic model which takes into account all link dynamic parameters,
- To demonstrate that the LPJTS singularities impact the robot effort transmission, as this point is usually bypassed in the literature, and
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- The trajectory is considered perfectly tracked \Rightarrow impossible in reality
- Robustness to model errors?

A short overview of IRCCyN	Introduction	Computation of the dynamic model of pkm	Case study	Conclusions
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Conclusion			ĨR	Cyny

- To fulfil the lacks of the previous studies and to analyze all degeneracy conditions of the full parallel robot dynamic model which takes into account all link dynamic parameters,
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