From Singularities in Parallel Mechanisms to Singularities in Visual Servoing

Towards the discovery of common issues and potential methods for solving them

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Introduction

Serial robots vs. Parallel robots

Introduction

Singularities of serial robots

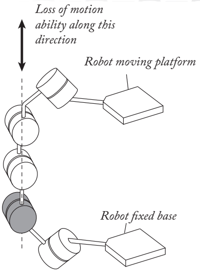
TRAVERSEE Type 2

Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities



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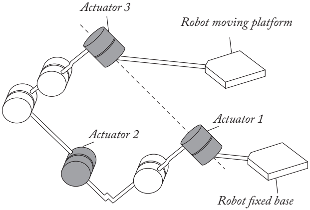

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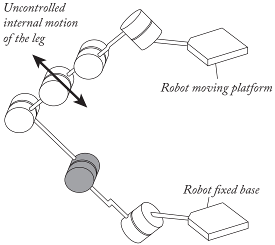

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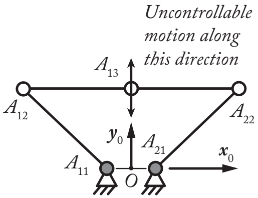

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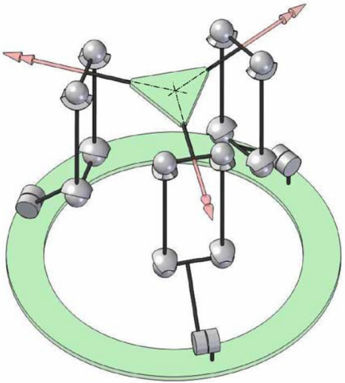

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- Platform singularities:
  - Type 2 singularities
  - Constraint singularities
  - Other (not detailed because extremely rare)

Introduction

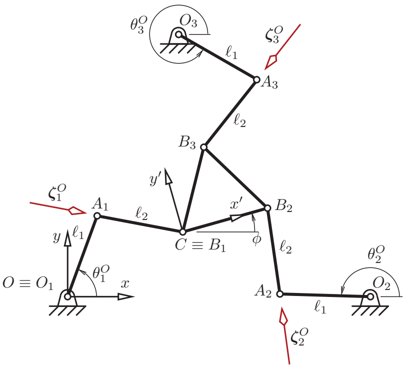
Special types of singularities

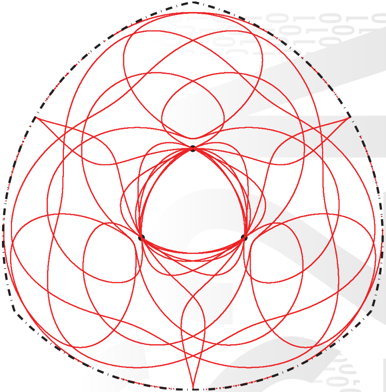
In Type 2, constraint and LPJTS singularities

- Loss of stiffness (uncontrollable / gained motions)
- Considerable decrease of performance (deformation, vibration, effort transmission, dynamics, positionning error, etc.)
- Singularities located IN the workspace (not on the boundaries)

Type 2 (parallel) singularities of *PKM*

Probably, the most important drawback of PKM





Type 2 Singularities of a 3–RRR planar robot [Bonev 2001]

Type 2 (parallel) singularities of *PKM*

Normally, impossible to cross these singularities

Because near these singularities, the input torques tend to infinity

TRAVERSEE Type 2

Type 2 (parallel) singularities of *PKM*

But...

By proper trajectory planning respecting a dynamics criterion [Briot et Arakelian 2008] and an adequate controller [Pagis et al, 2015]

TRAVERSEE Type 2

Singularities of parallel robots

How to find Type 2 or constraint singularities?

In the late 80's

- Type 2 singularities
  - Compute the I/O kinematic relationship:

$$\mathbf{A}(\mathbf{q}_a, \mathbf{x})\,{}^0\mathbf{t}_p + \mathbf{B}(\mathbf{q}_a, \mathbf{x})\dot{\mathbf{q}}_a = \mathbf{0} \qquad (1)$$

  - Compute the determinant of **A** and find the conditions for which it is equal to 0

⇒ Limited to simple cases

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  - Compute the determinant of **A** and find the conditions for which it is equal to 0

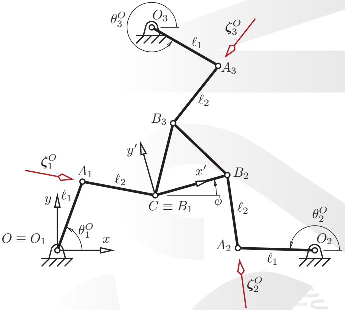
  ⇒ Limited to simple cases

- Constraint singularities:
  - Discovered at the early 2000's
  - Cannot be found using the previous method

How to find Type 2 or constraint singularities?

In the late 80's / early 90's, a method based on the Grassmann geometry

Type 2 or constraint sing. $\equiv$ singularities of the system of (static) wrenches applied by the legs on the platform

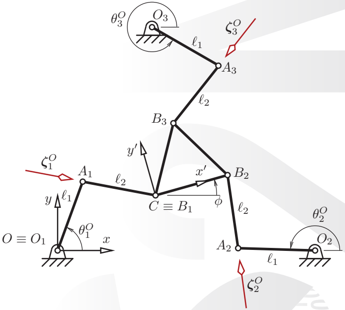


How to find Type 2 or constraint singularities?

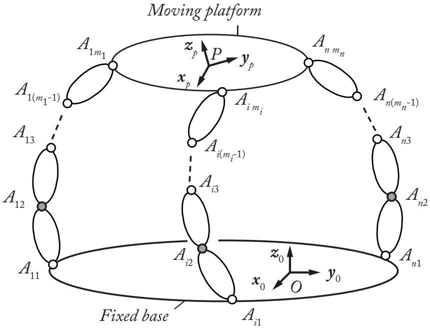
In the late 80's / early 90's, a method based on the Grassmann geometry

Type 2 or constraint sing. $\equiv$ singularities of the system of (static) wrenches applied by the legs on the platform

- Find the system of wrenches applied by the legs on the platform using the Screw Theory
- Analyze the degeneracy of this system of wrenches using the Grassmann geometry

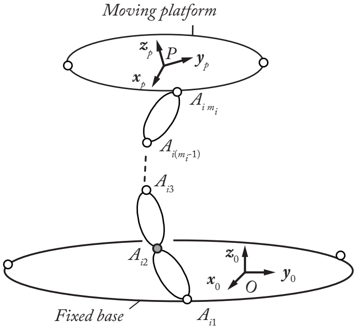


Determination of the system of wrenches



Determination of the system of wrenches

For serial leg (the *i*th leg of the parallel robot)



$$\mathbf{t}_p = \mathbf{J}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i \text{ where } \mathbf{J}_i = \begin{bmatrix} \$_{i1} & \dots & \$_{im_i} \end{bmatrix} \quad (2)$$

$\$_{ij}$ is unit a twist representing the twist of the platform when joint ij is moving only

Determination of the system of wrenches

We group, for the leg i,

- in a sub-matrix ${}^0\mathbf{\$}_{ia}$ the unit twists corresponding to the active joints of velocities $\dot{\mathbf{q}}_{ai}$,
- in a sub-matrix ${}^0\mathbf{\$}_{id}$ the unit twists corresponding to the passive joints of velocities $\dot{\mathbf{q}}_{di}$

and we express all equations in the base frame $\mathcal{F}_0$ (superscript "0" before the variables)

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and we express all equations in the base frame $\mathcal{F}_0$ (superscript "0" before the variables)

Thus

$$ {}^0\mathbf{t}_p = \begin{bmatrix} {}^0\$_{ia} & {}^0\$_{id} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_{ai} \\ \dot{\mathbf{q}}_{di} \end{bmatrix} = {}^0\$_{ia}\ \dot{\mathbf{q}}_{ai} + {}^0\$_{id}\ \dot{\mathbf{q}}_{di}. \quad (3) $$

Determination of the system of wrenches

For the leg i,

- The constraint wrenches (i.e. the wrenches applied by the leg even if it is not actuated) are the wrenches $\boldsymbol{\zeta}_{id}$ which are reciprocal to both ${}^0\$_{ia}$ and ${}^0\$_{id}$, i.e. they are defined such that

$$\boldsymbol{\zeta}_{id} \circ {}^0\$_{ia} = 0, \ \boldsymbol{\zeta}_{id} \circ {}^0\$_{id} = 0 \quad (4)$$

- The actuation wrenches (i.e. the wrenches applied by the leg because of the presence of the actuator) are the wrenches $\boldsymbol{\zeta}_{ia}$ which are reciprocal to ${}^0\$_{id}$ and are not included in the system of constraint wrenches $\boldsymbol{\zeta}_{id}$, i.e. they are defined such that

$$\boldsymbol{\zeta}_{ia} \circ {}^0\$_{id} = 0, \ \boldsymbol{\zeta}_{ia} \not\subset \boldsymbol{\zeta}_{id} \quad (5)$$

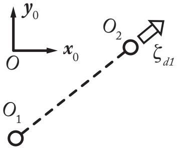
Determination of the system of wrenches

Example of a RR leg with R axes along z_0

- Motion is represented by two unit twists:

$$^{0}\$_{R1} = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (6)$$

$$^{0}\$_{R2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (7)$$



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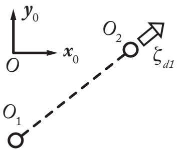

- If both joints are passive:

$$\zeta_{d1} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } \overrightarrow{O_1 O_2} \quad (8)$$

$$\zeta_{d2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } \boldsymbol{z}_0 \quad (9)$$

Determination of the system of wrenches

Example of a RR leg with R axes along z_0

- Motion is represented by two unit twists:

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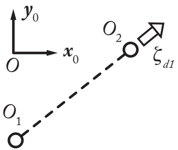


- If both joints are passive:

$$ \zeta_{d3} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } \boldsymbol{x}_0 \quad (8) $$

$$ \zeta_{d4} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } \boldsymbol{y}_0 \quad (9) $$

Determination of the system of wrenches

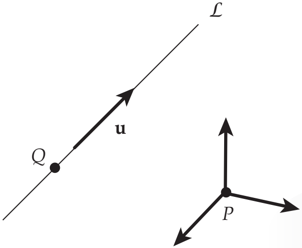
Then,

- Stack all constraint wrenches ζ_{id} in a matrix ζ_d
- Stack all actuation wrenches ζ_{ia} in a matrix ζ_a
- Analyze the degeneracy of ζ_a and ζ_d thanks to the Grassmann geometry

Singularities of parallel robots

Grassmann geometry

- Gives conditions on degeneracy of systems of lines
- Plücker representation of a line $\mathcal{L}$: $[\mathbf{u}^T \ (\overrightarrow{PQ} \times \mathbf{u})^T]^T$
  - A direction $\mathbf{u}$
  - Moment of the direction $\mathbf{u}$ wrt a given point P



Singularities of parallel robots

A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

$$\zeta_i = \begin{bmatrix} \mathbf{f} \\ \overrightarrow{PQ} \times \mathbf{f} \end{bmatrix} \quad (10)$$

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A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

$$\zeta_i = \begin{bmatrix} \mathbf{f} \\ \overrightarrow{PQ} \times \mathbf{f} \end{bmatrix} \tag{10}$$

A pure moment wrench is given by, for any application point

$$\zeta_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{m} \end{bmatrix} \tag{11}$$

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A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

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A pure moment wrench is given by, for any application point

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These expressions are Plücker representations of lines

- the pure force wrench: a line of direction $\mathbf{f}$ passing through point P
- the pure moment wrench: a line of direction $\mathbf{m}$ but in the projective plane at infinity

Singularities of parallel robots

Thanks to Grassmann geometry

Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

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It is still quite complicated

However for 2 and 3DOF planar robots, the conditions are quite simple to analyze

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Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

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However for 2 and 3DOF planar robots, the conditions are quite simple to analyze

For planar robots

- **2 DOF:** degeneracy if the two lines are parallel
- **3 DOF:** degeneracy if the three (coplanar) lines intersect in the same point (that may be at infinity) $\Rightarrow$ instantaneous center of rotation

Singularities of parallel robots

A few notations

- **a**, **b**: two points located at the position **a** and **b** in the Cartesian space (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to $w \neq 0$)
- **A**, **B**: two points located at the position **A** and **B** in the projective plane at infinity (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to $w = 0$)
- **ab**, the line passing through points **a** and **b**
- **abc**, the plane passing through points **a**, **b** and **c**
- [**abcd**]: the determinant of the (4×4) matrix whose columns are the expressions of the points **a**, **b**, **c** and **d** (in other words, the volume of the tetrahedron)
- $\bigwedge$: the “meet operator”

Singularities of parallel robots

Superbracket decomposition

$$[\mathbf{ab},\ \mathbf{cd},\ \mathbf{ef},\ \mathbf{gh},\ \mathbf{ij},\ \mathbf{kl}] = \sum_{i=1}^{24} y_i \qquad (12)$$

where

$$\begin{array}{lll}
y_1 = -[\mathbf{abcd}][\mathbf{efgi}][\mathbf{hjkl}] & y_2 = [\mathbf{abcd}][\mathbf{efhi}][\mathbf{gjkl}] & y_3 = [\mathbf{abcd}][\mathbf{efgj}][\mathbf{hikl}] \\
y_4 = -[\mathbf{abcd}][\mathbf{efhj}][\mathbf{gikl}] & y_5 = [\mathbf{abce}][\mathbf{dfgh}][\mathbf{ijkl}] & y_6 = -[\mathbf{abde}][\mathbf{cfgh}][\mathbf{ijkl}] \\
y_7 = -[\mathbf{abcf}][\mathbf{degh}][\mathbf{ijkl}] & y_8 = [\mathbf{abdf}][\mathbf{cegh}][\mathbf{ijkl}] & y_9 = -[\mathbf{abce}][\mathbf{dghi}][\mathbf{fjkl}] \\
y_{10} = [\mathbf{abde}][\mathbf{cghi}][\mathbf{fjkl}] & y_{11} = [\mathbf{abcf}][\mathbf{dghi}][\mathbf{ejkl}] & y_{12} = [\mathbf{abce}][\mathbf{dghj}][\mathbf{fikl}] \\
y_{13} = -[\mathbf{abdf}][\mathbf{cghi}][\mathbf{ejkl}] & y_{14} = -[\mathbf{abde}][\mathbf{cghj}][\mathbf{fikl}] & y_{15} = -[\mathbf{abcf}][\mathbf{dghj}][\mathbf{eikl}] \\
y_{16} = [\mathbf{abdf}][\mathbf{cghj}][\mathbf{eikl}] & y_{17} = [\mathbf{abcg}][\mathbf{defi}][\mathbf{hjkl}] & y_{18} = -[\mathbf{abdg}][\mathbf{cefi}][\mathbf{hjkl}] \\
y_{19} = -[\mathbf{abch}][\mathbf{defi}][\mathbf{gjkl}] & y_{20} = -[\mathbf{abcg}][\mathbf{defj}][\mathbf{hikl}] & y_{21} = [\mathbf{abdh}][\mathbf{cefi}][\mathbf{gjkl}] \\
y_{22} = [\mathbf{abdg}][\mathbf{cefj}][\mathbf{hikl}] & y_{23} = [\mathbf{abch}][\mathbf{defj}][\mathbf{gikl}] & y_{24} = -[\mathbf{abdh}][\mathbf{cefj}][\mathbf{gikl}]
\end{array} \qquad (13)$$

Determination of the system of wrenches

By an adequate choice of the points for representing the lines (intersection points, points are infinity, etc)

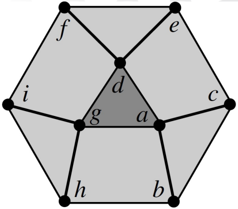
Many monomials y_i can be deleted

Example [Ben Horin and Shoham 2006]

$$[\mathbf{ab},\ \mathbf{ac},\ \mathbf{de},\ \mathbf{df},\ \mathbf{gh},\ \mathbf{gi}] = [\mathbf{ad\dot{f}g}][\mathbf{abc\dot{d}}][\mathbf{\dot{e}igh}] = \mathbf{edf} \wedge \mathbf{igh} \wedge \mathbf{abc} \wedge \mathbf{adg}$$

Geometric interpretation

Intersection of four planes



Singularities of parallel robots

Remarks

- These tools for singularity analysis are difficult to be used by non expert
- But a lot of scientific litterature $\Rightarrow$ If we know the general formulation of the system of wrenches, for instance
  - 3 forces + 3 moments
  - 6 forces, but only three points of applications, two forces by points

  geometric interpretation of results are already given (see the next slides)
- Sometimes, we still must do the analysis
- These tools were primarly used for singularities of PKM, we will show now that they can be used for other singularity analyses

What is visual servoing?

TRAVERSEE Type 2

What is visual servoing?

- to 3D features observed $\Rightarrow$ measures in the camera frame $\mathbf{s}$
- we can set a kinematic relationship between the twist $\boldsymbol{\tau}$ of the relative motion between the object and camera frames and the velocity of the measurements $\mathbf{s}$:

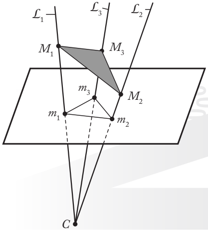
$$\dot{\mathbf{s}} = \mathbf{L}\boldsymbol{\tau} \tag{14}$$

- $\mathbf{L} = \mathbf{L}(\mathbf{s}, \mathbf{x})$ is called the interaction matrix, in which $\mathbf{x}$: relative configuration between the object and camera frames
- standard controller (wishing an exponential decay $\dot{\mathbf{e}} = -\lambda\mathbf{e}$ of error $\mathbf{e} = \mathbf{s} - \mathbf{s}^* \Rightarrow \dot{\mathbf{s}} = -\lambda\mathbf{e}$):

$$\boldsymbol{\tau} = \mathbf{L}\boldsymbol{\tau} = -\lambda\mathbf{L}^+\mathbf{e} \tag{15}$$

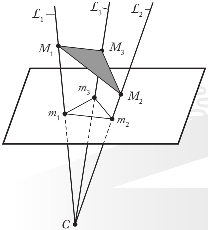
Introduction to singularities in visual servoing

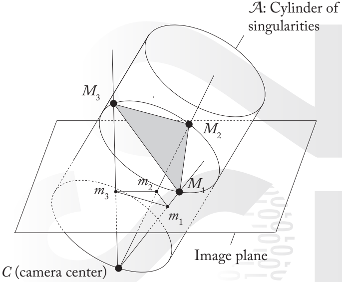
- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**



Introduction to singularities in visual servoing

- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**
- To the best of our knowledge, only known for three 3-D image points (*singularity cylinder*)
- Issue with singularities: interaction matrix cannot be inverted anymore = **loss of controllability**





Introduction to singularities in visual servoing

In order to avoid singularities

Increased number of image features (redundancy):

- Pb of local minima
- Proof that there is no singularity?

Determining the singularity cases stays an open problem

Introduction to singularities in visual servoing

Recently, the "Hidden Robot Concept" was developped

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM

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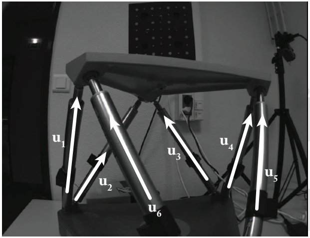

For instance, when observing the **leg directions** of the GS platform

- Real robot = 6–$U\underline{P}S$

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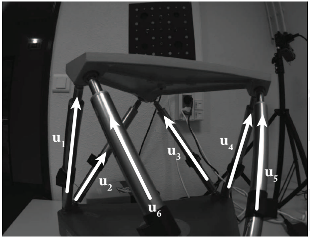
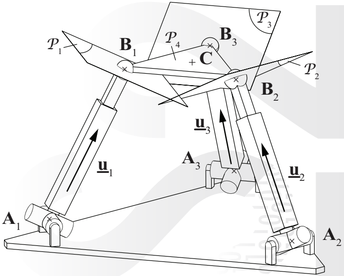
For instance, when observing the **leg directions** of the GS platform

- Real robot = $6\text{–}U\underline{P}S$
- Virtual robot = $6\text{–}\underline{U}PS$





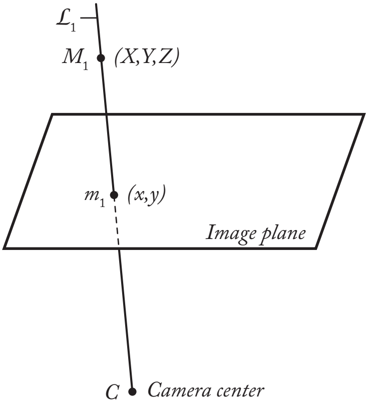
Introduction to singularities in visual servoing

Here

I show how we used the hidden robot concept in order to solve, for the first time, the singularities in

1. the observation of n image points ($n \geq 3$)
2. the observation of three lines
3. the leg-based visual servoing of parallel robots

Observation of an image point



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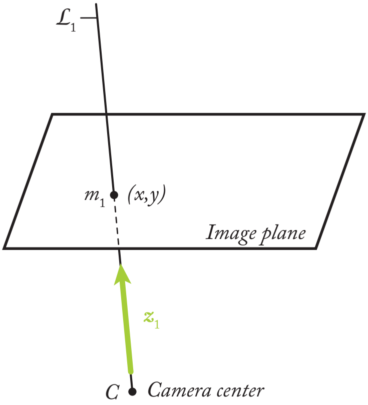

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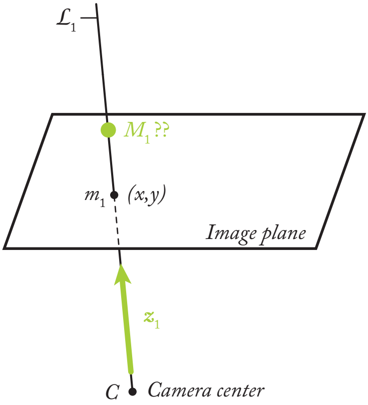
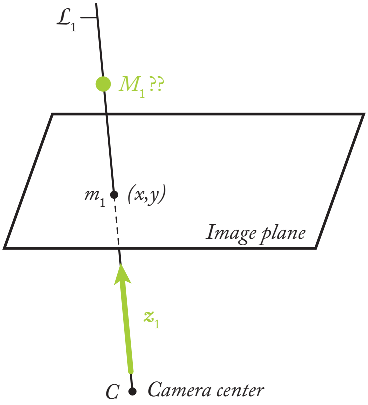


Observation of an image point



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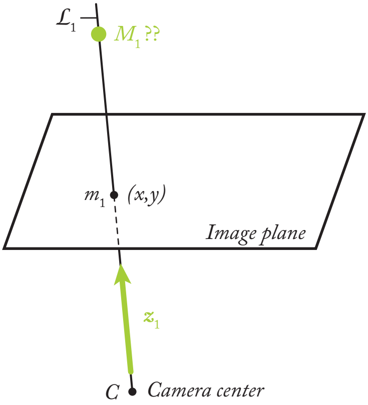
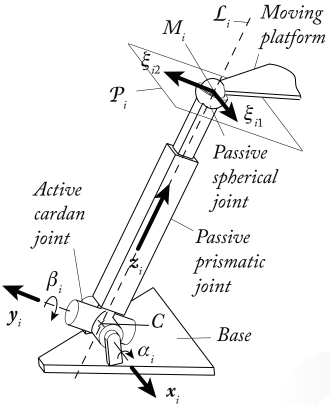
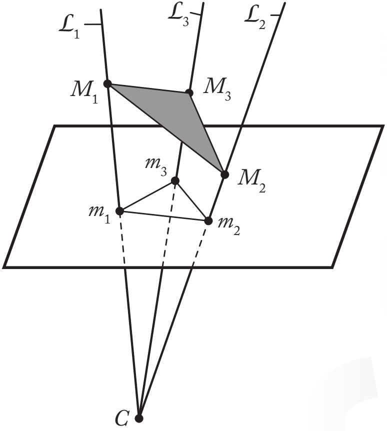


Observation of an image point

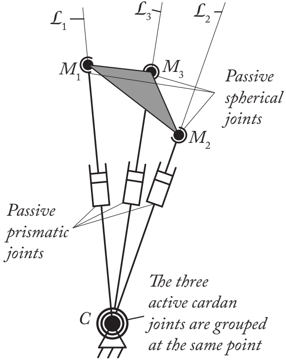


A *U̲PS* kinematic chain which allows for the same motion of the point M_i

Observation of three image points



Observation of three image points



A 3–$\underline{U}PS$ robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

$$\dot{\mathbf{s}} = \mathbf{L}\boldsymbol{\tau} \;//\; \dot{\mathbf{q}} = \mathbf{J}_{inv}\boldsymbol{\tau}$$

Singularities

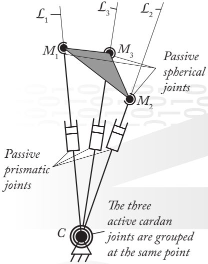
Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]



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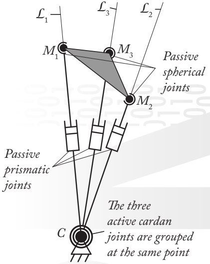
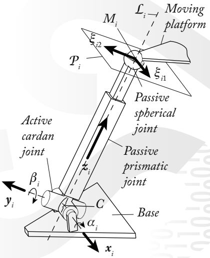
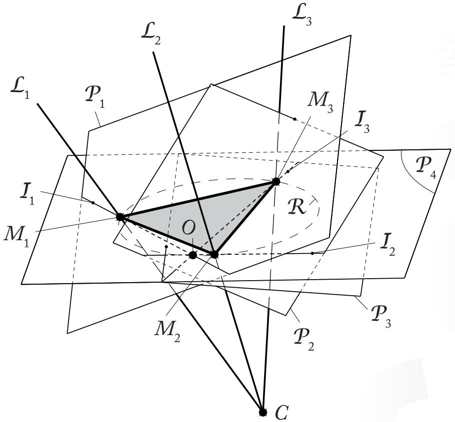
In our case (3 points), it can be proven that

The planes $\mathcal{P}_i$ ($i = 1, 2, 3$) and $\mathcal{P}_4$ (containing all 3-D points) have a non-null intersection

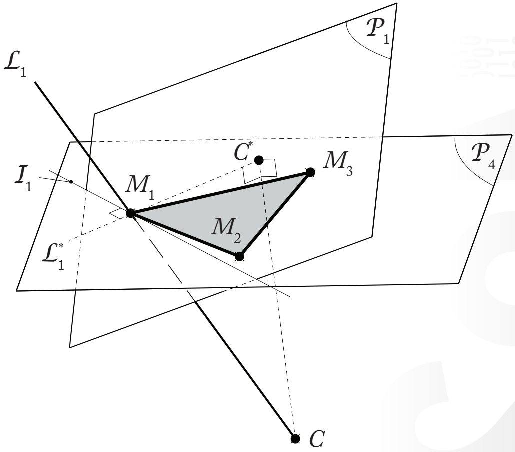




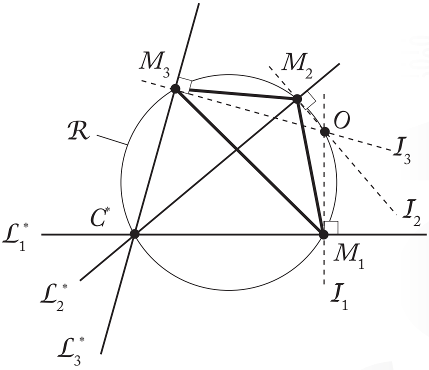
Singularities when observing 3 points



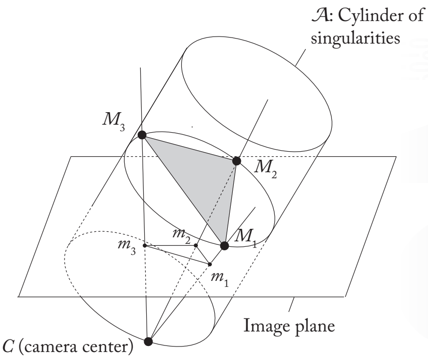
Singularities when observing 3 points



Singularities when observing 3 points



Singularities when observing 3 points



Singularities when observing n points ($n > 3$)

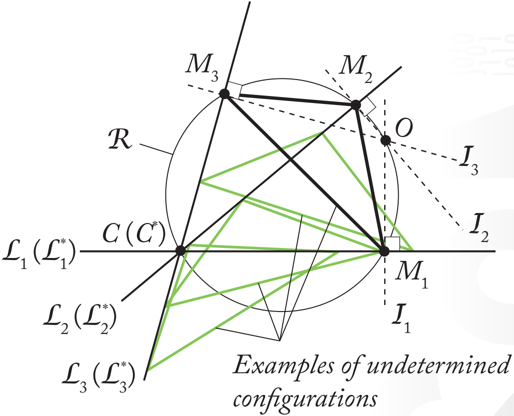
Possible if and only if

- All singularity cylinders associated with any subset of 3 points have a common intersection
- AND all kernels of the interaction matrices are identical

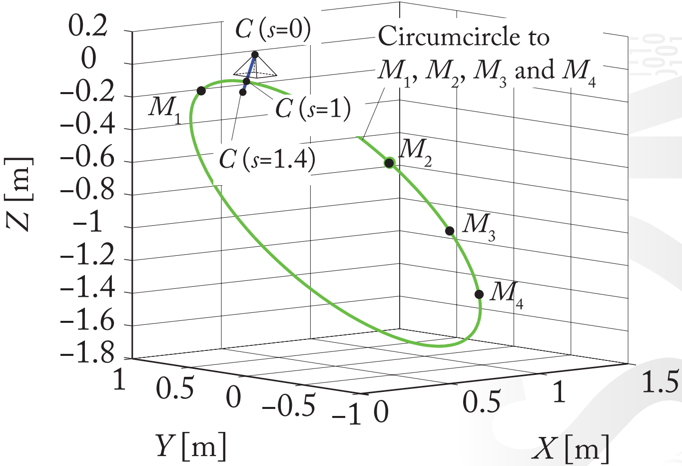
After (more complex) mathematical derivations, we proved that

The conditions of singularity when n coplanar points are observed only appear if and only if all 3-D points and the optical center are located on the same circle

Singularities when observing n points ($n > 3$)



Simulations



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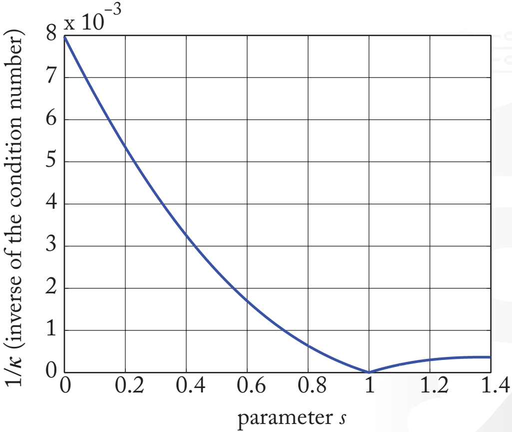
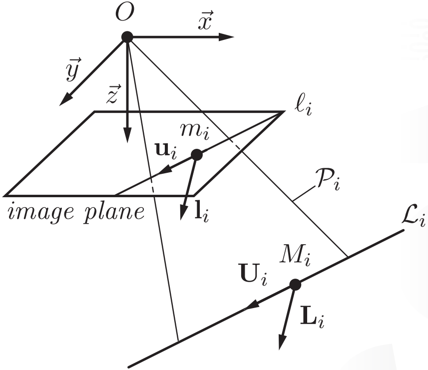


Observation of an image line



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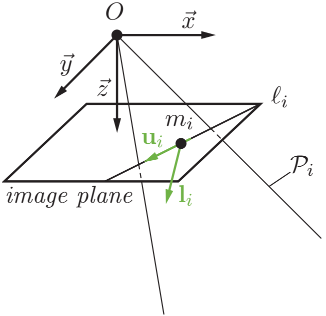
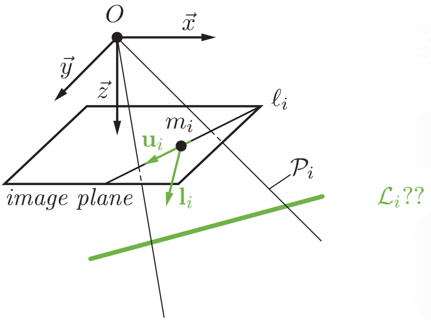
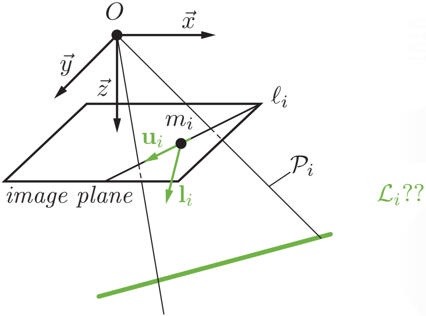


$\mathcal{L}_i$??

Observation of an image line



Observation of an image line



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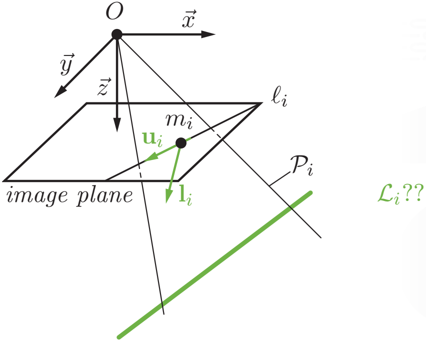

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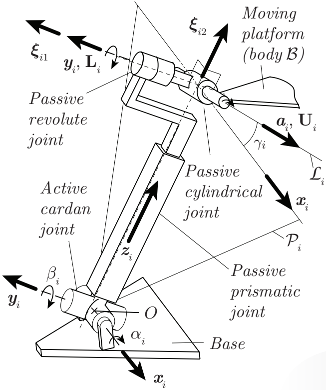
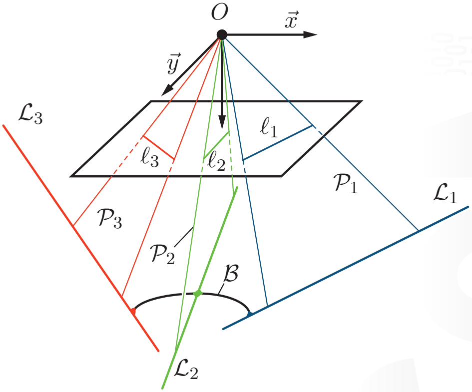
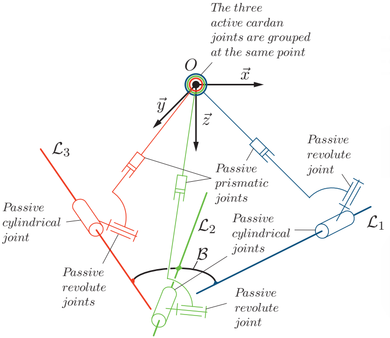


A $\underline{U}PRC$ kinematic chain which allows for the same motion of the line $\mathcal{L}_i$

Observation of three image lines



Observation of three image lines



A 3–$\underline{U}PRC$ robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

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Singularities of the interaction matrix = singularities of the virtual parallel robot

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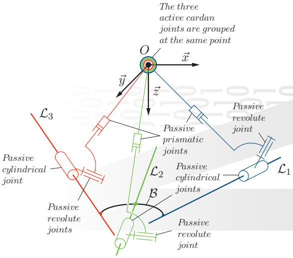

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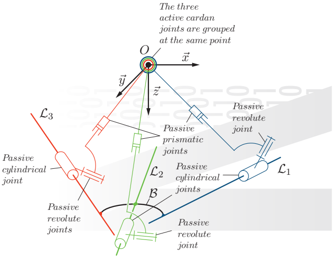
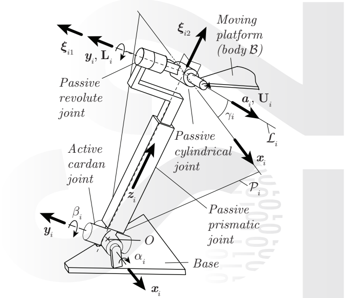
In our case (3 lines), singu. cond. iff

$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0$ or

$f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0$

where $\boldsymbol{\xi}_{ij} = [\mathbf{f}_{ij}^T \ \mathbf{m}_{ij}^T]^T$





Singularities

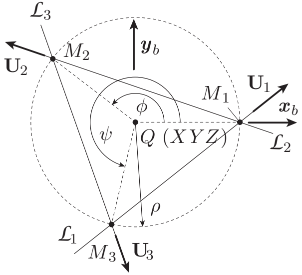
In order to simplify the problem

- Consider the “zero” platform orientation
- General case obtained by a simple rotation

$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T = \mathcal{R} \begin{bmatrix} X' & Y' & Z' \end{bmatrix}^T \quad (16)$$

where
X, Y and Z: position of the origin of the object frame $\mathcal{F}_b$ in the camera frame when considering the “zero” platform orientation
X', Y' and Z': position of the origin of the object frame for the considered “non-zero” platform orientation
$\mathcal{R}$ the rotation matrix between the two cases

Three coplanar lines with no common intersection point



$$\begin{aligned} f_1 = 0 \Leftrightarrow Z = 0 &\Rightarrow \text{Lines + optical center in the same plane} \\ f_2 = 0 \Leftrightarrow Z(X^2+Y^2-\rho^2) = 0 &\Rightarrow \text{ Singularity cylinder!} \end{aligned} \quad (17)$$

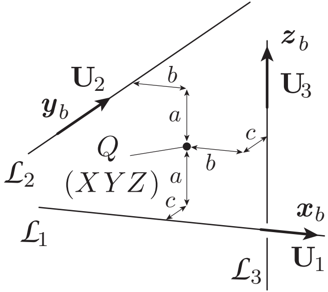
Three lines in space with a common intersection point

$$\begin{aligned} \overrightarrow{OQ} = [X\ Y\ Z]^T,\ \mathbf{U}_1 = [1\,0\,0]^T, \\ \mathbf{U}_2 = [a\,b\,0]^T,\ \mathbf{U}_3 = [c\,d\,e]^T \end{aligned} \quad (18)$$

$$\begin{aligned} f_1 = 0 \Rightarrow & \text{ For any object configuration} \\ f_2 = 0 \Leftrightarrow & b(adeY^3 + ((-ad^2 + bcd + ae^2)Z \\ & + (ac - bd)eX)Y^2 - e(bcX^2 + (ad - bc)Z^2 \\ & + 2beXZ)Y + ((-ad^2 + bcd - ae^2)X^2 Z \\ & + (bd + ac)eXZ^2)) = 0 \end{aligned} \quad (19)$$

$\Rightarrow$ The origin of the body frame belongs to a cubic surface parameterized by $f_2 = 0$.

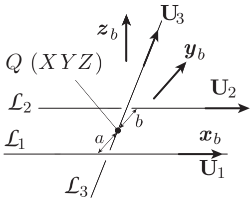
Three orthogonal lines in space



$$\begin{aligned} f_1 = 0 &\Leftrightarrow aXY + bYZ - cXZ - abc = 0 \\ f_2 = 0 &\Leftrightarrow acX - abY + bcZ - XYZ = 0 \end{aligned} \tag{20}$$

$\Rightarrow$ Expression f_1 represents a quadric surface while expression f_2 is a cubic surface

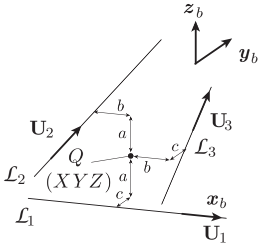
Three lines, two of them being parallel



$$
\begin{aligned}
f_1 = 0 &\Leftrightarrow Z(dZ - eY) = 0 \\
f_2 = 0 &\Leftrightarrow Z(X(d^2 + e^2) - cYd - cZe) = 0
\end{aligned}
\tag{21}
$$

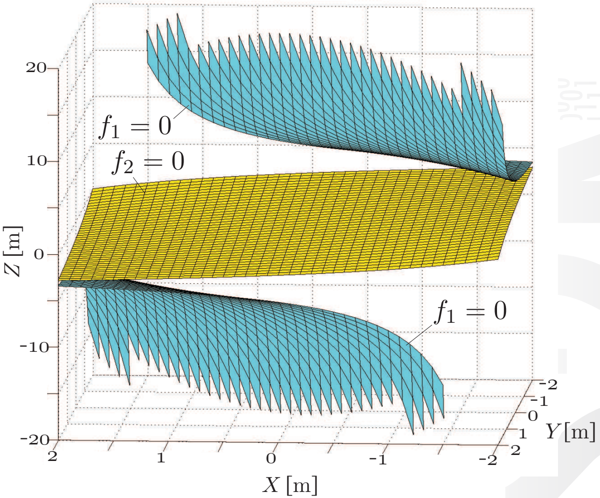
- $Z = 0$, which occur when the plane $\mathcal{P}$ containing $\mathcal{L}_1$ and $\mathcal{L}2$ also contains the optical center,
- $eY - dZ = 0$ is the plane containing $\mathbf{U}_1$, $\mathbf{U}_3$ and the optical center,
- $X(d^2 + e^2) - cdY - ceZ = 0$ is the plane containing $(\mathbf{U}_1 \times \mathbf{U}_3)$, $\mathbf{U}_3$ and the optical center.

Three general lines in space

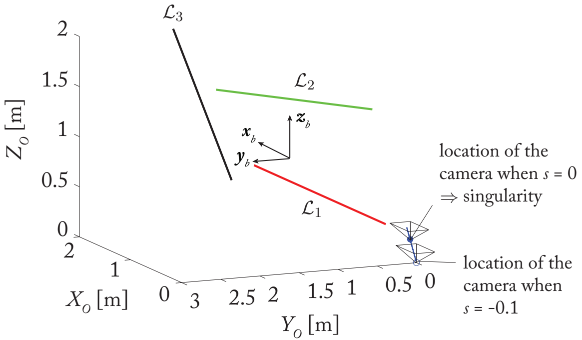


Condition $f_1 = 0$ provides the expression of a quadric surface while $f_2 = 0$ leads to a cubic surface.

Example for three general lines in space



Simulation 1 (general case)



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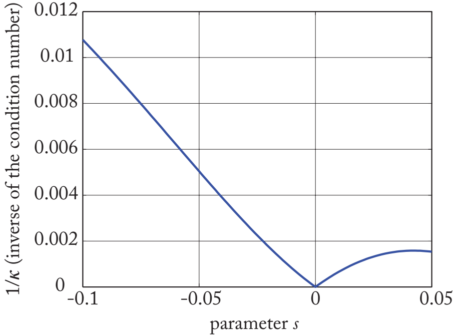
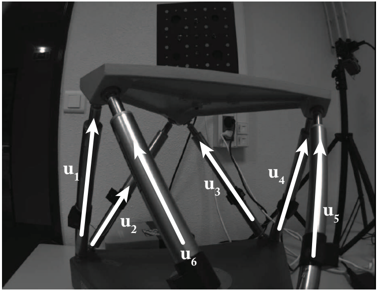


Leg-based visual servoing of parallel robots

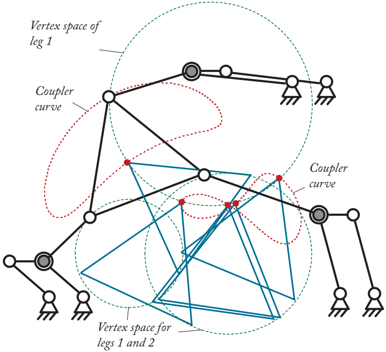
Generalisation to families of parallel robots



Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Planar robots: Example of the 3–RRR robot



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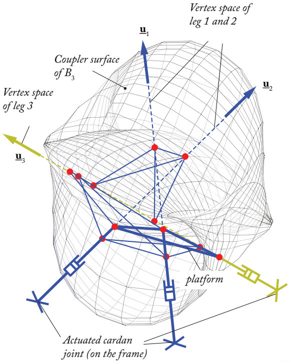
Spatial robots: Example of the GS Platform



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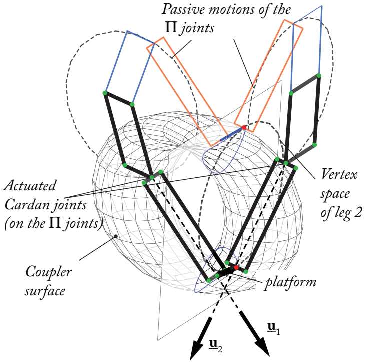
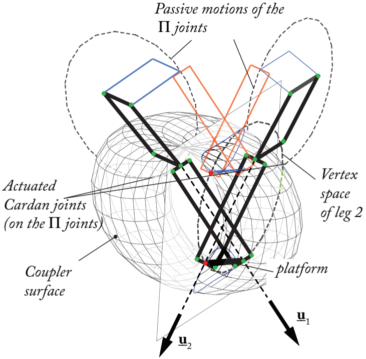
Spatial robots: Example of the Quattro





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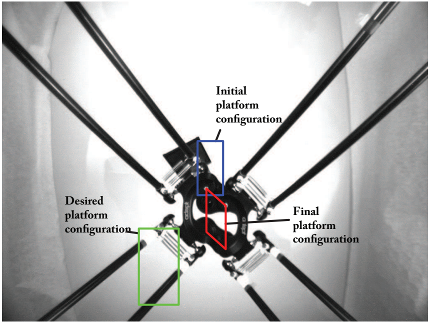
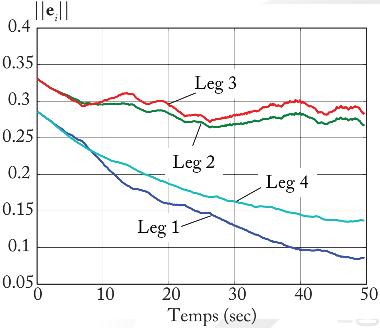
Generalisation to families of parallel robots

Experimental validation

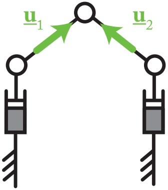




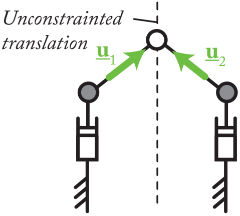
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 1: Robots which are uncontrollable with the observation of the leg directions



A *PRRRP* robot

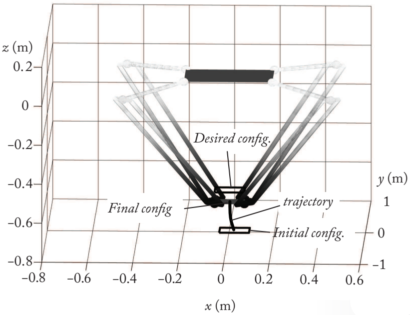


Hidden robot: a *PRRRP* robot

Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

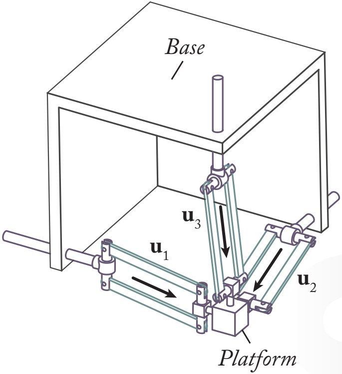
Class 2: Robots which are partially controllable (in their workspace) with the observation of the leg directions



Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

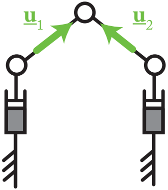
Class 3: Robots which are fully controllable (in their workspace) with the observation of the leg directions



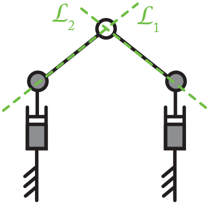
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 4: Robots which are fully controllable (in their workspace) thanks to additional measurements



A *PRRRP* robot



Hidden robot:
a *PRRRP* robot

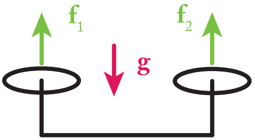
Singularities appear in many systems

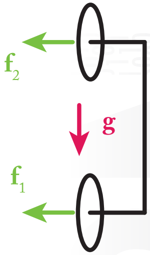
Fleets of agents

TRAVERSEE Type 2

Singularities appear in many systems

UAVs, ROVs

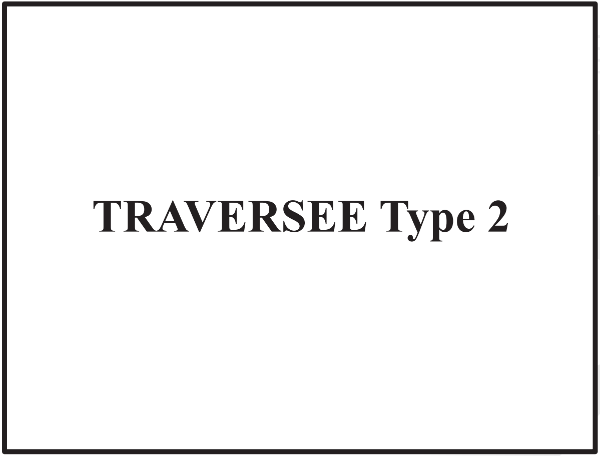




dynamics singularity

Singularities appear in many systems

UAVs, ROVs



Singularities appear in many systems

Reconfigurable drones

TRAVERSEE Type 2

Singularities appear in many systems

GG and AGC needs adaption

Because propellers apply force and torque which are linked (non zero and non infinite pitch screws)

Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot! 🙂

Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot! 🙂

In this talk,

- I presented a tool named the "hidden robot concept" able to solve the determination of the singularity cases visual servoing based on the observation of geometric features
- we proved the conditions of singularity for n coplanar points and 3 lines
- we discussed about the generalization of the "hidden robot concept" to other case studies

Conclusions

The hidden robot concept

- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (control community / mechanical engineering community $\Rightarrow$ dual problems)

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The hidden robot concept

- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (control community / mechanical engineering community $\Rightarrow$ dual problems)

Tools used here

- Easily extendable to the rigidity-based control theory
- And maybe other problems
- But useful for you?

Concluding remarks

P. Martinet
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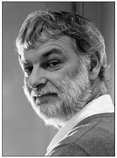

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