From Singularities in Parallel Mechanisms to Singularities in Control Problems

Towards the discovery of common issues and potential methods for solving them

LABORATOIRE
DES SCIENCES
DU NUMÉRIQUE
DE NANTES

Sébastien Briot

Laboratoire des Sciences du Numérique de Nantes (LS2N)

April 26, 2017

Introduction

Serial robots vs. Parallel robots

Introduction

Singularities of serial robots

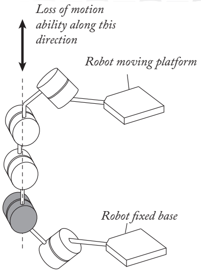
TRAVERSEE Type 2

Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities

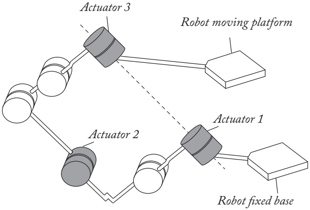


Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities
  - Leg Active Joint Twist System Singularities (LAJTS)

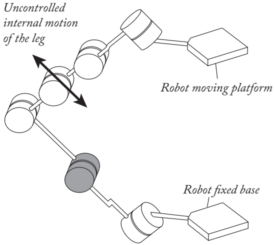


Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities
  - Leg Active Joint Twist System Singularities (LAJTS)
  - Leg Passive Joint Twist System Singularities (LPJTS)

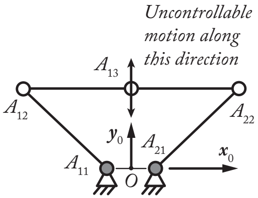


Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities
  - Leg Active Joint Twist System Singularities (LAJTS)
  - Leg Passive Joint Twist System Singularities (LPJTS)
- Platform singularities:
  - Type 2 singularities

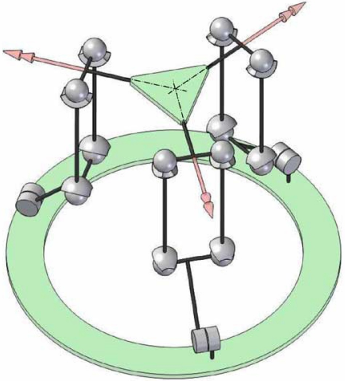


Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities
  - Leg Active Joint Twist System Singularities (LAJTS)
  - Leg Passive Joint Twist System Singularities (LPJTS)
- Platform singularities:
  - Type 2 singularities
  - Constraint singularities



Introduction

Singularities of parallel robots

Much more complex because of the architecture made of both active and passive joints

- Leg singularities:
  - "Usual" Leg (or Type 1) Singularities
  - Leg Active Joint Twist System Singularities (LAJTS)
  - Leg Passive Joint Twist System Singularities (LPJTS)
- Platform singularities:
  - Type 2 singularities
  - Constraint singularities
  - Other (not detailed because extremely rare)

Introduction

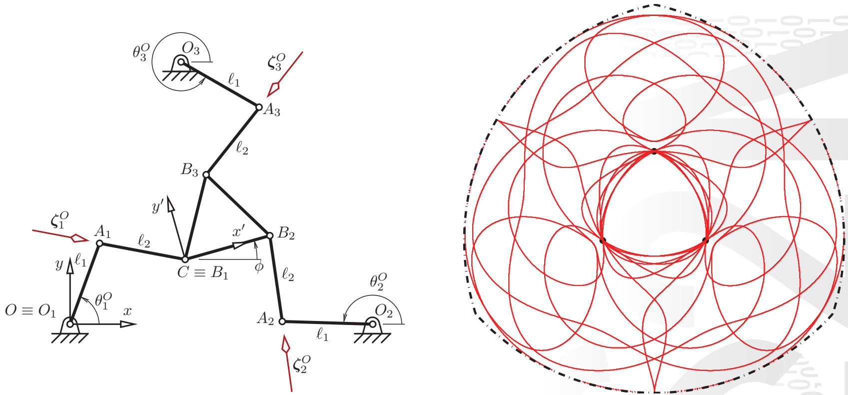
Special types of singularities

In Type 2, constraint and LPJTS singularities

- Loss of stiffness (uncontrollable / gained motions)
- Considerable decrease of performance (deformation, vibration, effort transmission, dynamics, positionning error, etc.)
- Singularities located IN the workspace (not on the boundaries)

Type 2 (parallel) singularities of *PKM*

Probably, the most important drawback of PKM



Type 2 Singularities of a 3–<u>R</u>RR planar robot [Bonev 2001]

Type 2 (parallel) singularities of *PKM*

Normally, impossible to cross these singularities

Because near these singularities, the input torques tend to infinity

TRAVERSEE Type 2

Type 2 (parallel) singularities of *PKM*

But...

By proper trajectory planning respecting a dynamics criterion [Briot et Arakelian 2008] and an adequate controller [Pagis et al, 2015]

TRAVERSEE Type 2

Singularities of parallel robots

How to find Type 2 or constraint singularities?

In the late 80's

- Type 2 singularities
  - Compute the I/O kinematic relationship:

$$\mathbf{A}(\mathbf{q}_a, \mathbf{x})^0\mathbf{t}_p + \mathbf{B}(\mathbf{q}_a, \mathbf{x})\dot{\mathbf{q}}_a = \mathbf{0} \tag{1}$$

  - Compute the determinant of **A** and find the conditions for which it is equal to 0

⇒ Limited to simple cases

Singularities of parallel robots

How to find Type 2 or constraint singularities?

In the late 80's

- Type 2 singularities
  - Compute the I/O kinematic relationship:

$$\mathbf{A}(\mathbf{q}_a, \mathbf{x})^0\mathbf{t}_p + \mathbf{B}(\mathbf{q}_a, \mathbf{x})\dot{\mathbf{q}}_a = \mathbf{0} \quad (1)$$

  - Compute the determinant of **A** and find the conditions for which it is equal to 0

  ⇒ Limited to simple cases

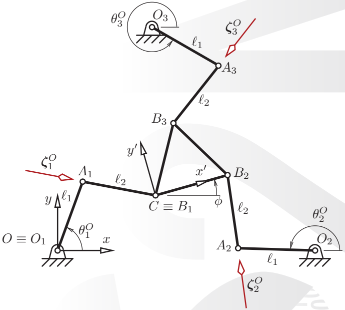
- Constraint singularities:
  - Discovered at the early 2000's
  - Cannot be found using the previous method

Introduction

How to find Type 2 or constraint singularities?

In the early 90's, a new method based on the Grassmann geometry

- Type 2 or constraint sing. $\equiv$ singularities of the system of (static) wrenches applied by the legs on the platform

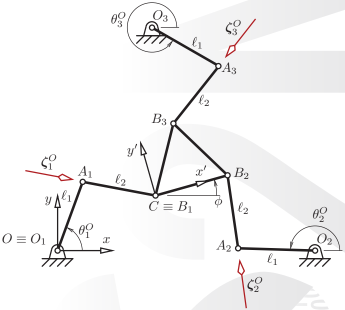


Introduction

How to find Type 2 or constraint singularities?

In the early 90's, a new method based on the Grassmann geometry

- Type 2 or constraint sing. ≡ singularities of the system of (static) wrenches applied by the legs on the platform
  - Find the system of wrenches applied by the legs on the platform using the Screw Theory
  - Analyze the degeneracy of this system of wrenches using the Grassmann geometry



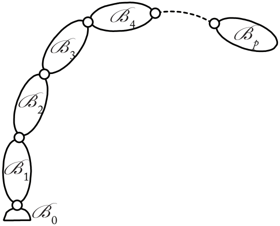
Determination of the system of wrenches

Basic idea for finding the wrenches applied by the leg i

Come back to the definition of the *kinematic* Jacobian matrix of a single serial leg

Determination of the system of wrenches

For serial leg (the *i*th leg of the parallel robot)



$$\mathbf{t}_p = \mathbf{J}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i \text{ where} \tag{2}$$

$$\mathbf{J}_i = \begin{bmatrix} \sigma_{i1}\mathbf{a}_{i1} + \bar{\sigma}_{i1}(\mathbf{a}_{i1} \times \mathbf{r}_{A_{i1}P}) & \dots & \sigma_{im_i}\mathbf{a}_{im_i} + \bar{\sigma}_{im_i}(\mathbf{a}_{im_i} \times \mathbf{r}_{A_{im_i}P}) \\ \bar{\sigma}_{i1}\mathbf{a}_{i1} & \dots & \bar{\sigma}_{im_i}\mathbf{a}_{im_i} \end{bmatrix}$$

$$\mathbf{J}_i = \begin{bmatrix} \$_{i1} & \dots & \$_{im_i} \end{bmatrix}$$

Determination of the system of wrenches

By definition

- $\mathbf{a}_{ij}$ is a 3D vector which represents the direction of the joint axis at A_{ij}
- $\sigma_{ij} = 1$ ($\bar{\sigma}_{ij} = 0$) if the joint located at A_{ij} is a P joint
- $\sigma_{ij} = 0$ ($\bar{\sigma}_{ij} = 1$) if the joint located at A_{ij} is a R joint
- $\$_{ij}$ is the unit twist characterizing the motion of the platform when the joint located at A_{ij} is moving

Determination of the system of wrenches

By definition

- $\mathbf{a}_{ij}$ is a 3D vector which represents the direction of the joint axis at A_{ij}
- $\sigma_{ij} = 1$ ($\bar{\sigma}_{ij} = 0$) if the joint located at A_{ij} is a P joint
- $\sigma_{ij} = 0$ ($\bar{\sigma}_{ij} = 1$) if the joint located at A_{ij} is a R joint
- $\$_{ij}$ is the unit twist characterizing the motion of the platform when the joint located at A_{ij} is moving

Thus

$$\mathbf{t}_p = \sum_{j=1}^{m_i} \$_{ij} \dot{q}_{ij} \qquad (3)$$

where $\dot{q}_{ij}$ is the velocity of the joint at A_{ij}

Determination of the system of wrenches

We group, for the leg *i*,

- in a sub-matrix ${}^0\$_{ia}$ the unit twists corresponding to the active joints of velocities $\dot{\mathbf{q}}_{ai}$,
- in a sub-matrix ${}^0\$_{id}$ the unit twists corresponding to the passive joints of velocities $\dot{\mathbf{q}}_{di}$

and we express all equations in the base frame $\mathcal{F}_0$ (superscript "0" before the variables)

Determination of the system of wrenches

We group, for the leg i,

- in a sub-matrix ${}^0\$_{ia}$ the unit twists corresponding to the active joints of velocities $\dot{\mathbf{q}}_{ai}$,
- in a sub-matrix ${}^0\$_{id}$ the unit twists corresponding to the passive joints of velocities $\dot{\mathbf{q}}_{di}$

and we express all equations in the base frame $\mathcal{F}_0$ (superscript “0” before the variables)

Thus

$$ {}^0\mathbf{t}_p = \begin{bmatrix} {}^0\$_{ia} & {}^0\$_{id} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_{ai} \\ \dot{\mathbf{q}}_{di} \end{bmatrix} = {}^0\$_{ia} \, \dot{\mathbf{q}}_{ai} + {}^0\$_{id} \, \dot{\mathbf{q}}_{di}. \quad (4) $$

Determination of the system of wrenches

For the leg i,

- The constraint wrenches (i.e. the wrenches applied by the leg even if it is not actuated) are the wrenches $\boldsymbol{\zeta}_{id}$ which are reciprocal to both ${}^{0}\$_{ia}$ and ${}^{0}\$_{id}$, i.e. they are defined such that

$$\boldsymbol{\zeta}_{id} \circ {}^{0}\$_{ia} = 0, \ \boldsymbol{\zeta}_{id} \circ {}^{0}\$_{id} = 0 \quad (5)$$

- The actuation wrenches (i.e. the wrenches applied by the leg because of the presence of the actuator) are the wrenches $\boldsymbol{\zeta}_{ia}$ which are reciprocal to ${}^{0}\$_{id}$ and are not included in the system of constraint wrenches $\boldsymbol{\zeta}_{id}$, i.e. they are defined such that

$$\boldsymbol{\zeta}_{ia} \circ {}^{0}\$_{id} = 0, \ \boldsymbol{\zeta}_{ia} \not\subset \boldsymbol{\zeta}_{id} \quad (6)$$

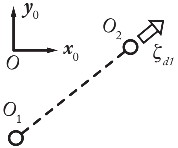
Determination of the system of wrenches

Example of a RR leg

- Motion is represented by two unit twists:

$$^{0}\$_{R1} = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (7)$$

$$^{0}\$_{R2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (8)$$



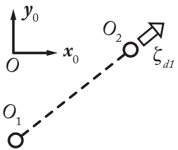
Determination of the system of wrenches

Example of a RR leg

- Motion is represented by two unit twists:

$$ {}^{0}\$_{R1} = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (7)$$

$$ {}^{0}\$_{R2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (8)$$



- If both joints are passive:

$$ \zeta_{d1} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } \overrightarrow{O_1 O_2} \quad (9)$$

$$ \zeta_{d2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure force along } \boldsymbol{z}_0 \quad (10)$$

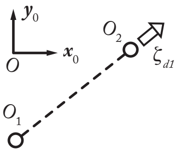
Determination of the system of wrenches

Example of a RR leg

- Motion is represented by two unit twists:

$$ {}^{0}\$_{R1} = \begin{bmatrix} -(y_2 - y_1) & x_2 - x_1 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (7)$$

$$ {}^{0}\$_{R2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (8)$$



- If both joints are passive:

$$ \zeta_{d3} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } \boldsymbol{x}_0 \quad (9)$$

$$ \zeta_{d4} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \Rightarrow \text{a pure moment along } \boldsymbol{y}_0 \quad (10)$$

Determination of the system of wrenches

Then,

- Stack all constraint wrenches ζ_{id} in a matrix ζ_d
- Stack all actuation wrenches ζ_{ia} in a matrix ζ_a
- Analyze the degeneracy of ζ_a and ζ_d thanks to the Grassmann geometry

Singularities of parallel robots

Grassmann geometry

- Gives conditions on degeneracy of systems of lines
- Plücker representation of a line
  - A direction $\mathbf{u}$
  - Moment of the direction $\mathbf{u}$ wrt a given point

Singularities of parallel robots

Grassmann geometry

- Gives conditions on degeneracy of systems of lines
- Plücker representation of a line
  - A direction **u**
  - Moment of the direction **u** wrt a given point

Any wrench is a Plücker representation of a line

Because it is composed of a

- a force **f** applied at a given point Q (direction of the line)
- a moment whose expression at any point P is given by $\mathbf{m}_P = \mathbf{m}_Q + \overrightarrow{PQ} \times \mathbf{f}$

Singularities of parallel robots

A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

$$\zeta_i = \begin{bmatrix} \mathbf{f} \\ \overrightarrow{PQ} \times \mathbf{f} \end{bmatrix} \tag{11}$$

Singularities of parallel robots

A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

$$\zeta_i = \begin{bmatrix} \mathbf{f} \\ \overrightarrow{PQ} \times \mathbf{f} \end{bmatrix} \tag{11}$$

A pure moment wrench is given by, for any application point

$$\zeta_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{m} \end{bmatrix} \tag{12}$$

Singularities of parallel robots

A pure force wrench is given by (at point P, if $\mathbf{f}$ is applied at point Q)

$$\zeta_i = \begin{bmatrix} \mathbf{f} \\ \overrightarrow{PQ} \times \mathbf{f} \end{bmatrix} \quad (11)$$

A pure moment wrench is given by, for any application point

$$\zeta_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{m} \end{bmatrix} \quad (12)$$

These expressions are Plücker representations of lines

- the pure force wrench: a line of direction $\mathbf{f}$ passing through point P
- the pure moment wrench: a line of direction $\mathbf{m}$ but in the projective plane at infinity

Singularities of parallel robots

Thanks to Grassmann geometry

Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

Singularities of parallel robots

Thanks to Grassmann geometry

Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

It is still quite complicated

However for 2 and 3DOF planar robots, the conditions are quite simple to analyze

Singularities of parallel robots

Thanks to Grassmann geometry

Possibility to analyze the conditions of deficiency of a system whose basis is represented by a set of lines

It is still quite complicated

However for 2 and 3DOF planar robots, the conditions are quite simple to analyze

For planar robots

- **2 DOF:** degeneracy if the two lines are parallel
- **3 DOF:** degeneracy if the three (coplanar) lines intersect in the same point (that may be at infinity) $\Rightarrow$ instantaneous center of rotation

Singularities of parallel robots

A few notations

- **a**, **b**: two points located at the position **a** and **b** in the Cartesian space (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to $w \neq 0$)
- **A**, **B**: two points located at the position **A** and **B** in the projective plane at infinity (if applying coordinates, using the Plücker representation with 4 coordinates, the last one is equal to $w = 0$)
- **ab**, the line passing through points **a** and **b**
- **abc**, the plane passing through points **a**, **b** and **c**
- [**abcd**]: the determinant of the (4×4) matrix whose columns are the expressions of the points **a**, **b**, **c** and **d** (in other words, the volume of the tetrahedron)
- $\bigwedge$: the "meet operator"

Singularities of parallel robots

Superbracket decomposition

$$[\mathbf{ab},\ \mathbf{cd},\ \mathbf{ef},\ \mathbf{gh},\ \mathbf{ij},\ \mathbf{kl}] = \sum_{i=1}^{24} y_i \tag{13}$$

where

$$\begin{array}{lll}
y_1 = -[\mathbf{abcd}][\mathbf{efgi}][\mathbf{hjkl}] & y_2 = [\mathbf{abcd}][\mathbf{efhi}][\mathbf{gjkl}] & y_3 = [\mathbf{abcd}][\mathbf{efgj}][\mathbf{hikl}] \\
y_4 = -[\mathbf{abcd}][\mathbf{efhj}][\mathbf{gikl}] & y_5 = [\mathbf{abce}][\mathbf{dfgh}][\mathbf{ijkl}] & y_6 = -[\mathbf{abde}][\mathbf{cfgh}][\mathbf{ijkl}] \\
y_7 = -[\mathbf{abcf}][\mathbf{degh}][\mathbf{ijkl}] & y_8 = [\mathbf{abdf}][\mathbf{cegh}][\mathbf{ijkl}] & y_9 = -[\mathbf{abce}][\mathbf{dghi}][\mathbf{fjkl}] \\
y_{10} = [\mathbf{abde}][\mathbf{cghi}][\mathbf{fjkl}] & y_{11} = [\mathbf{abcf}][\mathbf{dghi}][\mathbf{ejkl}] & y_{12} = [\mathbf{abce}][\mathbf{dghj}][\mathbf{fikl}] \\
y_{13} = -[\mathbf{abdf}][\mathbf{cghi}][\mathbf{ejkl}] & y_{14} = -[\mathbf{abde}][\mathbf{cghj}][\mathbf{fikl}] & y_{15} = -[\mathbf{abcf}][\mathbf{dghj}][\mathbf{eikl}] \\
y_{16} = [\mathbf{abdf}][\mathbf{cghj}][\mathbf{eikl}] & y_{17} = [\mathbf{abcg}][\mathbf{defi}][\mathbf{hjkl}] & y_{18} = -[\mathbf{abdg}][\mathbf{cefi}][\mathbf{hjkl}] \\
y_{19} = -[\mathbf{abch}][\mathbf{defi}][\mathbf{gjkl}] & y_{20} = -[\mathbf{abcg}][\mathbf{defj}][\mathbf{hikl}] & y_{21} = [\mathbf{abdh}][\mathbf{cefi}][\mathbf{gjkl}] \\
y_{22} = [\mathbf{abdg}][\mathbf{cefj}][\mathbf{hikl}] & y_{23} = [\mathbf{abch}][\mathbf{defj}][\mathbf{gikl}] & y_{24} = -[\mathbf{abdh}][\mathbf{cefj}][\mathbf{gikl}]
\end{array} \tag{14}$$

Determination of the system of wrenches

By an adequate choice of the points for representing the lines (intersection points, points are infinity, etc)

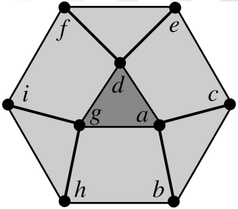
Many monomials y_i can be deleted

Example [Ben Horin and Shoham 2006]

$$[\mathbf{ab},\ \mathbf{ac},\ \mathbf{de},\ \mathbf{df},\ \mathbf{gh},\ \mathbf{gi}] =$$
$$[\mathbf{ad\dot{f}g}][\mathbf{abc\dot{d}}][\mathbf{\dot{e}igh}] = \mathbf{edf} \wedge \mathbf{igh} \wedge \mathbf{abc} \wedge \mathbf{adg}$$

Geometric interpretation

Intersection of four planes



Singularities of parallel robots

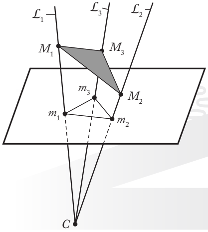
Remarks

- These tools for singularity analysis are difficult to be used by non expert
- But a lot of scientific litterature ⇒ If we know the general formulation of the system of wrenches, for instance
  - 3 forces + 3 moments
  - 6 forces, but only three points of applications, two forces by points

  geometric interpretation of results are already given (see the next slides)
- Sometimes, we still must do the analysis
- These tools were primarly used for singularities of PKM, we will show now that they can be used for other singularity analyses

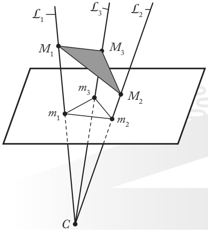
Introduction

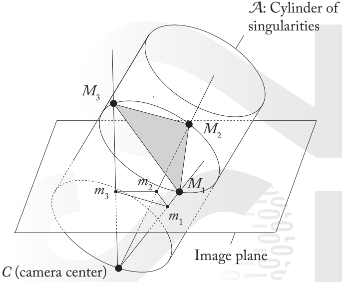
- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**



Introduction

- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**
- To the best of our knowledge, only known for three 3-D image points (*singularity cylinder*)
- Issue with singularities: interaction matrix cannot be inverted anymore = **loss of controllability**





Introduction

In order to avoid singularities

Increased number of image features (redundancy):

- Pb of local minima
- Proof that there is no singularity?

Determining the singularity cases stays an open problem

Introduction

Recently, the "Hidden Robot Concept" was developped

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea $\Rightarrow$ Interaction matrix $\equiv$ Inv. Jacobian matrix of a virtual PKM

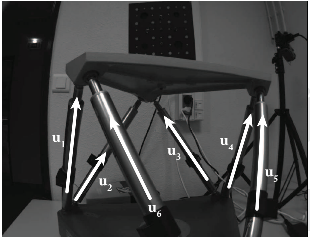
Introduction

Recently, the "Hidden Robot Concept" was developped

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM

For instance, when observing the **leg directions** of the GS platform

- Real robot = 6–$U\underline{P}S$



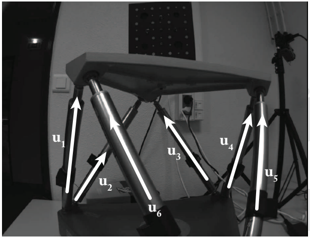
Introduction

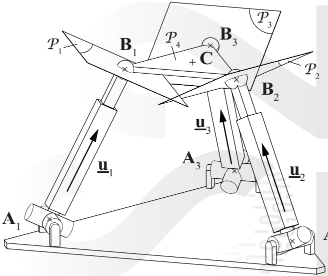
Recently, the "Hidden Robot Concept" was developped

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea ⇒ Interaction matrix ≡ Inv. Jacobian matrix of a virtual PKM

For instance, when observing the **leg directions** of the GS platform

- Real robot = $6\text{–}U\underline{P}S$
- Virtual robot = $6\text{–}\underline{U}PS$





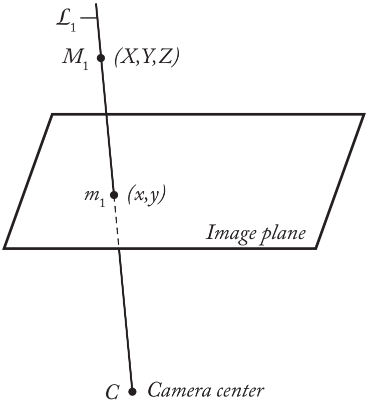
Introduction

Here

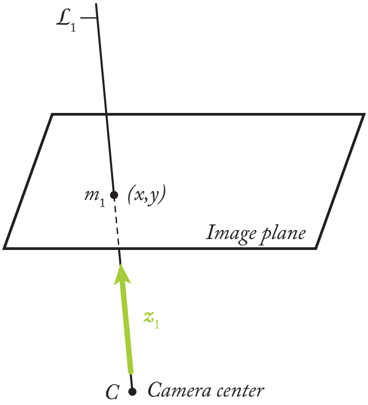
I show how we used the hidden robot concept in order to solve, for the first time, the singularities in

1. the observation of n image points ($n \geq 3$)
2. the observation of three lines
3. the leg-based visual servoing of parallel robots

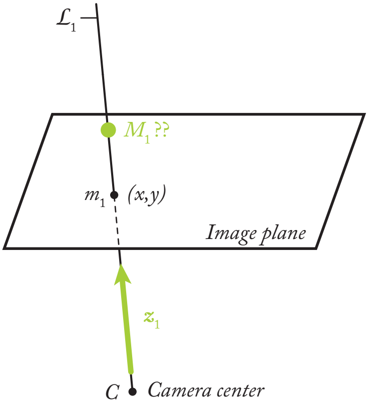
Observation of an image point



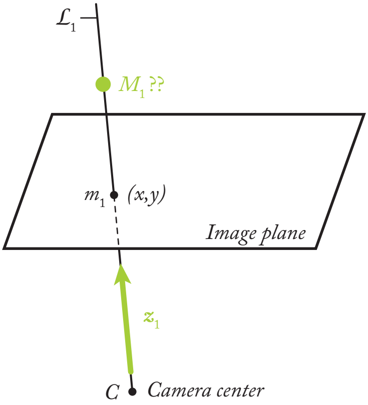
Observation of an image point



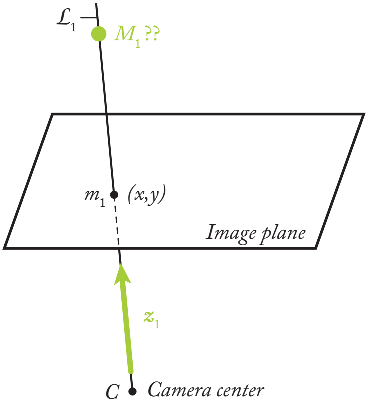
Observation of an image point



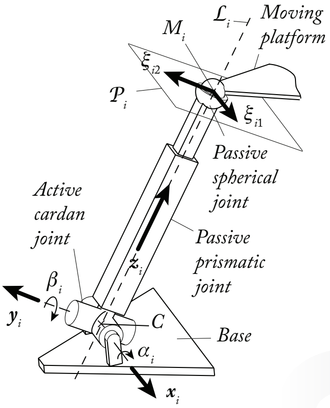
Observation of an image point



Observation of an image point

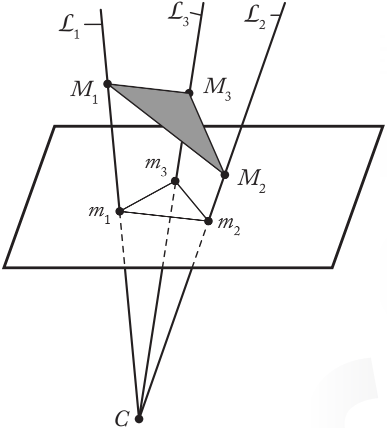


Observation of an image point

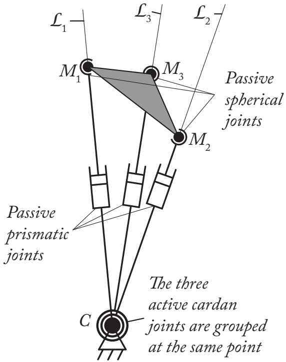


A $\underline{U}PS$ kinematic chain which allows for the same motion of the point M_i

Observation of three image points



Observation of three image points



A 3–*<u>U</u>PS* robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

$$\dot{\mathbf{s}} = \mathbf{L}\boldsymbol{\tau} \; // \; \dot{\mathbf{q}} = \mathbf{J}_{inv}\boldsymbol{\tau}$$

Singularities

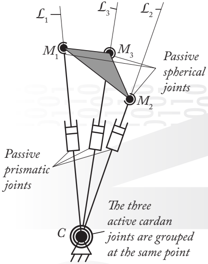
Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]



Singularities

Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

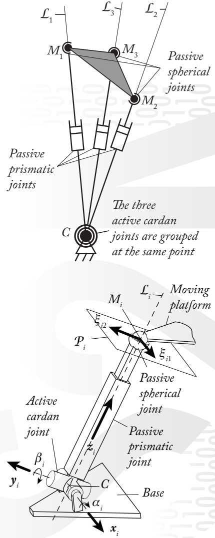
Singularities of parallel robots

Can be studied by using several (complementary) tools

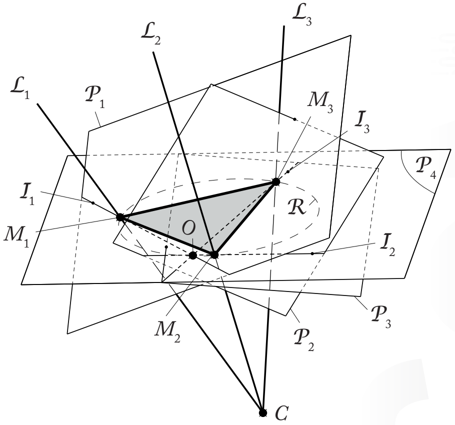
- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]

In our case (3 points), it can be proven that

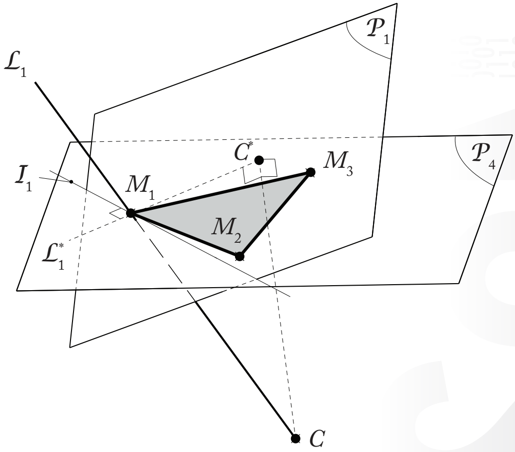
The planes $\mathcal{P}_i$ ($i = 1, 2, 3$) and $\mathcal{P}_4$ (containing all 3-D points) have a non-null intersection



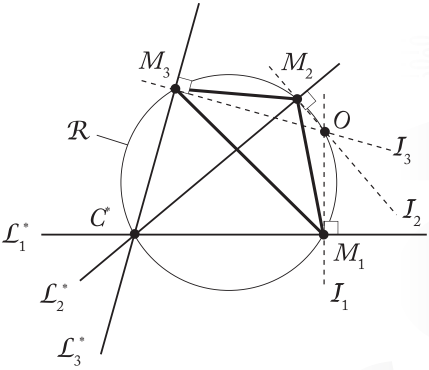
Singularities when observing 3 points



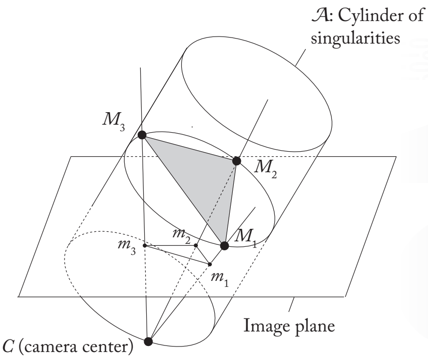
Singularities when observing 3 points



Singularities when observing 3 points



Singularities when observing 3 points



Singularities when observing n points ($n > 3$)

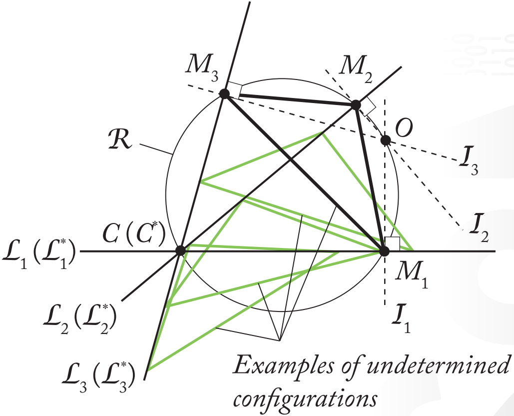
Possible if and only if

- All singularity cylinders associated with any subset of 3 points have a common intersection
- AND all kernels of the interaction matrices are identical

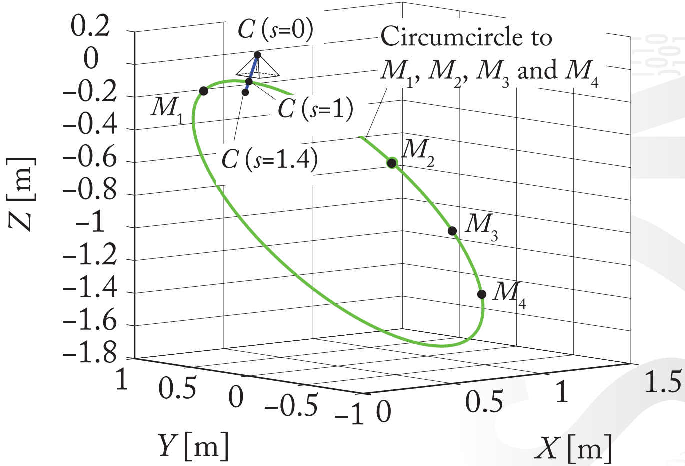
After (more complex) mathematical derivations, we proved that

The conditions of singularity when n coplanar points are observed only appear if and only if all 3-D points and the optical center are located on the same circle

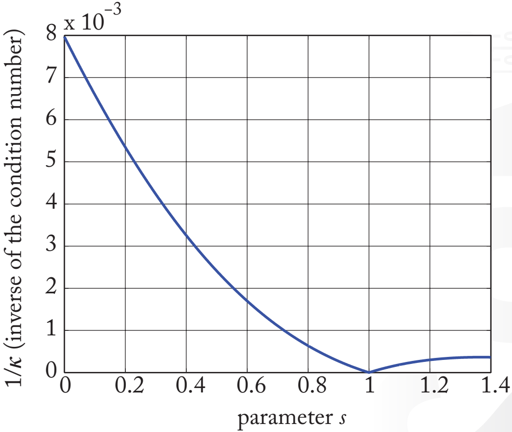
Singularities when observing n points ($n > 3$)



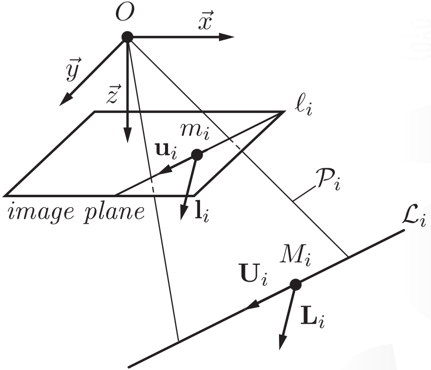
Simulations



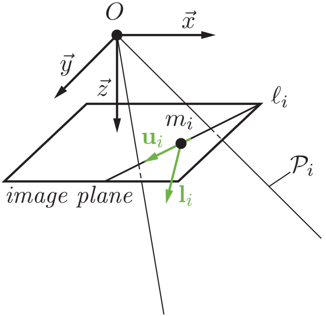
Simulations



Observation of an image line

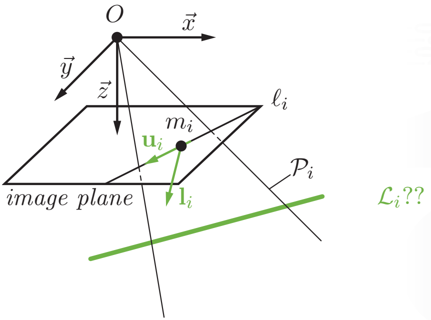


Observation of an image line

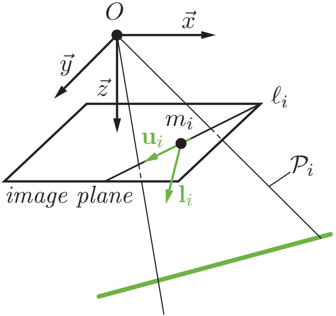


$\mathcal{L}_i$??

Observation of an image line

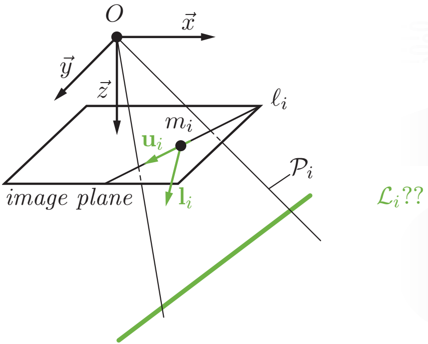


Observation of an image line

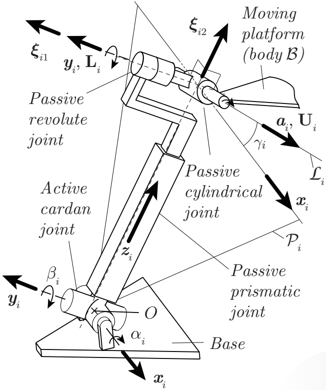


$\mathcal{L}_i$??

Observation of an image line

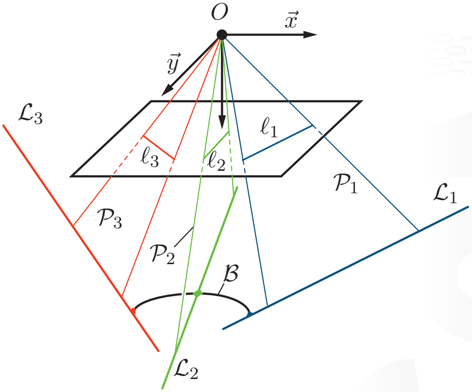


Observation of an image line

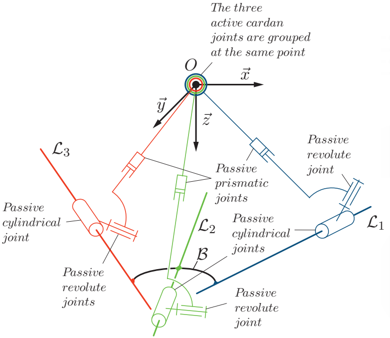


A *<u>U</u>PRC* kinematic chain which allows for the same motion of the line $\mathcal{L}_i$

Observation of three image lines



Observation of three image lines



A 3–*UPRC* robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

$$\dot{\mathbf{s}} = \mathbf{L}\boldsymbol{\tau} \;//\; \dot{\mathbf{q}} = \mathbf{J}_{inv}\boldsymbol{\tau}$$

Singularities

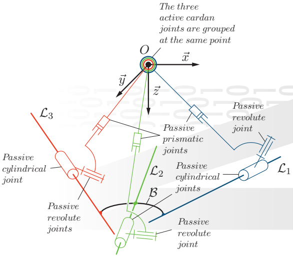
Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]



Singularities

Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

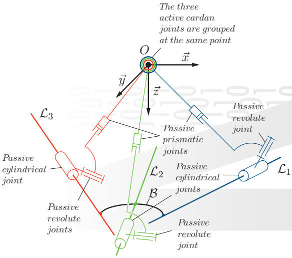
Singularities of parallel robots

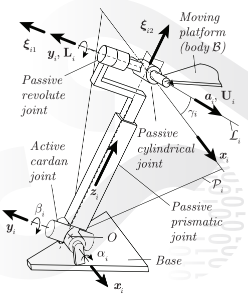
Can be studied by using several (complementary) tools

- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]

In our case (3 lines), singu. cond. iff

$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0$ or
$f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0$
where $\boldsymbol{\xi}_{ij} = [\mathbf{f}_{ij}^T \; \mathbf{m}_{ij}^T]^T$





Singularities

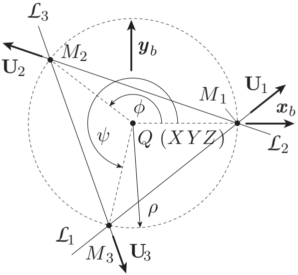
In order to simplify the problem

- Consider the "zero" platform orientation
- General case obtained by a simple rotation

$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T = \mathcal{R} \begin{bmatrix} X' & Y' & Z' \end{bmatrix}^T \quad (15)$$

where
X, Y and Z: position of the origin of the object frame $\mathcal{F}_b$ in the camera frame when considering the "zero" platform orientation
X', Y' and Z': position of the origin of the object frame for the considered "non-zero" platform orientation
$\mathcal{R}$ the rotation matrix between the two cases

Three coplanar lines with no common intersection point



$$
\begin{aligned}
f_1 = 0 &\Leftrightarrow Z = 0 \Rightarrow \text{Lines + optical center in the same plane} \\
f_2 = 0 &\Leftrightarrow Z(X^2 + Y^2 - \rho^2) = 0 \Rightarrow \text{ Singularity cylinder!}
\end{aligned}
\tag{16}
$$

Three lines in space with a common intersection point

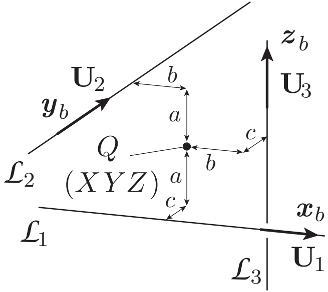
$$\overrightarrow{OQ} = [X\ Y\ Z]^T,\ \mathbf{U}_1 = [1\,0\,0]^T,\ \mathbf{U}_2 = [a\,b\,0]^T,\ \mathbf{U}_3 = [c\,d\,e]^T \tag{17}$$

$f_1 = 0 \Rightarrow$ For any object configuration

$$f_2 = 0 \Leftrightarrow b(adeY^3 + ((-ad^2 + bcd + ae^2)Z + (ac - bd)eX)Y^2 - e(bcX^2 + (ad - bc)Z^2 + 2beXZ)Y + ((-ad^2 + bcd - ae^2)X^2 Z + (bd + ac)eXZ^2)) = 0 \tag{18}$$

$\Rightarrow$ The origin of the body frame belongs to a cubic surface parameterized by $f_2 = 0$.

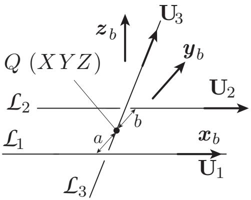
Three orthogonal lines in space



$$\begin{aligned} f_1 = 0 &\Leftrightarrow aXY + bYZ - cXZ - abc = 0 \\ f_2 = 0 &\Leftrightarrow acX - abY + bcZ - XYZ = 0 \end{aligned} \tag{19}$$

$\Rightarrow$ Expression f_1 represents a quadric surface while expression f_2 is a cubic surface

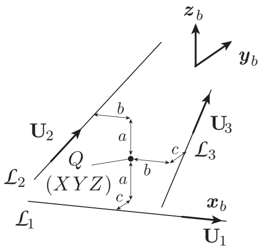
Three lines, two of them being parallel



$$
\begin{aligned}
f_1 = 0 &\Leftrightarrow Z(dZ - eY) = 0 \\
f_2 = 0 &\Leftrightarrow Z(X(d^2 + e^2) - cYd - cZe) = 0
\end{aligned} \tag{20}
$$

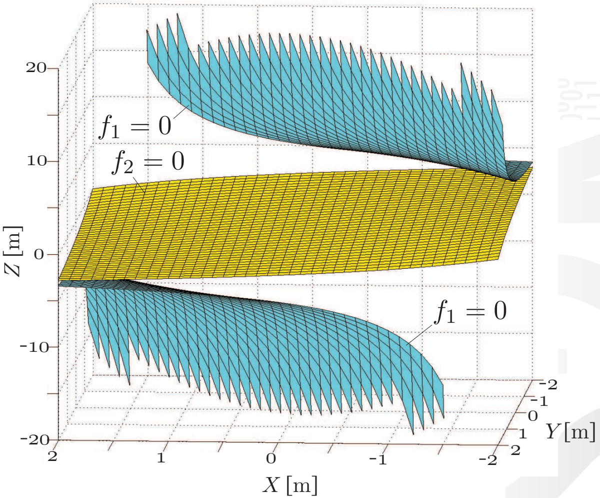
- $Z = 0$, which occur when the plane $\mathcal{P}$ containing $\mathcal{L}_1$ and $\mathcal{L}2$ also contains the optical center,
- $eY - dZ = 0$ is the plane containing $\mathbf{U}_1$, $\mathbf{U}_3$ and the optical center,
- $X(d^2 + e^2) - cdY - ceZ = 0$ is the plane containing $(\mathbf{U}_1 \times \mathbf{U}_3)$, $\mathbf{U}_3$ and the optical center.

Three general lines in space

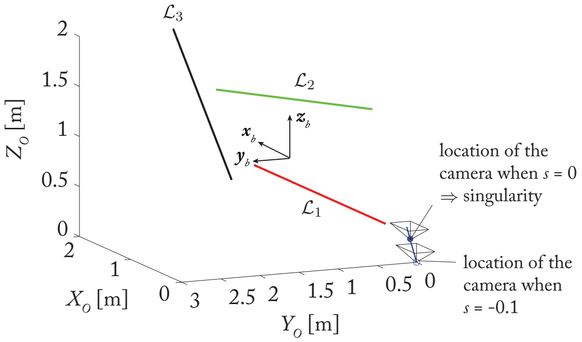


Condition $f_1 = 0$ provides the expression of a quadric surface while $f_2 = 0$ leads to a cubic surface.

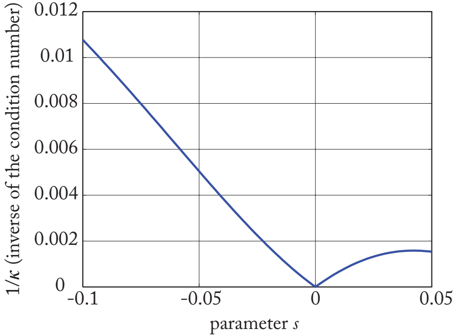
Example for three general lines in space



Simulation 1 (general case)



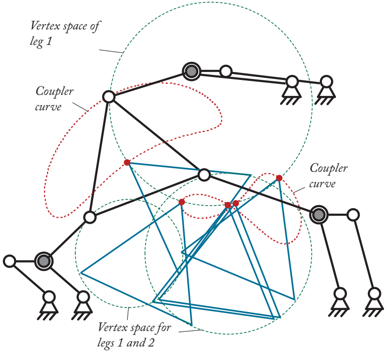
Simulation 1 (general case)



Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

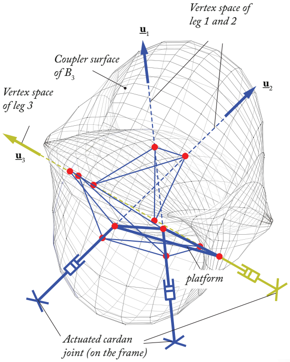
Planar robots: Example of the 3–RRR robot



Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

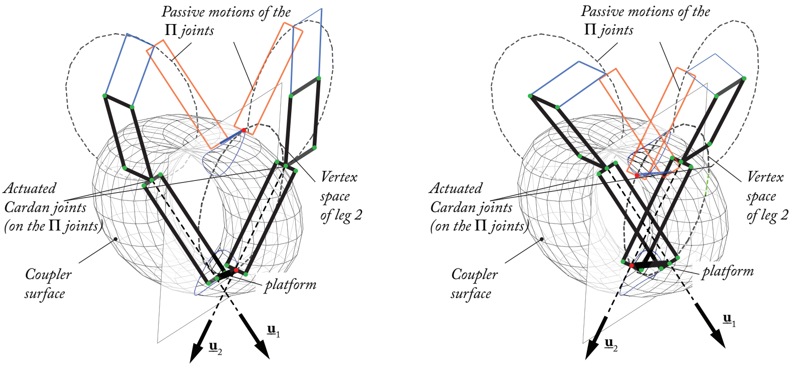
Spatial robots: Example of the GS Platform



Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

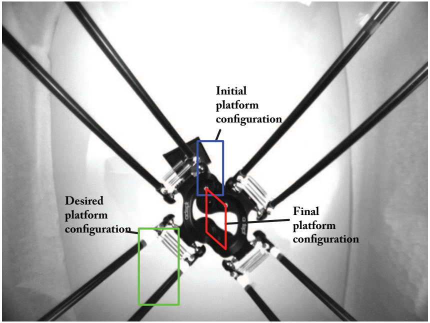
Spatial robots: Example of the Quattro

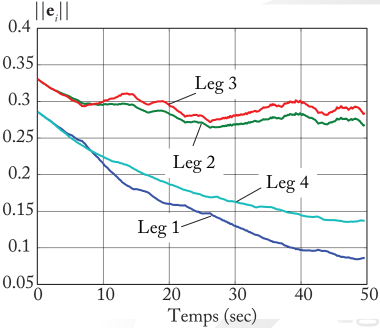


Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Experimental validation

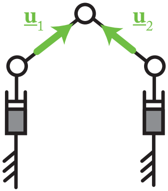




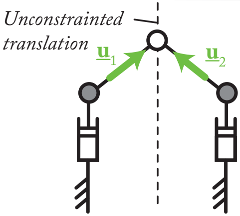
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 1: Robots which are uncontrollable with the observation of the leg directions



A *PRRRP* robot

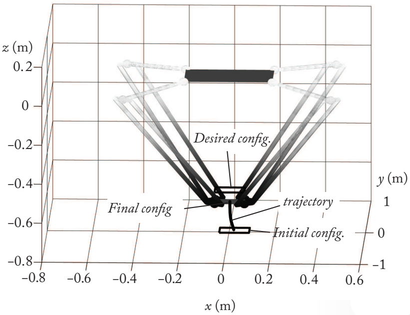


Hidden robot: a *PRRRP* robot

Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

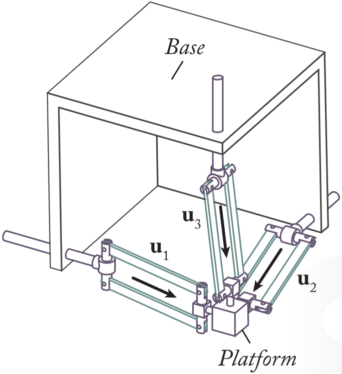
Class 2: Robots which are partially controllable (in their workspace) with the observation of the leg directions



Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

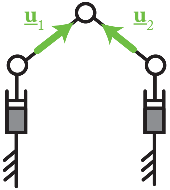
Class 3: Robots which are fully controllable (in their workspace) with the observation of the leg directions



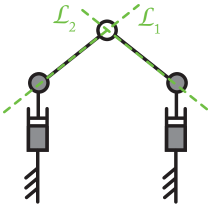
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 4: Robots which are fully controllable (in their workspace) thanks to additional measurements



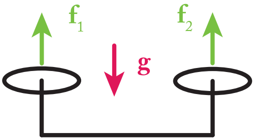
A *PRRRP* robot

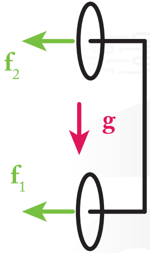


Hidden robot:
a *PRRRP* robot

Singularities appear in many systems

UAVs, ROVs

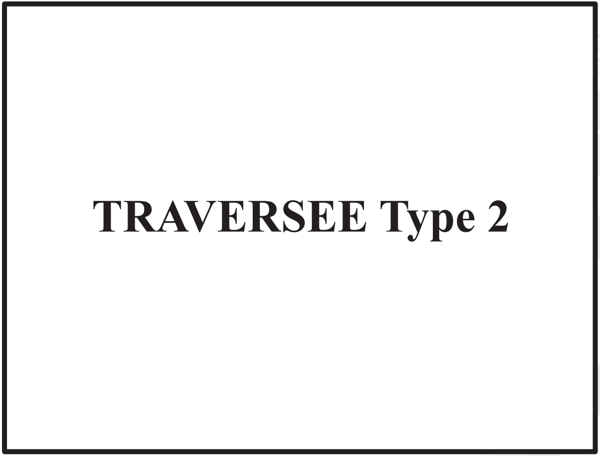




dynamics singularity

Singularities appear in many systems

UAVs, ROVs



Singularities appear in many systems

Reconfigurable drones

TRAVERSEE Type 2

Singularities appear in many systems

GG and AGC needs adaption

Because propellers apply force and torque which are linked (non zero and non infinite pitch screws)

Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot! 🙂

Conclusions

A new Theorem (to be proven)

The World IS a Parallel Robot! 🙂

In this talk,

- I presented a tool named the "hidden robot concept" able to solve the determination of the singularity cases visual servoing based on the observation of geometric features
- we rigorously proved the conditions of singularity for n coplanar points and 3 lines
- we discussed about the generalization of the "hidden robot concept" to other case studies

Conclusions

The hidden robot concept

- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (visual servoing community / mechanical engineering community ⇒ dual problems)

Conclusions

The hidden robot concept

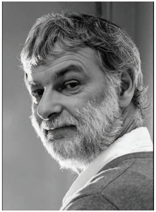
- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (visual servoing community / mechanical engineering community $\Rightarrow$ dual problems)

Tools used here

- Easilly extandable to the rigidity-based control theory
- And maybe other problems
- But useful for you?

Concluding remarks

Colleagues



Students