How theory on parallel robot singularities was used in order to solve sensor-based control problems

LABORATOIRE
DES SCIENCES
DU NUMÉRIQUE
DE NANTES

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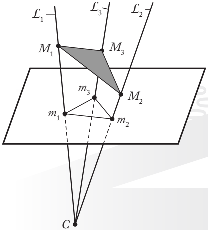
IFAC WC 2017, July 9

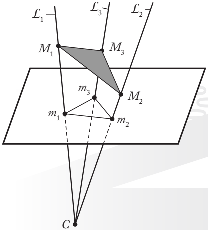
Introduction

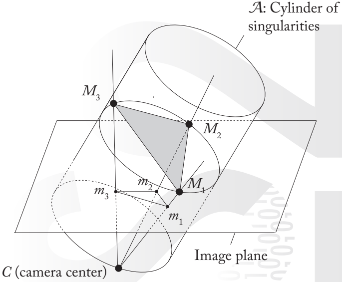
- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**



Introduction

- Singularities appearing when observing image features (e.g. with a camera) = **a huge challenge in visual servoing**
- To the best of our knowledge, only known for three 3-D image points (*singularity cylinder*)
- Issue with singularities: interaction matrix cannot be inverted anymore = **loss of controllability**





Introduction

In order to avoid singularities

Increased number of image features (redundancy):

- Pb of local minima
- Proof that there is no singularity?

Determining the singularity cases stays an open problem

Introduction

Recently, the "Hidden Robot Concept" was developped

- A tool made first for analyzing the singularities in visual servoing dedicated to PKMs
- Basic idea $\Rightarrow$ Interaction matrix $\equiv$ Inv. Jacobian matrix of a virtual PKM

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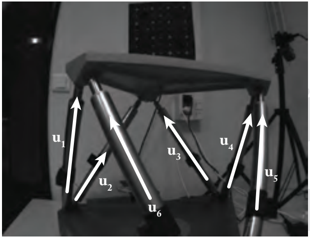

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For instance, when observing the **leg directions** of the GS platform

- Real robot = 6–$U\underline{P}S$

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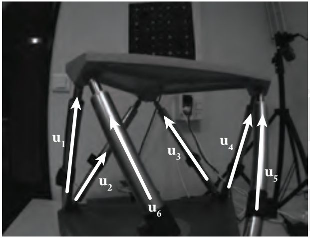
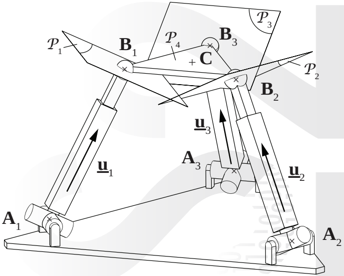
For instance, when observing the **leg directions** of the GS platform

- Real robot = $6\text{–}U\underline{P}S$
- Virtual robot = $6\text{–}\underline{U}PS$





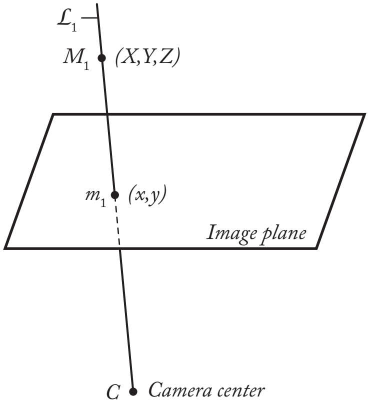
Introduction

Here

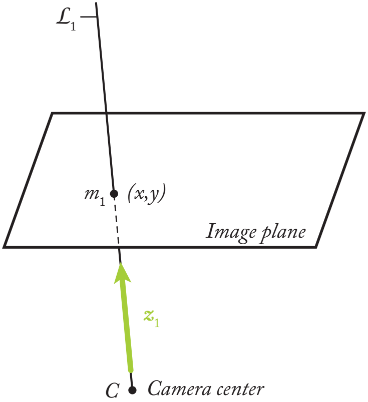
We show how we used the hidden robot concept in order to solve, for the first time, the singularity in

1. the observation of n image points ($n \geq 3$)
2. the observation of three lines
3. the leg-based visual servoing of parallel robots

Observation of an image point



Observation of an image point



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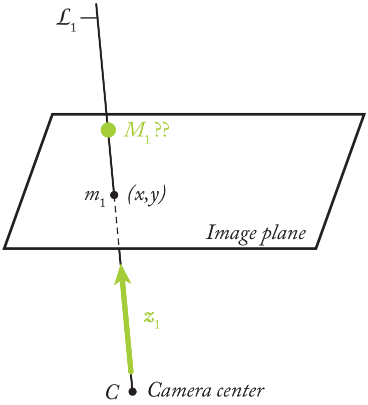
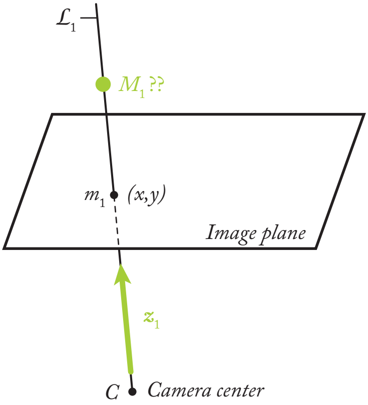


Observation of an image point



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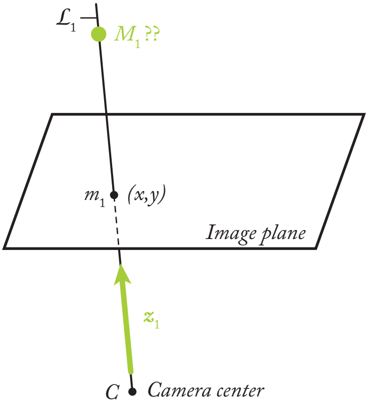

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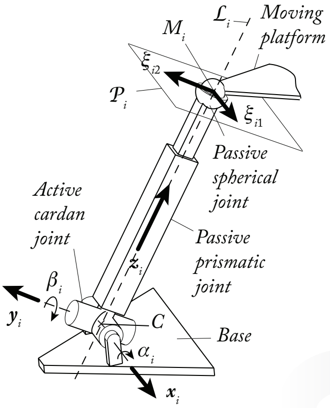
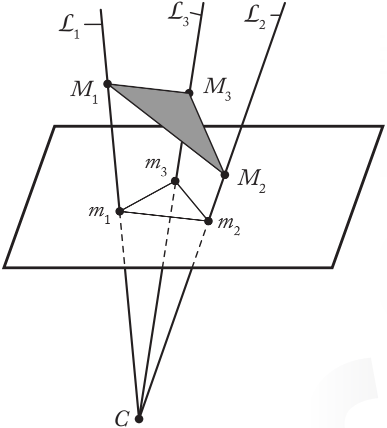
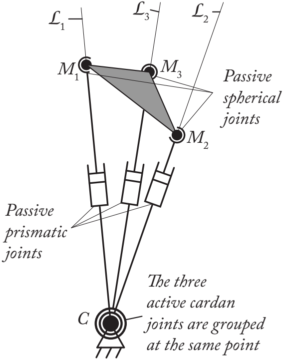


A $\underline{U}PS$ kinematic chain which allows for the same motion of the point M_i

Observation of three image points



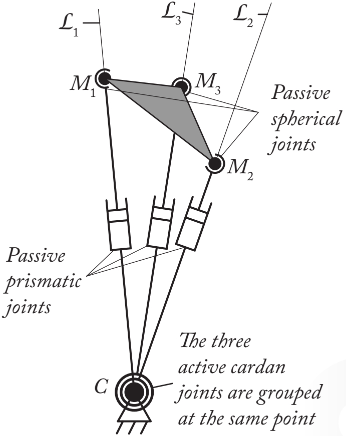
Observation of three image points



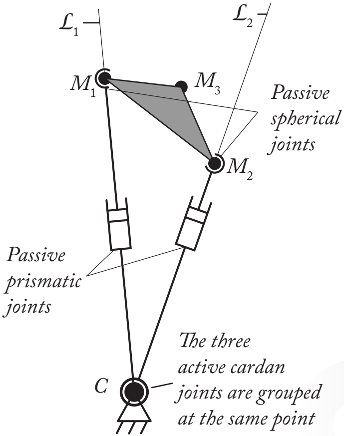
A 3–*<u>U</u>PS* robot which is the virtual robot architecture with its inverse kinematic Jacobian matrix similar to the interaction matrix

$$\dot{\mathbf{s}} = \mathbf{L}\boldsymbol{\tau} \; // \; \dot{\mathbf{q}} = \mathbf{J}_{inv}\boldsymbol{\tau}$$

P3P


$\mathcal{L}_1$
$\mathcal{L}_3$
$\mathcal{L}_2$
M_1
M_3
M_2
Passive spherical joints
Passive prismatic joints
C
The three active cardan joints are grouped at the same point

P3P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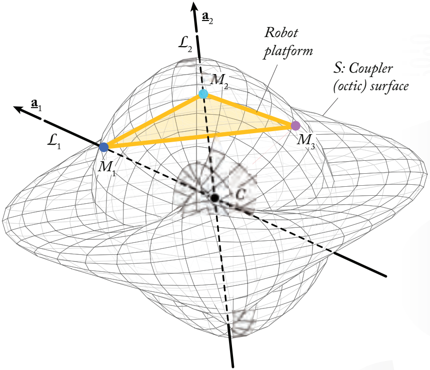

[Tischler et al., 1998]

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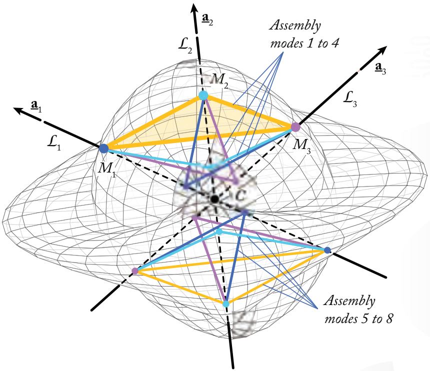

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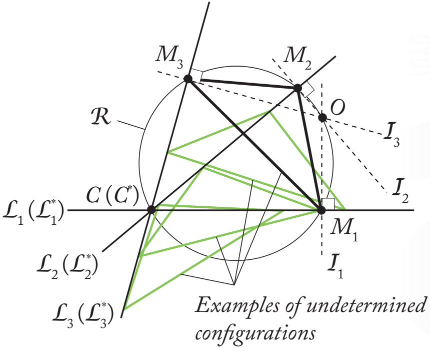
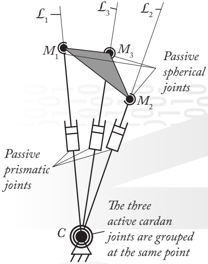


[Tischler et al., 1998]

Singularities



Thanks to the hidden robot analogy

Singularities of the interaction matrix = singularities of the virtual parallel robot

Singularities of parallel robots

Can be studied by using several (complementary) tools

- Screw Theory [Merlet 2006], Grassmann geometry [Merlet 2006], Grassmann-Cayley algebra [Ben-Horin and Shoham, 2006]

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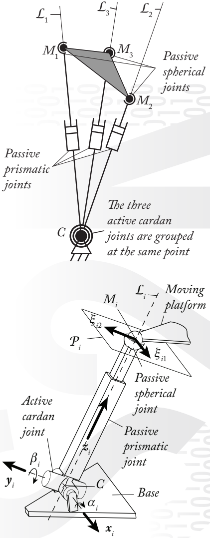
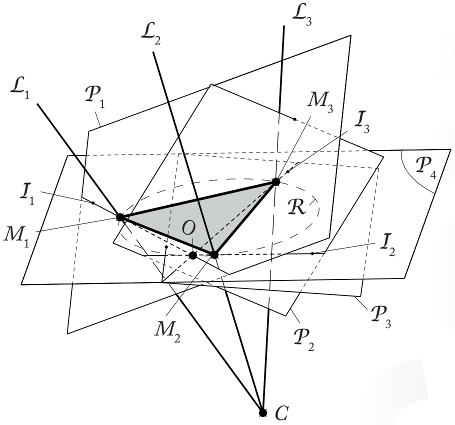
In our case (3 points), it can be proven that

The planes $\mathcal{P}_i$ ($i = 1, 2, 3$) and $\mathcal{P}_4$ (containing all 3-D points) have a non-null intersection



Singularities when observing 3 points



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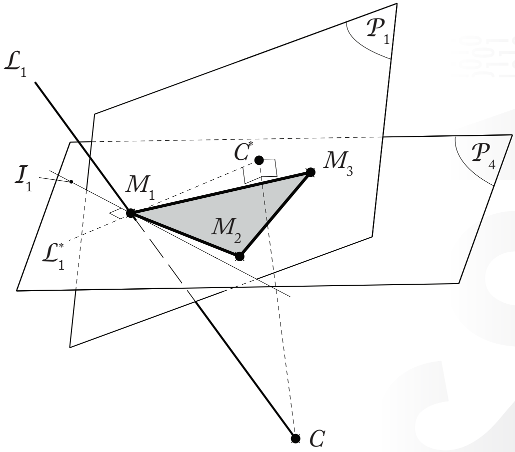

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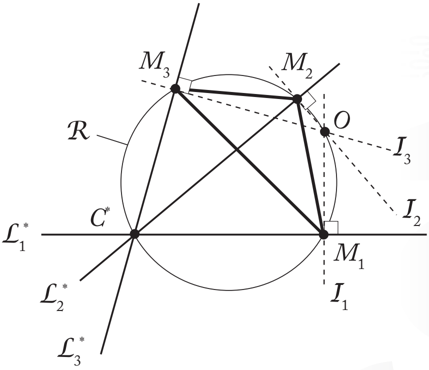

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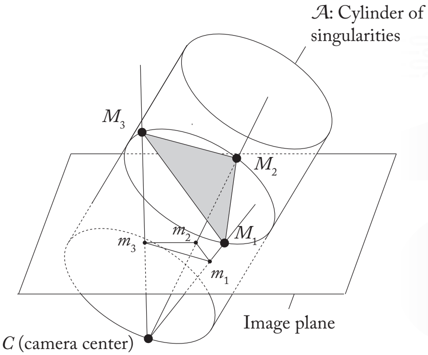


Singularities when observing n points ($n > 3$)

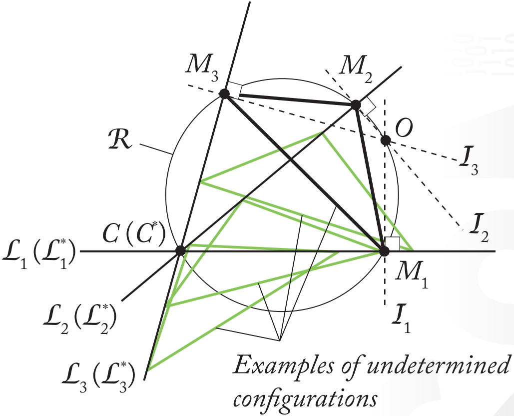
Possible if and only if

- All singularity cylinders associated with any subset of 3 points have a common intersection
- AND all kernels of the interaction matrices are identical

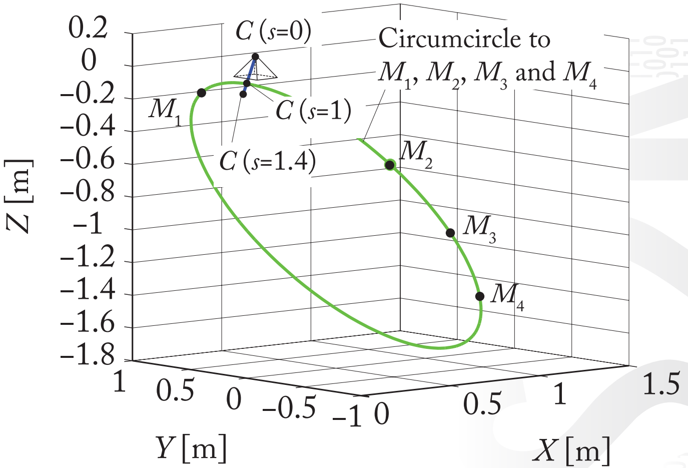
After (more complex) mathematical derivations, we proved that

The conditions of singularity when n coplanar points are observed only appear if and only if all 3-D points and the optical center are located on the same circle

Singularities when observing n points ($n > 3$)



Simulations



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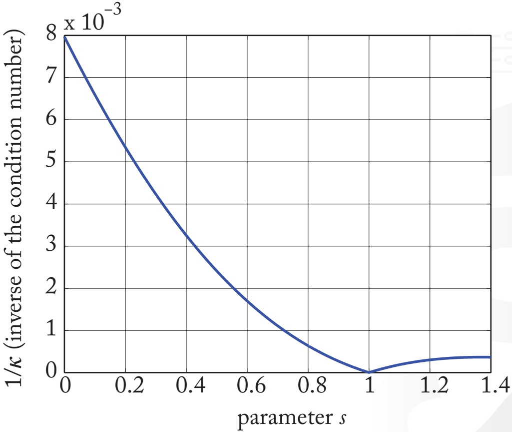
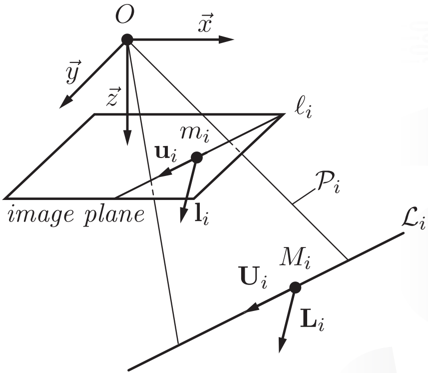


Observation of an image line



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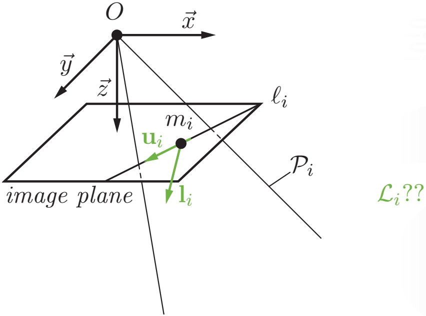

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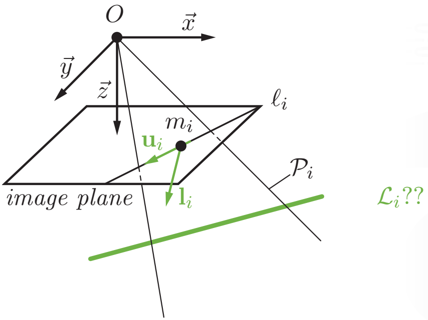

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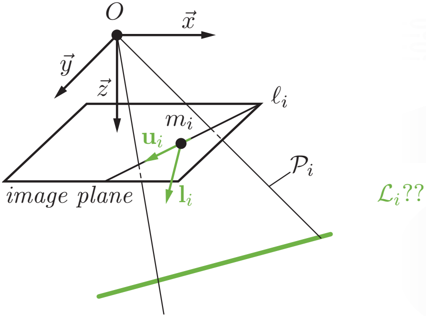

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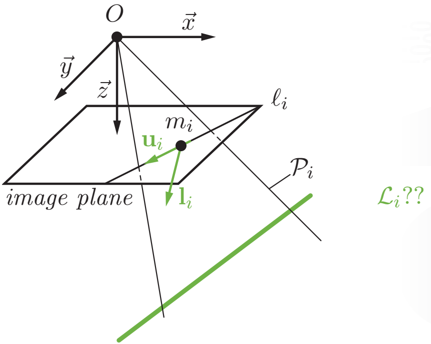

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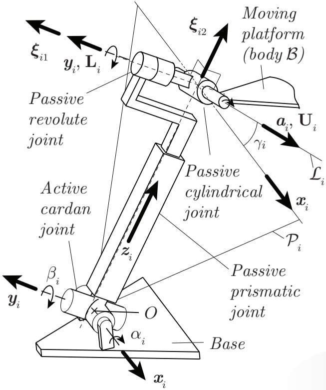

A *UPRC* kinematic chain which allows for the same motion of the line $\mathcal{L}_i$

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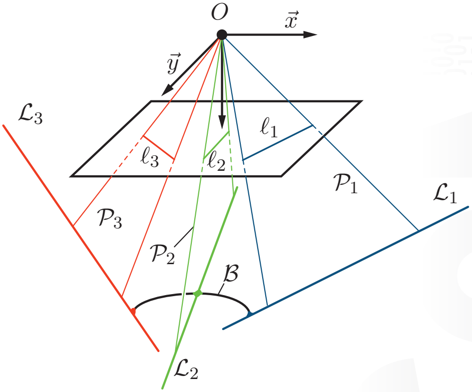

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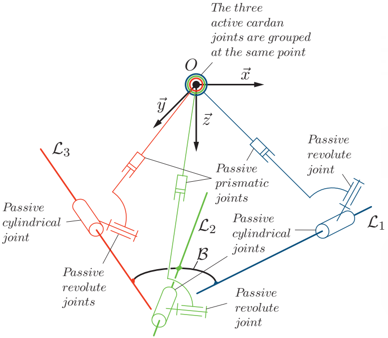

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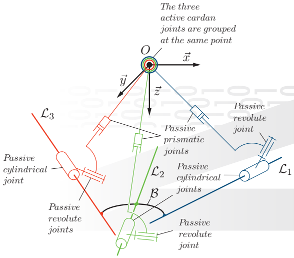

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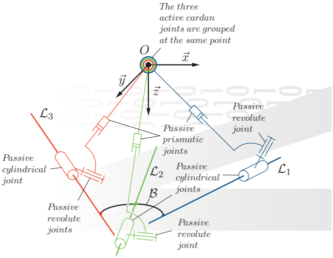
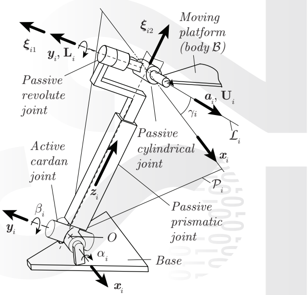
In our case (3 lines), singu. cond. iff

$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0$ or
$f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0$
where $\boldsymbol{\xi}_{ij} = [\mathbf{f}_{ij}^T \ \mathbf{m}_{ij}^T]^T$





Singularities

In order to simplify the problem

- Consider the "zero" platform orientation
- General case obtained by a simple rotation

$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T = \mathcal{R} \begin{bmatrix} X' & Y' & Z' \end{bmatrix}^T \tag{1}$$

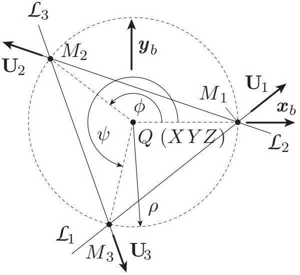
where

X, Y and Z: position of the origin of the object frame $\mathcal{F}_b$ in the camera frame when considering the "zero" platform orientation

X', Y' and Z': position of the origin of the object frame for the considered "non-zero" platform orientation

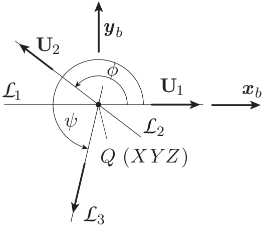
$\mathcal{R}$ the rotation matrix between the two cases

Three coplanar lines with no common intersection point



$$\begin{aligned} f_1 = 0 &\Leftrightarrow Z = 0 \Rightarrow \text{Lines + optical center in the same plane} \\ f_2 = 0 &\Leftrightarrow Z(X^2 + Y^2 - \rho^2) = 0 \Rightarrow \text{ Singularity cylinder!} \end{aligned} \tag{2}$$

Three coplanar lines with a common intersection point



$$\begin{aligned} f_1 = 0 &\Rightarrow \text{ Singular for any object configuration} \\ f_2 = 0 &\Leftrightarrow Z(X^2 + Y^2) = 0 \end{aligned} \tag{3}$$

$\Rightarrow$ Camera center O lies on the line which passes through Q and which is perpendicular to all vectors $\mathbf{U}_i$

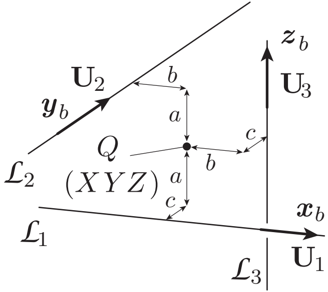
Three lines in space with a common intersection point

$$\overrightarrow{OQ} = [X\ Y\ Z]^T,\ \mathbf{U}_1 = [1\ 0\ 0]^T,\\ \mathbf{U}_2 = [a\ b\ 0]^T,\ \mathbf{U}_3 = [c\ d\ e]^T \tag{4}$$

$$\begin{aligned} f_1 = 0 \Rightarrow&\ \text{For any object configuration} \\ f_2 = 0 \Leftrightarrow&\, b(adeY^3 + ((-ad^2 + bcd + ae^2)Z \\ &+ (ac - bd)eX)Y^2 - e(bcX^2 + (ad - bc)Z^2 \\ &+ 2beXZ)Y + ((-ad^2 + bcd - ae^2)X^2 Z \\ &+ (bd + ac)eXZ^2)) = 0 \end{aligned} \tag{5}$$

$\Rightarrow$ The origin of the body frame belongs to a cubic surface parameterized by $f_2 = 0$.

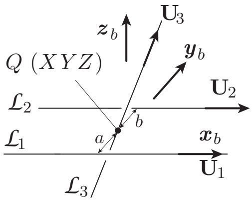
Three orthogonal lines in space



$$
\begin{aligned}
f_1 = 0 &\Leftrightarrow aXY + bYZ - cXZ - abc = 0 \\
f_2 = 0 &\Leftrightarrow acX - abY + bcZ - XYZ = 0
\end{aligned}
\quad (6)
$$

⇒ Expression f_1 represents a quadric surface while expression f_2 is a cubic surface

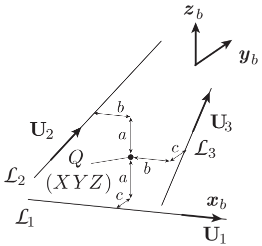
Three lines, two of them being parallel



$$
\begin{aligned}
f_1 = 0 &\Leftrightarrow Z(dZ - eY) = 0 \\
f_2 = 0 &\Leftrightarrow Z(X(d^2 + e^2) - cYd - cZe) = 0
\end{aligned} \tag{7}
$$

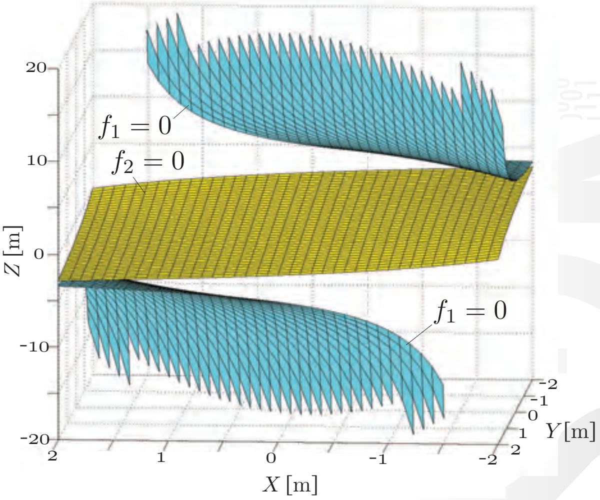
- $Z = 0$, which occur when the plane $\mathcal{P}$ containing $\mathcal{L}_1$ and $\mathcal{L}2$ also contains the optical center,
- $eY - dZ = 0$ is the plane containing $\mathbf{U}_1$, $\mathbf{U}_3$ and the optical center,
- $X(d^2 + e^2) - cdY - ceZ = 0$ is the plane containing $(\mathbf{U}_1 \times \mathbf{U}_3)$, $\mathbf{U}_3$ and the optical center.

Three general lines in space

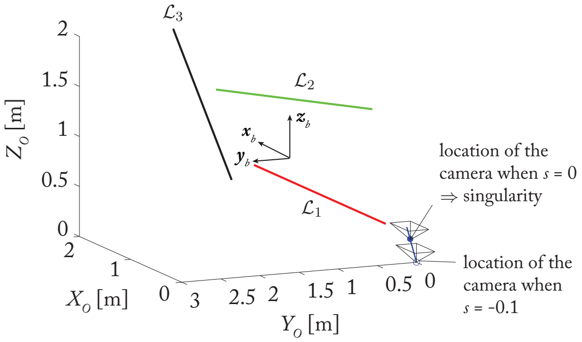


Condition $f_1 = 0$ provides the expression of a quadric surface while $f_2 = 0$ leads to a cubic surface.

Example for three general lines in space



Simulation 1 (general case)



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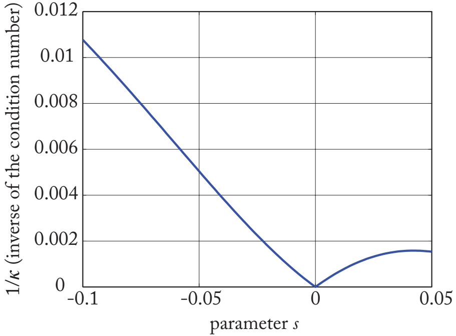
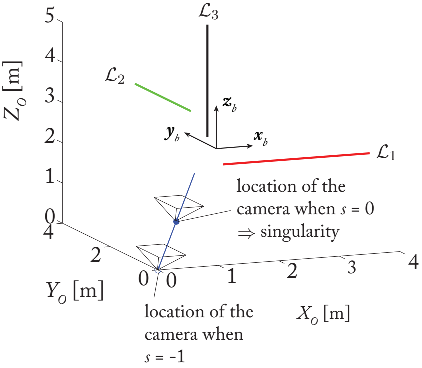


Simulation 2 (lines are perpendicular)



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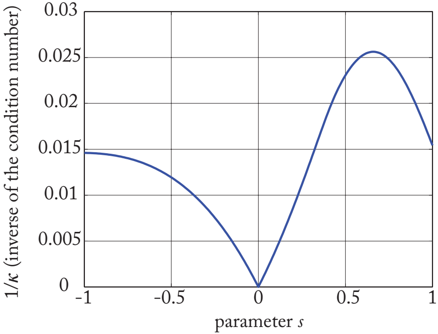


Leg-based visual servoing of parallel robots

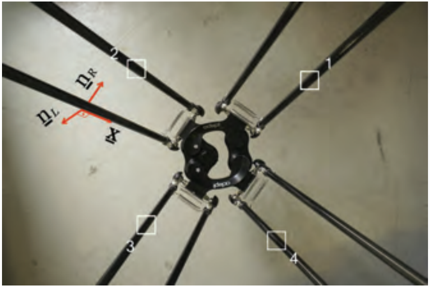
Many approaches, among which

- Direct observation of the end-effector [Paccot et al., 2008]

Leg-based visual servoing of parallel robots

Many approaches, among which

- Leg observation [Özgür et al., 2011]



Leg-based visual servoing of parallel robots

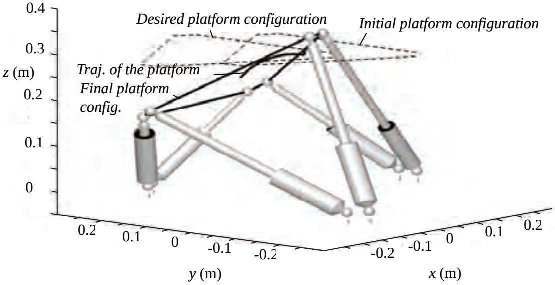
Problems / Questions

- The observation of m leg directions ($m < n$) among the n legs is enough,

Leg-based visual servoing of parallel robots

Problems / Questions

- The observation of m leg directions ($m < n$) among the n legs is enough,
- End-effector convergence issues, even if all leg directions did converge



Leg-based visual servoing of parallel robots

Problems / Questions

- The observation of m leg directions ($m < n$) among the n legs is enough,
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- Existence of local minima

Leg-based visual servoing of parallel robots

Problems / Questions

- The observation of m leg directions ($m < n$) among the n legs is enough,
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- Existence of local minima
- Interaction model singularities

Leg-based visual servoing of parallel robots

Answers thanks to the hidden robot concept

Leg-based visual servoing of parallel robots

Answers thanks to the hidden robot concept

Idea

We control a virtual robot architecture corresponding to the interaction model (different from the real robot)

Leg-based visual servoing of parallel robots

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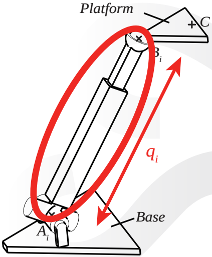
Usual encoder-based control

$\mathbf{q} \Rightarrow \mathbf{x}$ ($\mathbf{q}$: motor encoder measurements)



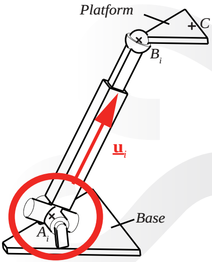
Leg-based visual servoing of parallel robots

Idea

We control a virtual robot architecture corresponding to the interaction model (different from the real robot)

Leg-based visual servoing

$\underline{\mathbf{u}} \Rightarrow \mathbf{x}$ ($\underline{\mathbf{u}}$: virtual actuator measurements)



Leg-based visual servoing of parallel robots

Leg-observation-based control

Gough-Stewart platform

- Real robot ⇒ 6–U$\underline{\text{P}}$S

Leg-based visual servoing of parallel robots

Leg-observation-based control

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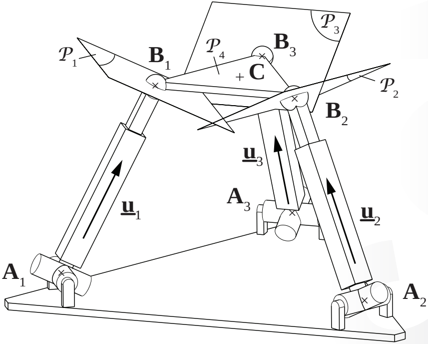
- Real robot ⇒ 6–UPS
- Hidden (virtual) robot ⇒ 3–UPS (case of the minimal observation)



Leg-based visual servoing of parallel robots

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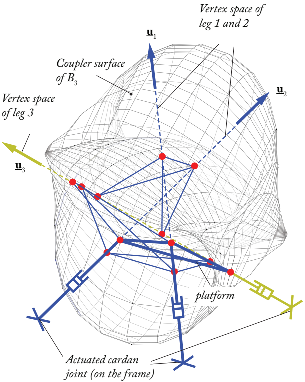

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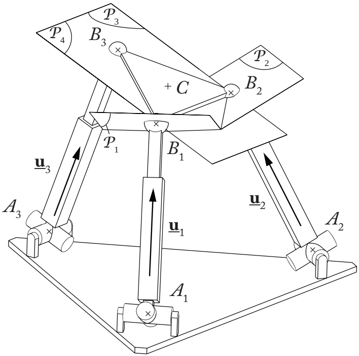
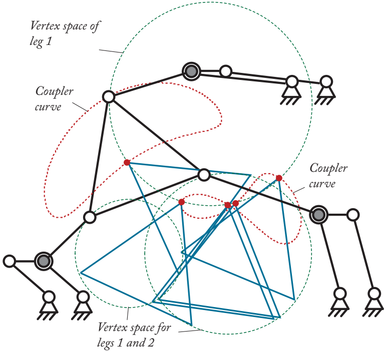


Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

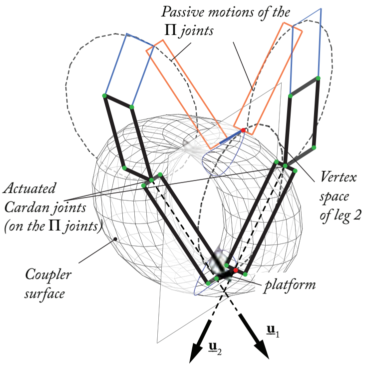
Planar robots: Example of the 3–RRR robot

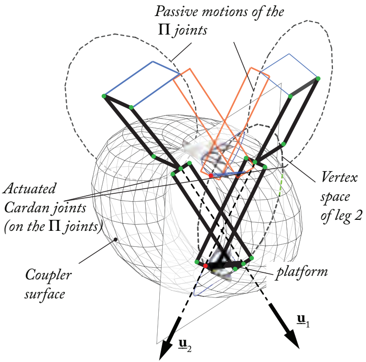


Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Spatial robots: Example of the Quattro

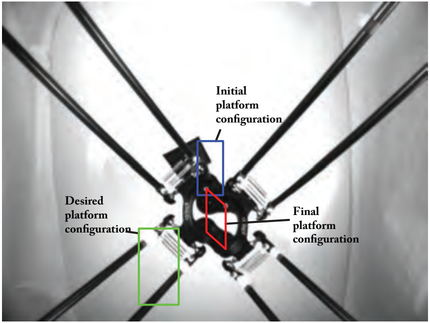


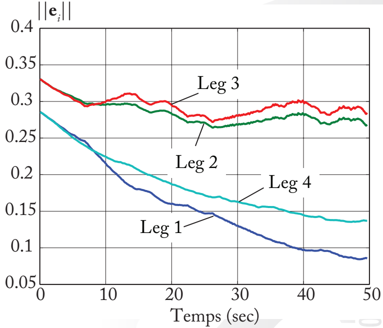


Leg-based visual servoing of parallel robots

Generalisation to families of parallel robots

Experimental validation

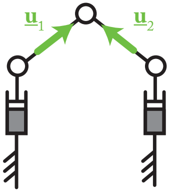




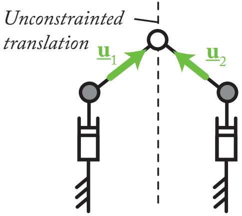
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 1: Robots which are uncontrollable with the observation of the leg directions



A *PRRRP* robot

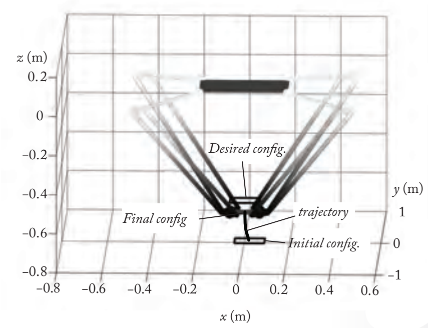


Hidden robot:
a *PRRRP* robot

Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

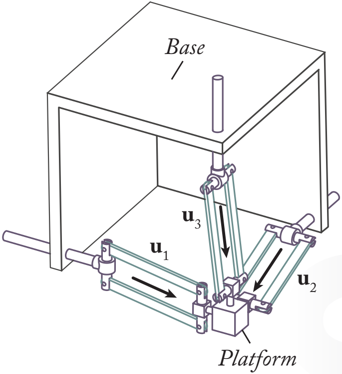
Class 2: Robots which are partially controllable (in their workspace) with the observation of the leg directions



Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

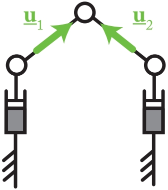
Class 3: Robots which are fully controllable (in their workspace) with the observation of the leg directions



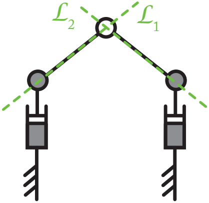
Leg-based visual servoing of parallel robots

Use of the hidden robot concept for analyzing the controllability

Class 4: Robots which are fully controllable (in their workspace) thanks to additional measurements



A *PRRRP* robot



Hidden robot:
a *PRRRP* robot

Conclusions

In this talk,

- I presented a tool named the "hidden robot concept" able to solve the determination of the singularity cases visual servoing based on the observation of geometric features
- we rigorously proved the conditions of singularity for n coplanar points and 3 lines
- we discussed about the generalization of the "hidden robot concept" to other case studies

Conclusions

The hidden robot concept

- a tangible visualization of the mapping between the observation space and the Cartesian space
- allowed to change the way we defined the problem (visual servoing community / mechanical engineering community $\Rightarrow$ dual problems)

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The hidden robot concept

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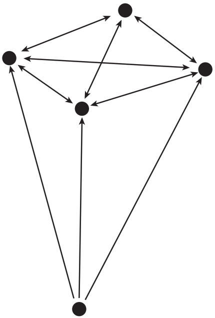
Tools used here

- Easily extendable to the rigidity-based control theory
- But useful for you?

Conclusions

Singularity when using bearing measurements



Conclusions

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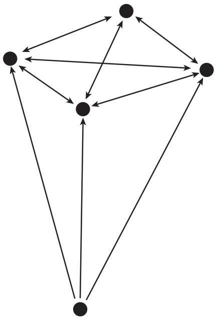

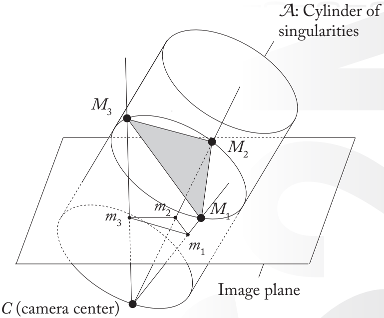

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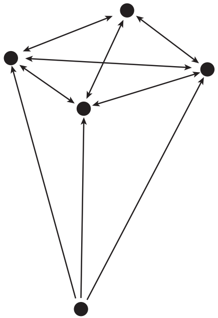

Uniqueness? ⇒ up to 8 solutions

Adding more measurements? ⇒ Bad choice still leads to singularities

Concluding remarks

Colleagues

Students