Technical Report Associated with the Paper: "Determining the Singularities for the Observation of Three Image Lines"

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Singularity cases in the visual servoing of three general lines in space

We consider here the case where the three observed lines have a general configuration. We define the frame $\mathcal{F}_b$: $(Q, \boldsymbol{x}_b, \boldsymbol{y}_b, \boldsymbol{z}_b)$ attached to the observed body $\mathcal{B}$ such that $\boldsymbol{x}_b$ is collinear to $\mathbf{U}_1$, $\boldsymbol{y}_b$ is lying in the plane $\mathcal{P}$ containing $\mathcal{L}_1$ and $\mathcal{L}_2$. We parameterize the lines as follows (see Fig. 1)

$$\begin{aligned}
\overrightarrow{OP_1} &= [(X-b)\ (Y-c)\ (Z-a)]^T,\ \mathbf{U}_1 = [1\,0\,0]^T \\
\overrightarrow{OP_2} &= [(X-b)\ (Y-c)\ (Z+a)]^T,\ \mathbf{U}_2 = [d\,e\,0]^T \\
\overrightarrow{OP_3} &= [(X+b)\ (Y+c)\ Z]^T,\ \mathbf{U}_3 = [f\,g\,h]^T
\end{aligned} \tag{1}$$

where d, e, f, g and h are variables parameterizing the direction of the lines $\mathcal{L}_2$ and $\mathcal{L}_3$.

Then, we have

$$\mathbf{f}_{i1} \propto \mathbf{U}_i \times \overrightarrow{OP_i},\ \mathbf{m}_{i2} \propto \mathbf{U}_i \times \overrightarrow{\mathbf{f}_{i1}} \tag{2}$$

which, from the singularity conditions which are recalled here for reasons of clarity,

$$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0 \text{ or } f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0 \tag{3}$$

leads to

$$\begin{aligned}
f_1 = 0 &\Leftrightarrow a_{2z}Z^2 + a_{1z}Z + a_{0z} = 0 \\
f_2 = 0 &\Leftrightarrow b_{2z}Z^2 + b_{1z}Z + b_{0z} = 0
\end{aligned} \tag{4}$$

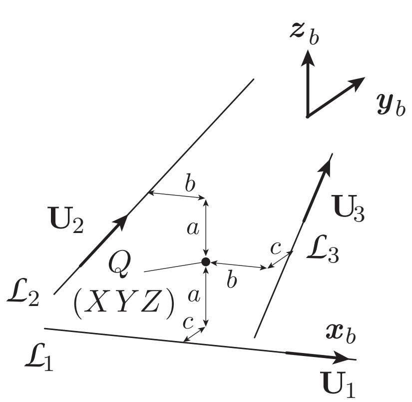

Fig. 1. Observation of three general lines in space

where

$$\begin{aligned}
a_{2z} &= 2e(cf - bg) \\
a_{1z} &= c_{1y}Y + c_{0y} \\
a_{0z} &= d_{2y}Y^2 + d_{1y}Y + d_{0y} \\
b_{2z} &= e_{1x}X + e_{1y}Y + e_0 \\
b_{1z} &= f_{2xy}(Y^2 + Y^2) + f_{1xy}XY + f_{1x}X + f_{1y}Y + f_0 \\
b_{0z} &= g_{3y}Y^3 + g_{21xy}X^2Y + g_{22xy}XY^2 + g_{2x}X^2 \\
&\quad + g_{2y}Y^2 + g_{1xy}XY + g_{1x}X + g_{1y}Y + g_0
\end{aligned} \tag{5}$$

in which

$$\begin{aligned}
c_{1y} &= 2adg - aef + 2beh \\
c_{0y} &= abeg - 2acdg + acef - (2ch - ag)eX \\
d_{2y} &= -2adh \\
d_{1y} &= 2ehaX - efa^2 \\
d_{0y} &= a^2beg - a^2cef + 2ac^2dh - 2abceh + a^2egX
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
e_{1x} &= (ghe + dfh)e \\
e_{1y} &= (fhe - dgh)e \\
e_0 &= -eh(bdf + beg - cdg + cef) \\
f_{2xy} &= -e(dg^2 - efg + dh^2) \\
f_{1xy} &= -2e^2h^2 \\
f_{1x} &= -e(2cef^2 + adfh - 2cdfg + aegh) \\
f_{1y} &= 2ahd^2f + 2cdef^2 - 2bdefg + 2cdeg^2 \\
&\quad + ahdeg + ahe^2f - 2be^2g^2 \\
f_0 &= b^2deg^2 + b^2deh^2 - b^2e^2fg + 2bce^2f^2 \\
&\quad + 2bce^2g^2 + 2bce^2h^2 + abdefh + abe^2gh \\
&\quad - 2c^2def^2 - c^2deg^2 - c^2deh^2 - c^2e^2fg \\
&\quad - 2acd^2fh - acdegh - ace^2fh \\
g_{3y} &= degh \\
g_{21xy} &= -e^2fh \\
g_{22xy} &= dfhe - ghe^2 \\
g_{2x} &= cfe^2h + adeg^2 + adeh^2 - afe^2g \\
g_{2y} &= 2agd^2f - adef^2 + bdefh - adeh^2 \\
&\quad - cgdeh + age^2f + bge^2h \\
g_{1xy} &= -2ad^2g^2 - 2ad^2h^2 - 2cdefh + ae^2f^2 \\
&\quad - ae^2g^2 \\
g_{1x} &= c^2defh + c^2e^2gh + 2acd^2g^2 + 2acd^2h^2 \\
&\quad - 2acdefg + ace^2f^2 + ace^2g^2 + 2ace^2h^2
\end{aligned}$$

$$
\begin{aligned}
g_{1y} &= b^2e^2fh - 2bcdefh - 2abd^2g^2 - 2abd^2h^2 \\
&+ 2abdefg - abe^2f^2 - abe^2g^2 - 2abe^2h^2 \\
&- c^2degh \\
g_0 &= b^2ce^2fh - ab^2deg^2 - ab^2deh^2 + ab^2e^2fg \\
&+ bc^2defh - bc^2e^2gh + 2abcd^2g^2 + 2abcd^2h^2 \\
&- abce^2f^2 + abce^2g^2 + c^3degh - 2ac^2d^2fg \\
&+ ac^2def^2 + ac^2deh^2 - ac^2e^2fg
\end{aligned}
$$