Complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs

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Abstract: This article deals with the complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs. The cancellation of dynamic loads transmitted to the ground is a challenge for these types of manipulators.

It is obvious that the classical methods based on the optimal redistribution of movable masses and additional counter-rotations can be used to cancel shaking force and shaking moment. However, balancing of parallel manipulators with prismatic pairs is attained via a considerably complicated design. This article shows that it is possible to balance planar parallel mechanisms by using Scott–Russell mechanisms. Such an approach enables counter-rotations to be divided by 2. Numerical simulations carried out using ADAMS software validate the obtained results and illustrate that the suggested balancing enables to create a parallel manipulator transmitting no inertial load to its base.

Keywords: shaking force, shaking moment, balancing, planar parallel robots with prismatic pairs

1 INTRODUCTION

Shaking force balancing is mostly obtained via an optimal redistribution of movable masses [**1–10**] or adjustment of kinematic parameters [**11**]. The cancellation of the shaking moment is a complicated task, and shaking moment balancing can be obtained by: (a) using counter-rotations [**12–17**] (Fig. 1(a)), (b) adding four bar linkages [**18–22**] (Fig. 1(b)), and (c) using optimal trajectory planning [**17**, **23**, **24**].

Previous works focused on the study of parallel manipulators with revolute joints, and until now, to the best of our knowledge, no study has been carried out on complete shaking force and shaking moment balancing of parallel manipulators with prismatic pairs.

In this article, for the first time, solutions for complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs are proposed. These are illustrated via a 3-RPR parallel manipulator. All obtained results are validated using ADAMS software simulations.

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2 COMPLETE SHAKING MOMENT AND SHAKING FORCE BALANCING BY ADDING AN IDLER LOOP BETWEEN THE BASE AND THE PLATFORM

Inertial force balancing by adding an idler loop is known to be used for one-degree-of-freedom (DOF) mechanisms [**25–29**]. With regard to planar manipulators, such approach has only been used in the balancing of gravitational and inertial forces [**9**, **10**, **30**, **31**].

In this section, the complete shaking force and shaking moment balancing of planar manipulators are discussed by adding an idler loop. The added balancing loop is mounted between the base and the platform of the mechanism. The suggested balancing technique on a 3-RPR mechanism is illustrated in Fig. 2. Note that the type of actuation of the mechanism is not mentioned as it has no influence on balancing.

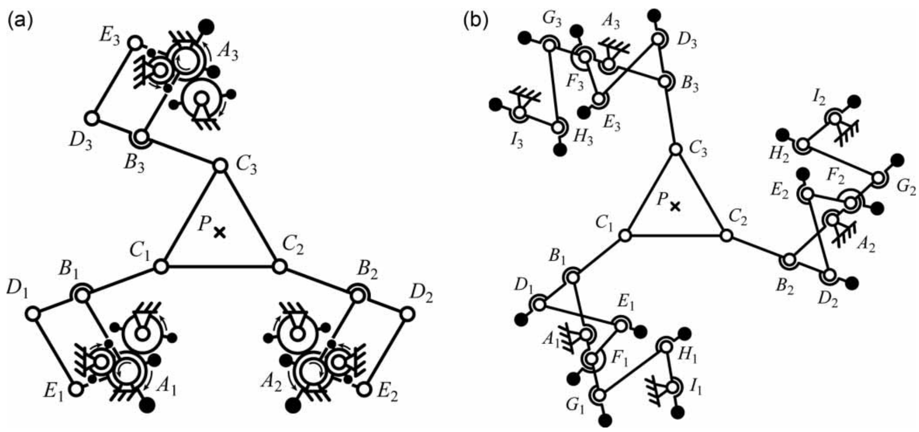

Fig. 1 Complete shaking force and shaking moment balanced 3-RRR planar parallel manipulators

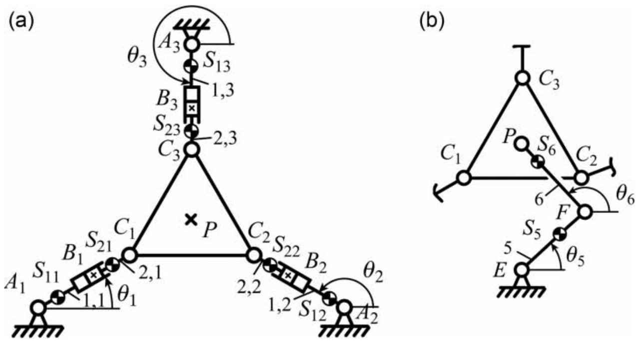

Fig. 2 Schematic of the 3-RPR robot under study

Initially, the cancellation of the dynamic reactions of the 3-RPR planar parallel mechanism is analysed (Fig. 2(a)). Such a mechanism has 3-DOF (two translations in the *Oxy* plane and one rotation of the moving platform around an axis perpendicular to *Oxy*) and is composed of three identical legs, each being composed of a revolute joint attached to the base at point A_i (in the remainder of this report, $i = 1$, 2, and 3), one moving prismatic guide located at point B_i, and the other revolute joint attached to the platform at C_i. The base and platform triangles denoted $A_1A_2A_3$ and $C_1C_2C_3$ are equilateral. On this manipulator, typically, the actuated joints are the first revolute joints at A_i or the linear guide at B_i.

Considering that the x-axis is directed along the line A_1A_2, the y axis being perpendicular to the x-axis, and the origin of the base frame located at point O, the centre of the circumcircle of triangle $A_1A_2A_3$, one can define the coordinate x, y, and ϕ of the platform as being, respectively, the coordinates of point P along the x and y axes and the angle between the lines C_1C_2 and A_1A_2.

The length B_iC_i is denoted as L_1, and S_{ji} is the centre of mass of link ij ($j = 1$, 2), which has a mass m_j and an axial moment of inertia I_j. The centre of the mass of the platform is located at point P. The mass of the platform is m_p and its axial moment of inertia I_p.

To cancel the shaking forces and shaking moment of the manipulator, an idler loop is added between the base and the platform (Fig. 2(b)). Lengths EF and FP are denoted as L_5 and L_6, respectively. The centre of mass of elements 5 and 6 of the idler loop is denoted as S_3 and S_4 with masses m_5 and m_6 and axial moments of inertia I_5 and I_6, respectively. The positions of the centre of masses are $\boldsymbol{d}_{AiS1i} = r_1L_1\boldsymbol{u}_i$, $\boldsymbol{d}_{CiS2i} = (r_2 - 1)L_1\boldsymbol{u}_i$, $\boldsymbol{d}_{\mathrm{ES5}} = r_5\boldsymbol{d}_{\mathrm{EF}}$, and $\boldsymbol{d}_{\mathrm{FS6}} = r_6\boldsymbol{d}_{\mathrm{FP}}$, where r_1, r_2, r_5, and r_6 are dimensionless coefficients and $\boldsymbol{u}_i$ is a unit vector directed along B_iC_i.

Thus, considering the shaking force F of leg 1, the expression is

$$F = \sum_{i=1}^{3}\sum_{j=1}^{2} m_j\ddot{\boldsymbol{d}}_{Sji} + m_\mathrm{p}\ddot{\boldsymbol{d}}_P + m_5\ddot{\boldsymbol{d}}_{S5} + m_6\ddot{\boldsymbol{d}}_{S6} \tag{1}$$

where $\ddot{\boldsymbol{d}}_{Sji}$, $\ddot{\boldsymbol{d}}_P$, $\ddot{\boldsymbol{d}}_{S5}$, and $\ddot{d}_{S6}$ are the accelerations of the centre of mass S_{ij}, P, S_5, and $S_{6,}$ respectively.

From expression (1), it can be seen that the shaking force F can be expressed as

$$F = (m_1r_1 - m_2(1 - r_2))\sum_{i=1}^{3}\boldsymbol{a}_i + (3m_2 + m_\mathrm{p} + m_6r_6)\boldsymbol{a}_4 + (3m_2 + m_\mathrm{p} + m_5r_5 + m_6)\ddot{\boldsymbol{d}}_F \tag{2}$$

with

$$\boldsymbol{a}_i = L_1\left(\ddot{\theta}_i\begin{bmatrix}-\sin\theta_i\\ \cos\theta_i\end{bmatrix} - \dot{\theta}_i^2\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}\right) \tag{3a}$$

$$\boldsymbol{a}_4 = L_6\left(\ddot{\theta}_6\begin{bmatrix}-\sin\theta_6\\ \cos\theta_6\end{bmatrix} - \dot{\theta}_6^2\begin{bmatrix}\cos\theta_6\\ \sin\theta_6\end{bmatrix}\right) \tag{3b}$$

and $\ddot{\boldsymbol{d}}_F$ is the acceleration of point F.

At this step, only five counterweights are required to cancel shaking force, but it could be seen after more derivations that three others are necessary to cancel the shaking moment. Therefore, direct

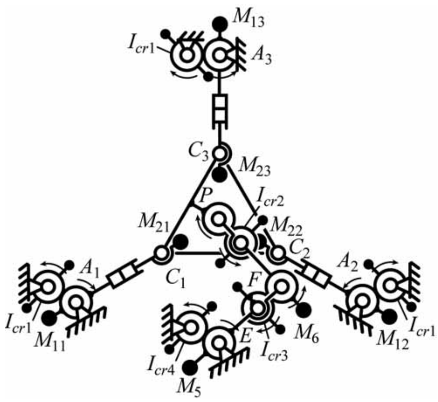

Fig. 3 Schematic of the 3-RPR mechanism with the added RRR chain used for the cancellation of the shaking force and shaking moment

adding of three supplementary counterweights is proposed (Fig. 3). The positions of the eight masses are $\boldsymbol{d}_{AiMcp1i} = r_{cp1}L_1\boldsymbol{u}_i$, $\boldsymbol{d}_{BiMcp2i} = r_{cp2}L_1\boldsymbol{u}_i$, $\boldsymbol{d}_{\mathrm{EM}cp5} = r_{cp5}\boldsymbol{d}_{\mathrm{EF}}$, and $\boldsymbol{d}_{\mathrm{FM}cp6} = r_{cp}\boldsymbol{d}_{\mathrm{FP}}$, where r_{cp1}, r_{cp2}, r_{cp5}, and r_{cp6} are dimensionless coefficients. Their masses are denoted by m_{cp1}, m_{cp2}, m_{cp5}, and m_{cp6}, respectively. With the addition of the counterweights, the shaking force becomes

$$F^{\mathrm{bal}} = F + (m_{cp1}r_{cp1} - m_{cp2}(1 - r_{cp2}))\sum_{i=1}^{3}\boldsymbol{a}_i + m_{cp6}r_{cp6}\boldsymbol{a}_4 + (m_{cp6} + m_{cp5}r_{cp5})\ddot{\boldsymbol{d}}_F \tag{4}$$

Thus, the shaking force is cancelled if

$$m_{cp1} = -\frac{m_1 r_1}{r_{cp1}} \tag{5a}$$

$$m_{cp2} = -\frac{m_2\,(1 - r_2)}{1 - r_{cp2}} \tag{5b}$$

$$m_{cp6} = -\frac{3(m_2 + m_{cp2}) + m_p + r_6 m_6}{r_{cp6}} \tag{5c}$$

and

$$m_{cp5} = -\frac{3(m_2 + m_{cp2}) + m_p + m_6 + m_{cp6} + r_5 m_5}{r_{cp5}} \tag{5d}$$

The expression of the shaking moment M_O of the modified structure (expressed at point O) can be written as

$$M_O = \frac{\mathrm{d}H_O}{\mathrm{d}t} \tag{6}$$

where H_O is the angular momentum of the leg (expressed at point O). Thus, to cancel the shaking moment, the angular momentum is held constant over time.

The expression of the angular momentum H_O is

$$\begin{aligned} H_O = {} & \sum_{i=1}^{3}\sum_{j=1}^{2}(m_j(x_{OSji}\dot{y}_{OSji} - y_{OSji}\dot{x}_{OSji}) + I_j\dot{\theta}_i) \\ & + \sum_{i=1}^{3}\sum_{j=1}^{2}(m_{cpj}(x_{OMji}\dot{y}_{OMji} - y_{OMji}\dot{x}_{OMji})) \\ & + I_{\mathrm{p}}\dot{\phi} + \sum_{j=5}^{6}(m_j(x_{OSj}\dot{y}_{OSj} - y_{OSj}\dot{x}_{OSj}) \\ & + m_{cpj}(x_{OMj}\dot{y}_{OMj} - y_{OMj}\dot{x}_{OMj}) + I_j\dot{\theta}_j) \end{aligned} \tag{7}$$

where x_{OQ}, y_{OQ}, $\dot{x}_{OQ}$, and $\dot{y}_{OQ}$ are the positions and velocities of any point Q along $x-$ and y-axes, respectively; Q being either point S_{ji}, $M_{ji}(j = 1, 2)$, S_j or M_j $(j = 5, 6)$.

Substituting equation (5) into equation (7) yields

$$\begin{aligned} H_O = {} & \sum_{i=1}^{3}(I_1 + I_2 + (m_1 r_1^2 + m_{cp1}r_{cp1}^2 \\ & + m_2(1 - r_2)^2)L_1^2)\dot{\theta}_i + \sum_{i=1}^{3}(m_{cp2}(1 - r_{cp2})^2L_1^2)\dot{\theta}_i \\ & + (I_{\mathrm{p}} + 3(m_2 + m_{cp2})R_{\mathrm{p}}^2)\dot{\phi} + (I_6 + (m_6 r_6^2 \\ & + m_{cp6}r_{cp6}^2 + m_{\mathrm{p}} + 3(m_2 + m_{cp2}))L_6^2)\dot{\theta}_6 \\ & + (I_5 + (m_5 r_5^2 + m_{cp5}r_{cp5}^2 + m_6 + m_{cp6} \\ & + m_{\mathrm{p}})L_5^2)\dot{\theta}_5 + 3(m_2 + m_{cp2})L_5^2\dot{\theta}_5 \end{aligned} \tag{8}$$

After such modifications of the RRR chain, the angular momentum of the legs of the mechanism and the RRR chain can be balanced using six counter-rotations (Fig. 3), which have an axial moment of inertia equal to

$$\begin{aligned} I_{cr1} = {} & I_1 + I_2 + \big(m_1 r_1^2 + m_{cp1}r_{cp1}^2 + m_2(1 - r_2)^2 \\ & + m_{cp2}(1 - r_{cp2})^2\big)L_1^2 \end{aligned} \tag{9a}$$

$$I_{cr2} = I_p + 3(m_2 + m_{cp2})R_{\mathrm{p}}^2 \tag{9b}$$

$$\begin{aligned} I_{cr3} = {} & \big(m_6 r_6^2 + m_{cp6}r_{cp6}^2 + m_{\mathrm{p}} \\ & + 3(m_2 + m_{cp2})\big)L_6^2 + 2I_{cr2} + I_6 \end{aligned} \tag{9c}$$

$$\begin{aligned} I_{cr4} = {} & \big(m_5 r_5^2 + m_{cp5}r_{cp5}^2 + m_6 + m_{cp6} + m_{\mathrm{p}} \\ & + 3(m_2 + m_{cp2})\big)L_5^2 + 2I_{cr3} + I_5 \end{aligned} \tag{9d}$$

2.1 Numerical application

The suggested balancing approach is illustrated by using numerical simulations carried out with ADAMS software. For this purpose, non-balanced and balanced 3-RPR parallel manipulators are compared.

The chosen trajectory for simulations is a straight line of the controlled point of the platform, achieved in $t_f = 0.25$ s, between $P_0 = (x_0, y_0) = (-0.05\,\mathrm{m}, 0)$ and

$P_f = (x_f, y_f) = (-0.2\,\text{m}, 0)$ with a rotation of the platform from $\phi_0 = 0°$ to $\phi_f = 30°$. For the displacement of the mechanism, fifth-order polynomial laws are used and therefore the trajectory is defined by the following expression

$$x(t) = x_0 + (x_f - x_0)\left(10\left(\frac{t}{t_f}\right)^3 - 15\left(\frac{t}{t_f}\right)^4 + 6\left(\frac{t}{t_f}\right)^5\right) \quad (10a)$$

$$y(t) = 0 \quad (10b)$$

$$\phi(t) = \phi_0 + (\phi_f - \phi_0)\left(10\left(\frac{t}{t_f}\right)^3 - 15\left(\frac{t}{t_f}\right)^4 + 6\left(\frac{t}{t_f}\right)^5\right) \quad (10c)$$

The parameters used for the simulations are:

(a) radii of the circumcircles of the base triangle $A_1A_2A_3$ and the platform triangle $C_1C_2C_3 - R_\text{b} = 0.35\,\text{m}$ and $R_\text{p} = 0.1\,\text{m}$;
(b) $L_1 = 0.05\,\text{m}$, $L_5 = 0.15\,\text{m}$, and $L_6 = 0.1581\,\text{m}$;
(c) $r_2 = r_5 = r_6 = 0.5$ and $r_1 = 2$;
(d) $m_1 = 0.75\,\text{kg}$, $m_2 = 0.37\,\text{kg}$, $m_5 = 0.42\,\text{kg}$, $m_6 = 0.47\,\text{kg}$, and $m_\text{p} = 1\,\text{kg}$;
(e) $I_1 = 0.003\,44\,\text{kg}\,\text{m}^2$, $I_2 = 0.000\,25\,\text{kg}\,\text{m}^2$, $I_5 = 0.001\,22\,\text{kg}\,\text{m}^2$, $I_6 = 0.001\,46\,\text{kg}\,\text{m}^2$, $I_\text{p} = 0.004\,36\,\text{kg}\,\text{m}^2$;
(f) point E is located at point O.

For such parameters and such a trajectory, the shaking force and shaking moment are computed using ADAMS software, and they are presented in Fig. 4 (solid line). Then, the counterweights and the idler loop EFP are added to the mechanism. The position coefficients of the counterweights are all equal to $r_{cpj} = 0.5$ ($j = 1$, 2, 5, and 6). Therefore, the added masses are equal to $m_{cp1} = 0.75\,\text{kg}$, $m_{cp2} = 0.37\,\text{kg}$, $m_{cp5} = 6.92\,\text{kg}$, and $m_{cp6} = 21.66\,\text{kg}$. The new values of the shaking force and moment are presented in Fig. 4 (dashed line). It is seen that with the added counterweights, the shaking efforts are cancelled, whereas the maximal value of shaking moment is increased by a factor 17. Finally, the counter-rotations are added. Their values are equal to $I_{cr1} = 0.019\,17\,\text{kg}\,\text{m}^2$, $I_{cr2} = 0.026\,65\,\text{kg}\,\text{m}^2$, $I_{cr3} = 0.181\,69\,\text{kg}\,\text{m}^2$, and $I_{cr4} = 0.727\,81\,\text{kg}\,\text{m}^2$. With such counter-rotations, the shaking moment is balanced (in grey line in Fig. 4(c)).

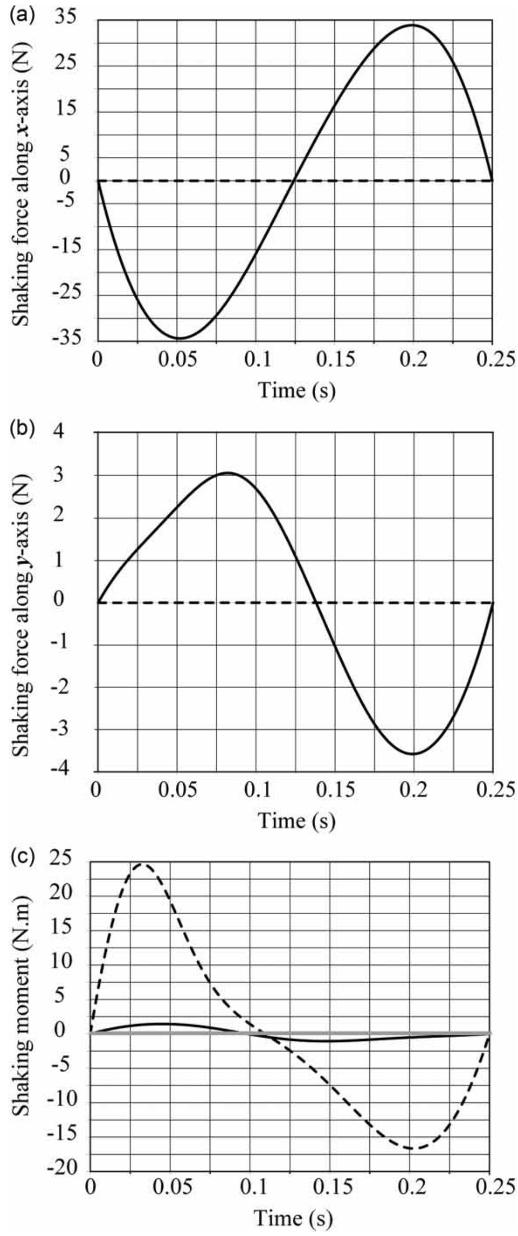

Fig. 4 Shaking force and shaking moment before (solid line) and after (dashed line) the addition of the counterweights and after the addition of the counter-rotations (grey line)

3 COMPLETE SHAKING FORCE AND SHAKING MOMENT BALANCING VIA SCOTT–RUSSELL MECHANISM

In this section, another approach for complete shaking force and shaking moment balancing is developed, which consists of adding Scott–Russell mechanisms to the initial architecture of a manipulator. This approach enables a reduction in the number of counter-rotations.

3.1 Properties of the Scott–Russell mechanism

Initially, a simple slider–crank mechanism is observed (Fig. 5). Lengths AB and BC are denoted by L_1 and L_2, respectively, and the centre of masses of link i ($i = 1$, 2, and 3) as S_i, which has a mass m_i and an

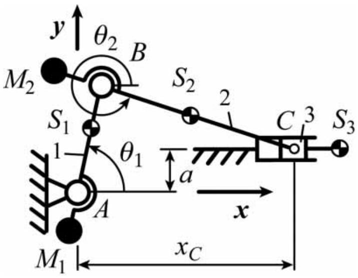

Fig. 5 A general slider–crank mechanism

axial moment of inertia I_i. The positions of the centres of mass are $\boldsymbol{d}_{\mathrm{AS1}} = r_1\boldsymbol{d}_{\mathrm{AB}}$, $\boldsymbol{d}_{\mathrm{AS2}} = \boldsymbol{d}_{\mathrm{AB}} + r_2\boldsymbol{d}_{\mathrm{BC}}$, and $\boldsymbol{d}_{\mathrm{AS3}} = \boldsymbol{d}_{\mathrm{AC}} + \boldsymbol{d}_{\mathrm{CS3}}$, where r_1 and r_2 are dimensionless coefficients and $\boldsymbol{d}_{\mathrm{CS3}} = L_3 r_3 x$ (L_3 is a constant).

It is known that the complete shaking force and shaking moment balancing of a general slider–crank mechanism can be obtained by adding two counterweights mounted on the links and two pairs of counter-rotation inertia-counterweights. However, it is possible to balance this mechanism without counter-rotation inertia-counterweights if it has specific geometrical parameters, as in Scott–Russell mechanisms ($a = 0$, $L_1 = L_2$).

The balancing of this mechanism is considered here. The expression of the shaking force F of a slider–crank mechanism can be written as

$$F = \sum_{i=1}^{3} m_i \ddot{\boldsymbol{d}}_{Si} \tag{11}$$

where $\ddot{\boldsymbol{d}}_{Si}$ is the acceleration of the centre of mass S_i.

Developing equation (11), the expression of F becomes

$$F = (m_1 r_1 + m_2)\ddot{\boldsymbol{d}}_B + (m_2 r_2 + m_3)\boldsymbol{a} \tag{12}$$

with

$$\boldsymbol{a} = L_2\left(\ddot{\theta}_2\begin{bmatrix}-\sin\theta_2\\ \cos\theta_2\end{bmatrix} - \dot{\theta}_2^2\begin{bmatrix}\cos\theta_2\\ \sin\theta_2\end{bmatrix}\right) \tag{13}$$

where $\ddot{\boldsymbol{d}}_B$ is the acceleration of point B.

The constant terms of equation (12) can be cancelled by the addition of two counterweights M_j ($j = 1$, 2) (Fig. 5), the masses of which are m_{cpj}. Their positions are $\boldsymbol{d}_{\mathrm{AM}cp1} = r_{cp1}\boldsymbol{d}_{\mathrm{AB}}$ and $\boldsymbol{d}_{\mathrm{BM}cp2} = r_{cp2}\boldsymbol{d}_{\mathrm{BC}}$, where r_{cp1} and r_{cp2} are dimensionless coefficients. With the addition of the counterweights, the shaking force becomes

$$F^{\mathrm{bal}} = F + (m_{cp1} r_{cp1} + m_{cp2})\ddot{\boldsymbol{d}}_B + m_{cp2} r_{cp2}\boldsymbol{a} \tag{14}$$

Thus, the shaking force vanishes if

$$m_{cp2} = -\frac{m_2 r_2 + m_3}{r_{cp2}} \quad \text{and}$$
$$m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3}{r_{cp1}} \tag{15}$$

The expression of the angular momentum H_{A} (expressed at point A) is

$$H_{\mathrm{A}} = \sum_{j=1}^{3}(m_j(x_{ASj}\dot{y}_{ASj} - \dot{x}_{ASj}y_{ASj})) + \sum_{j=1}^{2}(I_j\dot{\theta}_j + m_{cpj}(x_{MSj}\dot{y}_{MSj} - \dot{x}_{MSj}y_{MSj})) \tag{16}$$

where x_{AQ}, y_{AQ}, $\dot{x}_{AQ}$, and $\dot{y}_{AQ}$ are the positions and velocities of any point Q along x and y axes, respectively; Q being either point S_j or M_j ($j = 1$, 2, and 3).

Substituting equation (15) into equation (16)

$$H_{\mathrm{A}} = (I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2} + m_3)L_1^2)\dot{\theta}_1 + (I_2 + (m_2 r_2^2 + m_{cp2} r_{cp2}^2 + m_3)L_2^2)\dot{\theta}_2 \tag{17}$$

with

$$\dot{\theta}_2 = -\frac{\dot{y}_{\mathrm{AB}}(x_{\mathrm{AC}} - x_{\mathrm{AB}}) + (a - y_{\mathrm{AB}})(\dot{x}_{\mathrm{AC}} - \dot{x}_{\mathrm{AB}})}{L_2^2} \tag{18}$$

where x_{AC}, x_{AB}, and y_{AB} are the coordinates of points C and B, respectively, and $\dot{x}_{\mathrm{AC}}$, $\dot{x}_{\mathrm{AB}}$, and $\dot{y}_{\mathrm{AB}}$ are their velocities.

To cancel the shaking moment M_{A}, the angular momentum has to be constant or null. Developing equation (18), one notices that this can be obtained if

$$a = 0 \quad \text{and} \quad L_1 = L_2 \tag{19}$$

In such case, $\dot{\theta}_1 = -\dot{\theta}_2$. Therefore, the shaking moment is cancelled if

$$I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2})L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2)L_1^2 = 0 \tag{20}$$

3.2 Balancing of a manipulator's leg using a Scott–Russell mechanism

Now a manipulator's leg with an added Scott–Russell mechanism is considered (Fig. 6). The centre of mass of link 4 is denoted as S_4, which has a mass m_4 and an axial moment of inertia I_4. The position of S_4 is $\boldsymbol{d}_{CS4} = L_3 r_4 \boldsymbol{u}$; r_4 being a dimensionless coefficient and $\boldsymbol{u}$ a unit vector along $\boldsymbol{d}_{CS3}$.

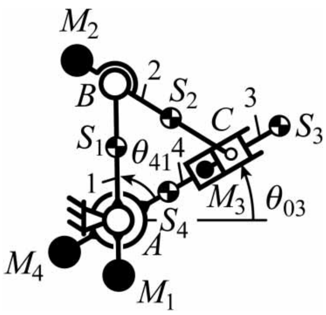

Fig. 6 A manipulator leg with added Scott–Russell mechanisms

Now the shaking force becomes

$$F = (m_1 r_1 + m_{cp1} r_{cp1} + m_2 + m_{cp2} + m_3)\ddot{\boldsymbol{d}}_B + (m_3 r_3 + m_4 r_4)\boldsymbol{a}_1 + (m_2 r_2 + m_{cp2} r_{cp2} + m_3)\boldsymbol{a}_2 \quad (21)$$

with

$$\boldsymbol{a}_1 = L_3 \left(\ddot{\theta}_{03} \begin{bmatrix} -\sin\theta_{03} \\ \cos\theta_{03} \end{bmatrix} - \dot{\theta}_{03}^2 \begin{bmatrix} \cos\theta_{03} \\ \sin\theta_{03} \end{bmatrix} \right) \quad (22)$$

and

$$\boldsymbol{a}_2 = L_1 \left((\ddot{\theta}_{03} - \ddot{\theta}_{41}) \begin{bmatrix} -\sin(\theta_{03} - \theta_{41}) \\ \cos(\theta_{03} - \theta_{41}) \end{bmatrix} - (\dot{\theta}_{03} - \dot{\theta}_{03})^2 \begin{bmatrix} \cos(\theta_{03} - \theta_{41}) \\ \sin(\theta_{03} - \theta_{41}) \end{bmatrix} \right) \quad (23)$$

At this step, only one counterweight is required to cancel shaking force; but it could be seen after more derivations that another counterweight is necessary for the cancellation of the shaking moment. Therefore, direct adding of this supplementary counterweight is proposed. The positions of the two masses are $\boldsymbol{d}_{AMcp3} = r_{cp3} L_3 \boldsymbol{u}$ and $\boldsymbol{d}_{CMcp4} = r_{cp4} L_3 \boldsymbol{u}$, where r_{cp3} and r_{cp4} are dimensionless coefficients. Their masses are denoted m_{cp3} and m_{cp4}, respectively. With the addition of the counterweights, the shaking force becomes

$$\mathbf{F}^{\text{bal}} = \mathbf{F} + m_{cp3}\ddot{\boldsymbol{d}}_B + m_{cp3}\boldsymbol{a}_1 + (m_{cp3} r_{cp3} + m_{cp4} r_{cp4})\boldsymbol{a}_2 \quad (24)$$

Thus, the shaking force is cancelled if

$$m_{cp4} = -\frac{m_4 r_4}{r_{cp4}} \quad (25a)$$

$$m_{cp3} = -\frac{m_3 r_3}{r_{cp3}} \quad (25b)$$

$$m_{cp2} = -\frac{m_2 r_2 + m_3 + m_{cp3}}{r_{cp2}} \quad (25c)$$

and

$$m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3 + m_{cp3}}{r_{cp1}} \quad (25d)$$

Simplifying the expression of the angular momentum yields

$$H_{\text{A}} = I_{\text{eq1}}\dot{\theta}_{03} + I_{\text{eq2}}\dot{\theta}_{41} \quad (26)$$

with

$$I_{\text{eq1}} = (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2(1 - r_2)^2 + m_{cp2}(1 - r_{cp2})^2)L_1^2 + I_1 + I_2 + I_3 + I_4 + (m_3 r_3^2 + m_{cp3} r_{cp3}^2 + m_4 r_4^2 + m_{cp4} r_{cp4}^2)L_3^2 \quad (27)$$

$$I_{\text{eq2}} = I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2})L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2)L_1^2 \quad (28)$$

From equation (20), $I_{\text{eq2}} = 0$. Therefore, the shaking moment of the slider–crank can be cancelled using a simple counter-rotation I_{cr} with an axial moment of inertia equal to I_{eq1}.

3.3 Shaking moment and shaking force balancing of the 3-RPR manipulator

Now, such an approach is applied to the 3-RPR mechanism. First, the platform mass is substituted by three points masses located at C_1, C_2, and C_3, with mass values equal to m_{p1}, m_{p2}, and m_{p3} respectively [**13**, **32**, **33**]. Such a condition can be obtained if

$$m_{\text{p}i} = \frac{m_{\text{p}}}{3} \quad \text{and} \quad I_{\text{p}} = 3 m_{\text{p}i} R_{\text{p}}^2 \quad (29)$$

where R_{p} is the radius of the circumcircle of $C_1C_2C_3$. Such a decomposition of the platform enables to consider the shaking force and shaking moment balancing of each leg of the mechanism. Then, for modifying each leg to obtain a mechanism similar to a slider–crank linkage (i.e. by adding an idler loop to each leg), the shaking force and shaking moment are cancelled if

$$m_{cp4} = -\frac{m_4 r_4}{r_{cp4}} \quad (30a)$$

$$m_{cp3} = -\frac{m_3 r_3 + m_{\text{p}i}}{r_{cp3}} \quad (30b)$$

$$m_{cp2} = -\frac{m_2 r_2 + m_3 + m_{cp3} + m_{\text{p}i}}{r_{cp2}} \quad (30c)$$

$$m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3 + m_{cp3} + m_{\text{p}i}}{r_{cp1}} \quad (30d)$$

$$0 = I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2})L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2)L_1^2 \quad (30e)$$

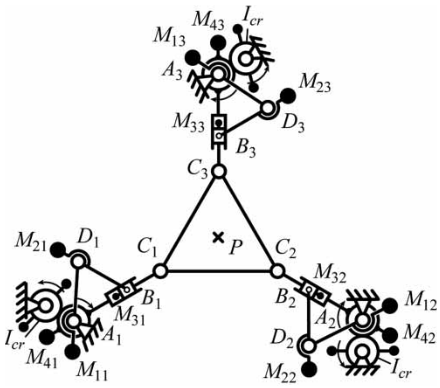

Fig. 7 Schematic of a shaking force and shaking moment balanced 3-RPR mechanism

and

$$
\begin{aligned}
I_{\mathrm{cr}} = {} & (m_1 r_1^2 + m_{cp1} r_{cp1}^2 \\
& + m_2(1 - r_2)^2 + m_{cp2}(1 - r_{cp2})^2)L_1^2 + I_1 + I_2 \\
& + I_3 + I_4 + (m_3 r_3^2 + m_{cp3} r_{cp3}^2 + m_4 r_4^2 \\
& + m_{cp4} r_{cp4}^2 + m_{pi})L_3^2
\end{aligned} \tag{30f}
$$

taking into account that I_{cr} is the axial moment of inertia of the counter-rotations (Fig. 7).

Thus, with this approach it is possible to create a fully balanced shaking force and shaking moment 3-RPR mechanism with only three counter-rotations (Fig. 7), i.e. this method enables a reduction in the number of counter-rotations by a factor of two.

3.4 Numerical application

The parameters used for the simulations are:

(a) radii of the circumcircles of the base triangle $A_1A_2A_3$ and the platform triangle $C_1C_2C_3$ – $R_{\mathrm{b}} = 0.35$ m and $R_{\mathrm{p}} = 0.1$ m;
(b) $L_1 = L_2 = 0.25$ m and $L_3 = 0.025$ m;
(c) $r_1 = r_2 = 0.5$, $r_3 = 0$, and $r_4 = 4$;
(d) $m_1 = 1.09$ kg, $m_2 = 1.1$ kg, $m_3 = 0.37$ kg, $m_4 = 0.75$ kg, $m_{\mathrm{p}} = 1$ kg;
(e) $I_1 = 0.007\,38$ kg m^2, $I_2 = 0.583\,89$ kg m^2, $I_3 = 0.003\,44$ kg m^2, $I_6 = 0.000\,25$ kg m^2, $I_{\mathrm{p}} = 0.01$ kg m^2.

For these new parameters and for the trajectory used previously, taking into account that the position coefficients of the counterweights are equal to $r_{cpj} = -0.5$ ($j = 1, 3$, and 4) and $r_{cp2} = -1$, the new values of the counterweights are $m_{cp1} = 3.17$ kg, $m_{cp2} = 11.71$ kg, $m_{cp3} = 0.33$ kg, and $m_{cp4} = 0.75$ kg. The shaking force and shaking moment are then computed (dashed line in Fig. 8). It is possible to see that, with the counterweights, the shaking efforts are cancelled, whereas the maximal value of the shaking moment is increased by a factor of 28. Finally, the counter-rotations are added. Their values are equal to $I_{\mathrm{cr}} = 1.569\,07$ kg m^2. With such counter-rotations, the shaking moment is balanced (grey line in Fig. 8(c)).

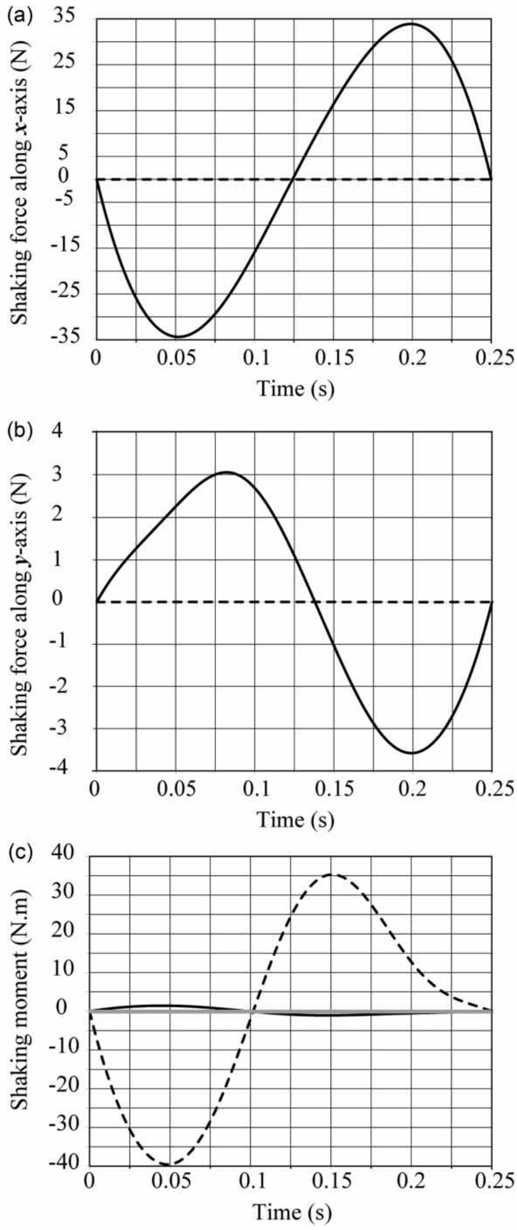

Fig. 8 Shaking force and shaking moment before (solid line) and after (dashed line) the addition of the counterweights and after the addition of the counter-rotations (grey line)

Finally, it should be noted that the combination of the proposed two techniques of balancing enables the creation of fully balanced parallel manipulators with modified legs. As an example, different structures of balanced manipulators are presented in Fig. 9 (3-RPR, 3-PRR, and 3-PRP), in which one leg with a prismatic pair is replaced by a leg with only revolute joints. Such a modification allows displacing the centre of mass of the manipulator to C_3 and then to balance the manipulator via the modified leg $C_3\,B_3\,A_3$.

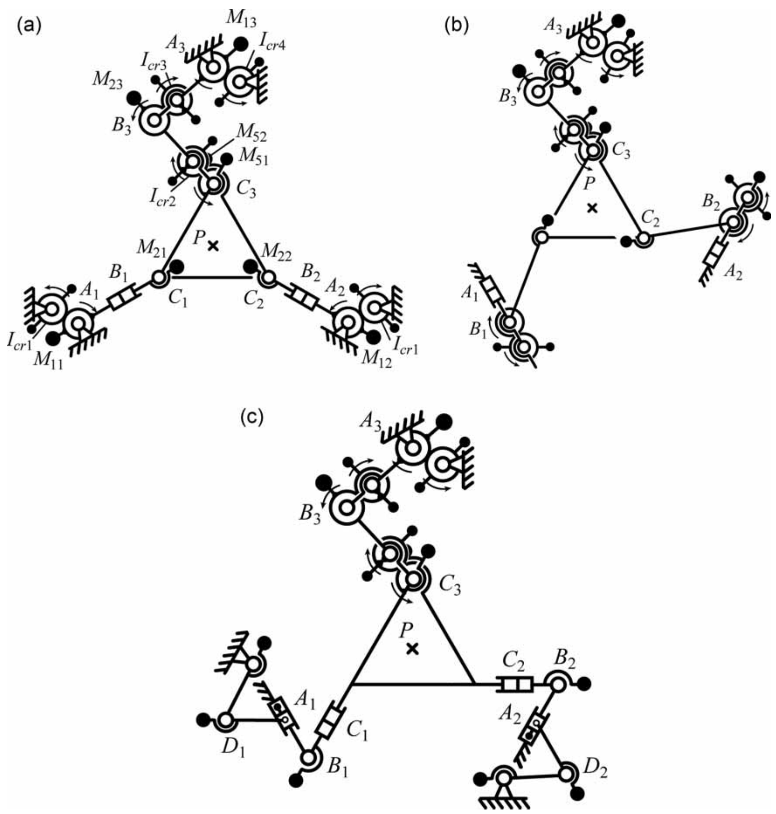

Fig. 9 Complete shaking force and shaking moment balancing of planar manipulators with prismatic pairs via structural modification of one leg

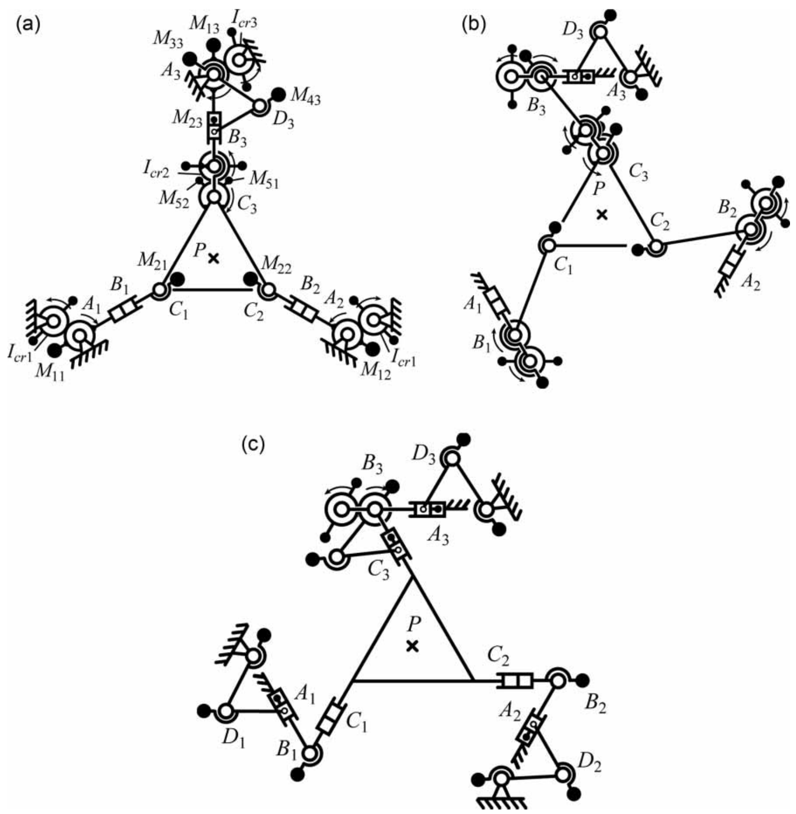

Fig. 10 Complete shaking force and shaking moment balancing of planar manipulators with prismatic with reduced number of Scott–Russell mechanisms

In the same way, it is possible to balance a parallel manipulator with prismatic pairs by adding fewer Scott–Russell mechanisms. The balancing schemes for several parallel manipulators are presented in Fig. 10.

4 CONCLUSIONS

This article presents the complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs. Two approaches are discussed: balancing via adding an idler loop mounted between the platform and the base of the manipulator and balancing via the Scott–Russell mechanism, which enables a reduction in the number of counter-rotations by a factor of two. All studied balancing techniques are validated by simulations carried out using ADAMS software. The obtained results show that parallel manipulators balanced by using the suggested methods transmit no inertia loads to their bases, i.e. the sum of all ground forces and their moments are zero.

Finally, it is mentioned that using Scott–Russel mechanisms remains using some modified 3-RRR manipulator. Thus, if the type of actuation is not considered, the prismatic guides could be suppressed and the work could be of no interest. In contrast, the goal of the study is to propose the complete shaking force and shaking moment balancing of manipulators for applications, in which actuation via a prismatic motor is required, such as in high load carrying (by using hydraulic devices). Therefore, the proposed solutions are of great interest to the scientific community.

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