A Controller for Avoiding Dynamic Model Degeneracy of Parallel Robots during Type 2 Singularity Crossing

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Abstract. Parallel robots presents singular configurations that divide the operational workspace into several aspects. It was proven that such singularities can be crossed under the constraint of a dynamic criterion. However, the development of a controller able to track such trajectories is up to now limited by a restrictive criterion, which decreases the total number of possible trajectories for crossing a singularity. In this paper, by finding a solution to the inverse dynamic model at the singularity locus, we were able to implement a controller in Cartesian space and track a trajectory that crosses a Type 2 singularity.

Key words: Parallel robot - Dynamic model - Singularity - Computed torque control

1 Introduction

Parallel robots present many advantages over serial robots, such as higher acceleration capacities, better stiffness and higher payload-to-weight ratios. However, their use in an industrial context remains limited partly due to the division of their workspace by singularities into several aspects [8]. Numerous singularities are prone to appear on a parallel mechanism. For a global overview the reader is referred to [13, 4]. This paper will focus on a specific singularity, called Type 2 singularities [5], where the inverse dynamic model (IDM) of the mechanism does not admit a finite solution without respecting a dynamic criterion [3].

Previous studies [6, 3] proposed a solution to increase the workspace of parallel robots by planning assembly mode changing trajectories directly through the singularity. This solution seems promising because a direct path between two points crossing a Type 2 singularity can be generated on any robot structure as long as the dynamic criterion is respected by the trajectory. However, the tracking of a trajectory, and so respecting the dynamic criterion is not always perfectly ensured in reality. Then, the design of a stable controller to cross the singularity becomes problematic. This issue was partially solved in [10] by the design of a multi-model computed torque control. However, the solution proposed requires a more restrictive criterion on the designed trajectory than the dynamic criterion computed in previous studies [6, 3], decreasing the total number of possible trajectories for crossing the singularity.

This paper aims to extend the solution proposed in [10] by the design of a generic controller in Cartesian space able to carry out the crossing of a Type 2 singularity as long as the trajectory design respects the crossing dynamic criterion. The validation of the designed controller is discussed through experiments in Section 4.

2 Dynamic model at singularity locus

In this section, we will briefly recall the dynamic equations of a parallel manipulator and discuss its degeneracy on a Type 2 singularity. For a more detailed analysis of the dynamic modeling of a parallel manipulator, the reader is referred to [2, 7]. We will then propose a general solution to the dynamic model that does not degenerate at singularity locus.

2.1 Dynamic modelling of parallel mechanisms, Type 2 singularity degeneracy condition and criteria for singularity crossing

The studied manipulator is composed of a fixed base, linked by several kinematic chains (the legs) to a mobile platform actuated along n_{dof} independent coordinates. Actuation is provided by n_{dof} active joints. The configuration and velocity of the manipulator can be described using:

- $\mathbf{q}_a$ and $\dot{\mathbf{q}}_a$ two n_{dof}-dimensional vectors of active joint variables and active joint velocities, respectively.
- $\mathbf{x}$ and $\dot{\mathbf{x}}$ two n_{dof}-dimensional vectors of the independent platform coordinates and their time derivatives, respectively.

Relations between these coordinates are found by writing the closed-loop equations. Using Lagrangian formalism, the IDM of the mechanism can be written as [1]

$$\boldsymbol{\tau} = \mathbf{w}_b - \mathbf{B}^T \boldsymbol{\lambda} \qquad \text{and} \qquad \mathbf{w}_d = \mathbf{A}_r^T \boldsymbol{\lambda} \tag{1}$$

Where,

- $\boldsymbol{\tau}$ is the n_{dof}-dimensional vector of the input efforts,
- $\boldsymbol{\lambda}$ is a n_{dof}-dimensional vector of Lagrange multipliers,
- $\mathbf{A}_r$ and $\mathbf{B}$ are two $(n_{dof} \times n_{dof})$ matrices characterizing the first-order input/output kinematic constraints
$$\mathbf{A}_r \dot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{q}}_a = \mathbf{0} \tag{2}$$
- $\mathbf{w}_b$ and $\mathbf{w}_d$ are respectively n_{dof}-dimensional vectors related to the Lagrangian L of the system (which can be explicitly expressed as a function of $\mathbf{q}_a, \mathbf{x}, \dot{\mathbf{q}}_a,$ and $\dot{\mathbf{x}}$) by [1]

$$\mathbf{w}_b = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right)^T - \left(\frac{\partial L}{\partial \mathbf{q}_a}\right)^T \qquad \text{and} \qquad \mathbf{w}_d = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{x}}}\right)^T - \left(\frac{\partial L}{\partial \mathbf{x}}\right)^T \tag{3}$$

Fig. 1: Example of a five-bar mechanism in a Type 2 singularity configuration. The vector $\mathbf{t}_s$ represents the uncontrolled motion.

From equations (1), considering the matrix $\mathbf{A}_r$ to be full rank, the dynamic model of a parallel manipulator is obtained

$$\boldsymbol{\tau} = \mathbf{w}_b - \mathbf{B}^T \mathbf{A}_r^{-T} \mathbf{w}_d \tag{4}$$

This model remains valid as long as the matrix $\mathbf{A}_r$ is invertible. The degeneracy of the matrix $\mathbf{A}_r$ corresponds to a kinematic singularity, named Type 2 singularity [5]. In such a singularity, one (or more) of the degrees of freedom of the platform becomes uncontrollable (see Fig. 1). For clarity on this article, we will limit the study to one uncontrollable degree of freedom[1]. Approaching a singularity, the determinant of the matrix $\mathbf{A}_r^T$ tends toward zero. The computed input efforts (Eq. (4)) are related to the inverse of this determinant and may tend towards infinite values in the neighborhood of a singularity. A previous study [3], showed this issue can be avoided by respecting a dynamical criterion at the singularity locus. As the matrix $\mathbf{A}_r$ is degenerated, a non-null vector $\mathbf{t}_s$ exists in its kernel. Multiplying the equation (1) by $\mathbf{t}_s^T$ gives us

$$\mathbf{t}_s^T \mathbf{A}_r^T \boldsymbol{\lambda} = 0 \Rightarrow \mathbf{t}_s^T \mathbf{w}_d = 0 \tag{5}$$

This equation is a necessary condition for the inverse dynamic equations (1) to remain consistent in a Type 2 singularity. $\mathbf{w}_d$ represents the sum of the wrenches applied on the platform by the legs, inertia/gravitational effects, and the external environment and $\mathbf{t}_s$ represents the direction of the uncontrollable motion of the platform inside the singularity in Cartesian space (see Fig. 1). Equation (5) implies that these two vectors must be reciprocal when crossing a Type 2 singularity so that the input torque remains finite.

2.2 Solution to the inverse dynamic model at singularity locus

Equation (4) gives a solution to the dynamic model away from singularities. We show below that it is possible to find solutions to the IDM at a singularity if and only if the criterion (5) is respected. From basic knowledge in linear algebra, the rank degeneracy of the matrix $\mathbf{A}_r$ implies that the equation (1) admits at least one exact solution $\boldsymbol{\lambda}$ if and only if $\mathbf{w}_d$ is included in the image of the matrix $\mathbf{A}_r^T$. The image of $\mathbf{A}_r^T$ is spanned by all the total wrenches $\mathbf{w}_d$ that can be applied by the legs through actuation and external forces on the platform. From linear algebra, we know also that the image of the matrix $\mathbf{A}_r^T$ is the orthogonal complement of the kernel of the matrix $\mathbf{A}_r$

[1] Even if the case rarely appears on existing parallel robots, the results can be extended to a higher order degeneracy.

$$\ker(\mathbf{A}_r)^{\perp} = \mathrm{Im}(\mathbf{A}_r^T) \tag{6}$$

This equation can be interpreted as “Any total wrench that can be generated on the platform by the legs and the external forces is reciprocal to the uncontrolled motion”. It is equivalent to the criterion already expressed to avoid the degeneracy of the IDM on a Type 2 singularity (5). If the criterion is respected the inverse dynamic equations admits at least an exact solution. In order to find it, let us consider the dynamic equations under an other form. Multiplying (1) by $\mathbf{A}_r^T\mathbf{B}^{-T}$ gives[2]

$$\mathbf{A}_r^T\mathbf{B}^{-T}\boldsymbol{\tau} = \mathbf{A}_r^T\mathbf{B}^{-T}\mathbf{w}_b - \mathbf{A}_r^T\boldsymbol{\lambda} \tag{7}$$

Then, $\mathbf{A}_r^T\boldsymbol{\lambda}$ can be replaced in equation (7) from equation (1).

$$\mathbf{J}_{inv}^T\boldsymbol{\tau} = \mathbf{J}_{inv}^T\mathbf{w}_b + \mathbf{w}_d \tag{8}$$

with $\mathbf{J}_{inv} = -\mathbf{B}^{-1}\mathbf{A}_r$. Knowing that the IDM (1) admits at least an exact solution, if the criterion (5) is respected, the solution minimizing the input torques can be expressed using the Moore-Penrose pseudo-inverse of $\mathbf{J}_{inv}^T$:

$$\boldsymbol{\tau} = \mathbf{J}_{inv}^{T\,+}(\mathbf{J}_{inv}^T\mathbf{w}_b + \mathbf{w}_d) \tag{9}$$

3 Design of a computed torque control law in Cartesian space

The computed torque control (CTC) [9] is a natural controller to enforce a dynamical criterion such as the criterion computed for singularity crossing (5). To implement it, the IDM (9) of the robot must be expressed as a function of the acceleration of the controlled coordinates. From [2], $\mathbf{w}_b$ and $\mathbf{w}_d$ can be expressed as function of the robot active joint and platform accelerations

$$\mathbf{w}_b = \mathbf{M}_a\ddot{\mathbf{q}}_a + \mathbf{c}_a \qquad \text{and} \qquad \mathbf{w}_d = \mathbf{M}_x\ddot{\mathbf{x}} + \mathbf{c}_x \tag{10}$$

where

- $\mathbf{M}_a$ and $\mathbf{M}_x$ are $(n_{dof} \times n_{dof})$ matrices depending on the robot configuration coordinates $\mathbf{q}_a$ and $\mathbf{x}$.
- $\mathbf{c}_a$ and $\mathbf{c}_x$ are n_{dof}-dimensional vectors depending on the robot configuration coordinates $\mathbf{q}_a$, $\mathbf{x}$ and their time derivative $\dot{\mathbf{q}}_a$, $\dot{\mathbf{x}}$.

Moreover, the time derivative of the loop-closure equations (2) gives

$$\mathbf{A}_r\ddot{\mathbf{x}} + \mathbf{B}\ddot{\mathbf{q}}_a + \mathbf{b} = \mathbf{0} \tag{11}$$

with $\mathbf{b} = \dot{\mathbf{A}}_r\dot{\mathbf{x}} + \dot{\mathbf{B}}\dot{\mathbf{q}}_a$. Eq. (11) links the acceleration of the active joints to the platform acceleration.

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[2] We consider that $\mathbf{B}$ is full rank. This hypothesis is taken as the case of the coincidence of two singularities is extremely rare and generally avoided in the design of a parallel robot. Note that the computation of $\mathbf{B}^{-T}$ is not necessary in the inverse dynamic model away from a type 2 singularity.

Robot controllers are usually established in joint space. However, in a Type 2 singularity, the matrix $\mathbf{A}_r$ is not invertible anymore, and then $\mathbf{w}_d$ cannot be expressed as a function of the joint accelerations $\ddot{\mathbf{q}}_a$. In order to avoid this issue, a previous study [10] chose to impose a criterion $\mathbf{w}_d = \mathbf{0}$ to cross the singularity , allowing the computation of the IDM as function of the joints accelerations only. However, this criterion is more restrictive than $\mathbf{t}_s^T \mathbf{w}_d = 0$ (5), thus limiting the number of achievable trajectories. In order allow all trajectories respecting the criterion $\mathbf{t}_s^T \mathbf{w}_d = 0$, in this paper, we will express the IDM as function of the platform acceleration and, consequently, design a CTC controller in Cartesian space. Even at singularity locus, from (11) the joint accelerations can be expressed as a function of the platform acceleration

$$\ddot{\mathbf{q}}_a = -\mathbf{B}^{-1}\mathbf{A}_r\ddot{\mathbf{x}} - \mathbf{B}^{-1}\mathbf{b} \tag{12}$$

Introducing (10) into the IDM (9) gives

$$\boldsymbol{\tau} = \mathbf{J}_{inv}^{T\,+}\mathbf{J}_{inv}^{T}\mathbf{M}_a\ddot{\mathbf{q}}_a + \mathbf{J}_{inv}^{T\,+}\mathbf{M}_x\ddot{\mathbf{x}} + \mathbf{J}_{inv}^{T\,+}(\mathbf{J}_{inv}^{T}\mathbf{c}_a + \mathbf{c}_x) \tag{13}$$

Then combining (13) and (12) gives the expression of the IDM as a function of the platform acceleration

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{x}} + \mathbf{h} \tag{14}$$

where $\mathbf{M} = \mathbf{J}_{inv}^{T\,+}(-\mathbf{J}_{inv}^{T}\mathbf{M}_a\mathbf{B}^{-1}\mathbf{A}_r + \mathbf{M}_x)$ and $\mathbf{h} = \mathbf{J}_{inv}^{T\,+}(\mathbf{J}_{inv}^{T}(\mathbf{c}_a - \mathbf{B}^{-1}\mathbf{b}) + \mathbf{c}_x)$. Away from a singularity $\mathbf{J}_{inv}^{T}$ is full rank, so $\mathbf{J}_{inv}^{T\,+} = \mathbf{J}_{inv}^{-T}$. In the close neighborhood of a singularity locus, the computation of the torques from the IDM given in (14) remains bounded only if the dynamic criterion established (5) is exactly respected. Unfortunately, tracking errors make the exact respect of the criterion impossible in a real experiment. In our CTC application, the IDM is computed from an auxiliary control signal $\mathbf{u}$ corresponding to a desired platform acceleration. The enforcement of the criterion is carried out by projecting $\mathbf{w}_u = \mathbf{M}_x\mathbf{u} + \mathbf{c}_x$ in the image of the matrix $\mathbf{A}_r^T$. This projection ensures that the IDM admits a unique solution and that the desired input efforts remain bounded.

From the expression (14) of the dynamic model, a classical CTC controller *in Cartesian space*, as described in Fig. 2, is established for the tracking of a trajectory with a parallel mechanism. The proof of convergence and stability of such controllers in Cartesian space is discussed in [11]. An exact linearization is performed by the inner loop of the controller and a double integrator is obtained between the auxiliary control signal $\mathbf{u}$ and the joints coordinates $\ddot{\mathbf{q}}_a$. Then, a PD-control loop is sufficient to ensure the convergence. An integral term may be added to compensate residual modeling errors. It should be noted that the controlled system output is the platform coordinates vector. This output may be directly measured with an external sensor or estimated via computation, this point is discussed in section 4.2.

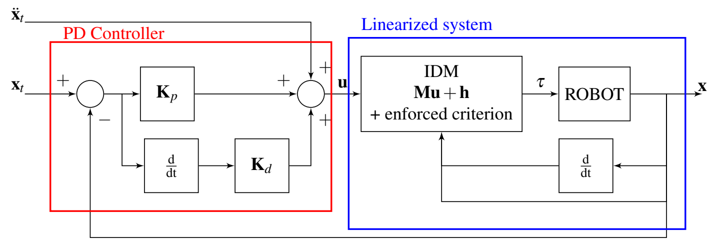

Fig. 2: Computed Torque controller in Cartesian space.

Fig. 3: Prototype of the five-bar mechanism and parametrization scheme

(a) (b)

Fig. 4: (a) Trajectory tracked crossing a Type 2 singularity (to scale).
(b) Input torques and estimated tracking errors along the trajectory. Vertical lines represent the planned singularity crossing.

4 Case study

4.1 Experiment

The proposed approach was validated on the crossing of Type 2 singularities with a planar five-bar mechanism designed by Mecademics (see Fig. 3). This mechanism is able to generate a motion of the end-effector located at A_{13} ($\mathbf{x} = (x, y)^T$) through the actuation of the joints q_{11} and q_{21}. To test the controller a trajectory that cross a Type 2 singularity twice has been generated for the five-bar mechanism. This trajectory is a return trip between point A and point B (see Fig. 4), optimized in order to respect the dynamic criterion to be enforced when crossing a Type 2 singularity (5) for this mechanism. A controller was implemented as defined in section 3. However, only active joint coordinates were available on our experimental platform. Hence, an estimation of the platform coordinates, using the computation of the direct geometric model (DGM) of the robot, has been introduced in the feedback loop [12]. The controller implemented enabled the crossing of a Type 2 singularity in experimental condition. The input torque and tracking error on the end-effector pose estimation are given in Fig. 4.

4.2 Discussion about the practical estimation of the platform coordinates for the controller

For our implementation, we used the DGM as an estimator of the platform coordinates in the implementation of our controller. This implementation generates several issues:

- The estimator uncertainties, issued from calibration errors and DGM modeling errors, are not corrected by the controller, leading to a steady state error of the end-effector position in Cartesian space.
- Around a Type 2 singularity, any error in joint position is amplified in Cartesian space along the direction of the uncontrollable motion. Specific precautions have been taken in the estimation of the end-effector pose around the singularity at the cost of estimation uncertainty around the singularity locus. Those uncertainties in the estimator causes an increase of the tracking error just after the singularity is crossed by the robot (see Fig. 4).

A better implementation of this controller should be based on an external measure of the end-effector position. However, the ability of our controller to cross the singularity even with those uncertainties demonstrated the robustness of our implementation. The repeatability of the process was tested on a trajectory crossing the Type 2 singularities several times on a five-bar mechanism at different points with a successful crossing at every point. A video of the DexTAR robot following this trajectory is available online at http://www.irccyn.ec-nantes.fr/∼six/videos.

5 Conclusions

In a Type 2 singular configuration, the IDM of a parallel robot cannot be computed in its general form due to the degeneracy in its equation system. In this paper we showed that, under the constraint of a dynamic criterion, the IDM equations can be solved even at singular configurations. This expression allowed us to design a controller able to track a trajectory respecting the dynamic criterion defined while crossing a Type 2 singularity. The theoretical results were confirmed through experimentation. The ability to cross Type 2 singularities is promising to increase the reachable workspace of any parallel robot that has such singularities in its workspace. However, the designed controller can only be implemented in Cartesian space. The implementation of a joint space CTC controller is currently under investigation.

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