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ON THE DYNAMIC PROPERTIES OF FLEXIBLE PARALLEL MANIPULATORS IN THE PRESENCE OF PAYLOAD AND TYPE 2 SINGULARITIES

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ABSTRACT
It is known that a parallel manipulator at a singular configuration can gain one or more degrees of freedom and become uncontrollable. In our recent work [1], the dynamic properties of rigid-link parallel manipulators, in the presence of Type 2 singularities, have been studied. It was shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts is orthogonal to the twist along the direction of the uncontrollable motion. This condition was obtained using symbolic approach based on the inverse dynamics and the study of the Lagrangian of a general rigid-link parallel manipulator. It was validated by experimental tests carried out on the prototype of a four-degrees-of-freedom parallel manipulator. However, it is known that the flexibility of the mechanism may not always been neglected. Indeed, for robots, joint flexibility can be the main source contributing to overall manipulator flexibility and can lead to trajectory distortion. Therefore, in our second paper [2], the condition of passing through a Type 2 singularity for parallel manipulators with flexible joints has been studied.

In the present paper, we expand information about the dynamic properties of parallel manipulators in the presence of Type 2 singularity by including in the studied problem the link flexibility and the payload. The suggested technique is illustrated by a 5*R* parallel manipulator with flexible elements (actuated joints and moving links) and a payload. The obtained results are validated by numerical simulations carried out using the software ADAMS.

1 INTRODUCTION
There are many papers dealing with the singularity analysis of parallel manipulators. We present some of them in the remaining.

The analysis of singular configurations has been first discussed from a kinematic point of view [3]–[12]. However, it is also known that, when parallel manipulators have Type 2 singularities [3], they lose their stiffness and their quality of motion transmission, and as a result, their payload capability. Therefore, the singularity zones in the workspace of manipulators may be analyzed not only in terms of kinematic criterions, from the theoretically perfect model of manipulators, but also in terms of kinetostatic approaches [13]–[19].

The further study of singularity in parallel manipulators has revealed an interesting problem that concerns the path planning of parallel manipulators under the presence of singular positions, i.e. the motion feasibility in the neighbourhood of singularities. In this case the dynamic conditions can be considered in the path planning process. One of the most evident solutions for the stable motion generation in the neighbourhood of singularities is to use redundant sensors and actuators [20]–[24]. However, it is an expensive solution to the problem because of the additional actuators and the complicated control of the manipulator caused by actuation redundancy. Another approach concerns with motion planning to pass through singularity [25]–[30],

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i.e. a parallel manipulator may track a path through singular poses if its velocity and acceleration are properly constrained. This is a promising way for the solution of this problem. However, only a few research papers on this approach have addressed the path planning for obtaining a good tracking performance. But they have not adequately addressed the physical interpretation of dynamic aspects.

In [1], optimal force generation in parallel manipulators for passing through the singular positions has been studied. It was shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts of the manipulator are orthogonal to the twist along the direction of the uncontrollable motion. This approach has been generalised in the case of rigid-link flexible-joints parallel manipulators [2].

This study is the continuation of works [1], [2]. The purpose of this paper is to study the dynamic properties of parallel manipulators with flexible links and joints in the presence of preponderant payload, which can be the main source of elastic deformations.

The paper is organized as follows. The next section presents theoretical aspects of the examined problem, using the Lagrangian formulation. The condition of force distribution is defined, that allows the passing of any parallel manipulator through the Type 2 singular positions. In section 3, the suggested solution is illustrated via 5*R* planar parallel manipulator. In section 4, the conclusions are given.

2 OPTIMAL DYNAMIC CONDITIONS FOR PASSING THROUGH TYPE 2 SINGULARITY

Let us consider a parallel manipulator of *m* links, *n* degrees of freedom and driven by *n* actuators, for which the effects of the payload and of the geared motors inertia are preponderant from the others.

The general Lagrangian dynamic formulation for a non-rigid manipulator can be expressed as [31]:

$$\boldsymbol{\tau} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right) - \frac{\partial L}{\partial \mathbf{q}_a}, \qquad (1a)$$

$$\mathbf{0} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_e}\right) - \frac{\partial L}{\partial \mathbf{q}_e}, \qquad (1b)$$

where,

- L is the Lagrangian of the manipulator; $L = T-V$, where T is the kinetic energy and V the potential energy due to gravitational forces, friction and elasticity;
- $\mathbf{q}_a = [q_1^a, q_2^a, ..., q_n^a]^T$ and $\dot{\mathbf{q}}_a = [\dot{q}_1^a, \dot{q}_2^a, ..., \dot{q}_n^a]^T$ represent the vectors of position and velocity of the actuators, respectively;
- $\mathbf{q}_e = [q_1^e, q_2^e, ..., q_n^e]^T$ and $\dot{\mathbf{q}}_e = [\dot{q}_1^e, \dot{q}_2^e, ..., \dot{q}_n^e]^T$ represent the vectors of position and velocity of the elastic coordinates (deformations of links and joints);
- $\boldsymbol{\tau}$ is the vector of the actuators efforts.

In the case where the load on the end-effector and the inertia of actuators are preponderant, the expressions of the kinetic energy T and potential energy V are given by:

$$2T = \mathbf{v}^T \mathbf{M}\,\mathbf{v} + \dot{\mathbf{q}}_a^T \mathbf{I}_a\,\dot{\mathbf{q}}_a, \qquad (2a)$$

$$V = V_g + V_e,\ V_g = m\,g\,z \text{ and } V_e = f(\mathbf{q}_e) \qquad (2b)$$

where V_g corresponds to the potential energy due to gravity, and V_e to the energy due to elastic deformations. $\mathbf{M}$ is the mass matrix of the payload (comprising both mass and rotational inertia terms), $\mathbf{I}_a$ the diagonal matrix containing the axial moments of inertia of the geared motors, m the mass of the load, g the gravity, $f(\mathbf{q}_e)$ being a function depending of the deformations $\mathbf{q}_e$. Moreover, $\mathbf{x} = [x, y, z, \phi, \psi, \theta]^T$ and $\mathbf{v} = [\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\psi}, \dot{\theta}]^T$ are vectors containing the trajectory parameters and their derivatives, respectively; x, y, z represent the position of the controlled point in the global frame and ϕ, ψ and θ the rotation of the platform about three axes $\mathbf{a}_\phi$, $\mathbf{a}_\psi$ and $\mathbf{a}_\theta$. Vector $\mathbf{x}$ depends on both rigid coordinates $\mathbf{q}_a$ and elastic coordinates $\mathbf{q}_e$.

Analyzing expression (2), the potential and kinetic energies do not explicitly depend both of the actuated variables $\mathbf{q}_a$ and elastic coordinates $\mathbf{q}_e$, but also from the positions $\mathbf{x}$ and velocities $\mathbf{v}$ of the payload. Therefore it is preferable to rewrite Eq. (1) using the Lagrange multipliers [31], as follows:

$$\boldsymbol{\tau} = \mathbf{W_b} + \mathbf{B}^T\boldsymbol{\lambda},\ \mathbf{W_b} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right) - \frac{\partial L}{\partial \mathbf{q}_a} = \mathbf{I}_a\ddot{\mathbf{q}}_a \qquad (3a)$$

$$\mathbf{0} = \mathbf{W_c} + \mathbf{C}^T\boldsymbol{\lambda},\ \mathbf{W_c} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_e}\right) - \frac{\partial L}{\partial \mathbf{q}_e} \qquad (3b)$$

where $\boldsymbol{\lambda}$ is the Lagrange multipliers vector, which is related to the wrench $\mathbf{W_p}$ applied on the platform by:

$$\mathbf{A}^T\boldsymbol{\lambda} = \mathbf{W_p},\ \mathbf{W_p} = \left(\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \mathbf{v}}\right) - \frac{\partial L}{\partial \mathbf{x}}\right) \qquad (4)$$

and,

- $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are three matrices relating the vectors $\mathbf{v}$, $\dot{\mathbf{q}}_e$ and $\dot{\mathbf{q}}_a$ according to $\mathbf{A}\mathbf{v} = \mathbf{B}\dot{\mathbf{q}}_a + \mathbf{C}\dot{\mathbf{q}}_e$. They can be found by differentiating the closure equations $f_i(\mathbf{x}, \mathbf{q}_a, \mathbf{q}_e) = 0$ (taking into account the rigid as well as the elastic coordinates [31]) with respect to time. In the hypothesis of small elastic displacements ($\mathbf{q}_e \approx \mathbf{0}$), matrices $\mathbf{A}$ and $\mathbf{B}$ may be found assuming that the robot is composed of rigid links **only**.
- $\mathbf{W_p}$ is the wrench applied on the platform by the legs and external forces expressed along axes $\mathbf{a}_\phi$, $\mathbf{a}_\psi$ and $\mathbf{a}_\theta$ [32].

Expressing $\mathbf{W_p}$ in the base frame, one can obtain:

$$\boldsymbol{\tau} = \mathbf{I}_a\ddot{\mathbf{q}}_a + \mathbf{J}_{\mathbf{q_a}}^T\ {}^{\mathbf{R_0}}\mathbf{W_p}, \qquad (5a)$$

$$\mathbf{0} = \mathbf{W_c} + \mathbf{J}_{\mathbf{q_e}}^T\ {}^{\mathbf{R_0}}\mathbf{W_p}, \qquad (5b)$$

where $\mathbf{J}_{\mathbf{q_a}} = \left({}^{\mathbf{R_0}}\mathbf{A}\right)^{-1}\mathbf{B}$ is the square Jacobian matrix between twist $\mathbf{t}$ of the platform (expressed in the base frame) and the vector $\dot{\mathbf{q}}_a$ of actuators velocities, $\mathbf{J}_{\mathbf{q_e}} = \left({}^{\mathbf{R_0}}\mathbf{A}\right)^{-1}\mathbf{C}$ is the non-square Jacobian matrix between twist $\mathbf{t}$ of the platform (expressed in the base frame) and the vector $\dot{\mathbf{q}}_e$ of deformations velocities, ${}^{\mathbf{R_0}}\mathbf{A} = \mathbf{A}\,\mathbf{D}$ is the expression of matrix $\mathbf{A}$ in the base frame, where $\mathbf{D}$ is a transformation matrix, of which expression is given in [33].

For any prescribed trajectory $\mathbf{x}(t)$, the values of vector $\mathbf{q}_a$ can be found using the inverse kinematics and dynamics. Thus, taking into account that the manipulator is not in a Type 1 singularity [3], the terms $\mathbf{W_c}$ and ${}^{\mathbf{R_0}}\mathbf{W_p}$ can be computed (using, or not, some recursive algorithm [34]). However, for a trajectory passing through a Type 2 singularity, the determinant of matrix ${}^{\mathbf{R_0}}\mathbf{A}$ vanishes. Numerically, the values of the efforts applied by the actuators become infinite. In practice, the manipulator either is locked in such a position of the end-effector or it can not follow the prescribed trajectory.

As previously mentioned, in a Type 2 singularity, the determinant of matrix ${}^{\mathbf{R_0}}\mathbf{A}$ vanishes. In other words, at least two of its columns are linearly dependant [33]. So, one may obtain such a relationship:

$$\sum_{j=1}^{6} \alpha_j \mathbf{A}_j = \mathbf{0}\,, \tag{6}$$

where $\mathbf{A}_j$ represents the j-th column of matrix ${}^{\mathbf{R_0}}\mathbf{A}$ and α_j are coefficients, which in general can be functions of $\mathbf{q}_a$ and $\mathbf{q}_e$. It should be noted that the vector $\mathbf{t_s} = [\alpha_1, \alpha_2, \ldots, \alpha_6]^T$ represents the direction of the uncontrollable motion of the platform in a Type 2 singularity.

By substituting (6) into (4), we obtain

$$\mathbf{A}_j^T \boldsymbol{\lambda} = W_j\,, j = 1, \ldots, 6 \tag{7}$$

where W_j is the j-th row of vector ${}^{\mathbf{R_0}}\mathbf{W_p}$.

Then, from (6) and (7) the following conditions are derived:

$$\sum_{j=1}^{6} \left(\alpha_j \mathbf{A}_j^T \boldsymbol{\lambda}\right) = \sum_{j=1}^{6} \left(\alpha_j W_j\right) = 0\,. \tag{8}$$

The right term of Eq. (8) corresponds to the scalar product of vectors $\mathbf{t_s}$ and ${}^{\mathbf{R_0}}\mathbf{W_p}$.

Thus, in the presence of a Type 2 singularity, it is possible to satisfy conditions (8) if **the wrench applied on the platform by the legs and external efforts ${}^{\mathbf{R_0}}\mathbf{W_p}$ are orthogonal to the direction of the uncontrollable motion $\mathbf{t_s}$**. Otherwise, the dynamic model is not consistent. Obviously, in the presence of a Type 2 singularity, the displacement of the end-effector of the manipulator has to be planned to satisfy (8). Therefore, our task will be to achieve a trajectory which will allow the manipulator passing trough the Type 2 singularities, i.e. which will allow the manipulator respecting condition (8).

In the next section, an example illustrates the obtained results discussed above. This example presents a planar 5*R* parallel manipulator.

3 ILLUSTRATIVE EXAMPLE

In the planar 5*R* parallel manipulator, as shown in Fig. 1, the output axis is connected to the base by two legs, each of which consists of three revolute joints and two links. In each of the two legs, the revolute joint connected to the base is actuated. Thus, such a manipulator is able to position its output axis in a plane.

As shown in Fig. 1, the input joints are denoted as *A* and *E*. The orientation of elements 1 and 4 are denoted q_1^e and q_2^e, respectively. The common joint of the two legs is denoted as *C*, which is also the output axis with controlled parameters *x* and *y*. A fixed global reference system ***xOy*** is located at the middle of segment *AE* with the ***y***-axis normal to **AE** and the ***x***-axis directed along **AE**. The lengths of the links *AB*, *BC*, *CD*, *DE* are respectively denoted as L_1, L_2, L_3 and L_4.

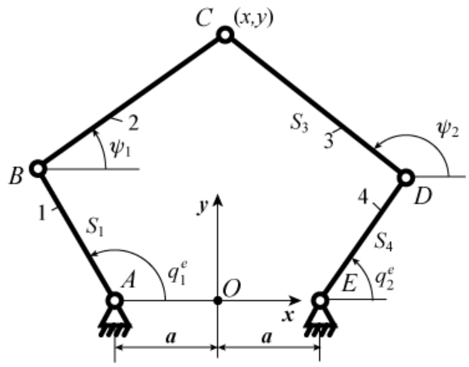


Figure 1. Kinematic chain of the planar 5*R* parallel manipulator.

Actuators 1 and 2 are connected to links 1 and 4, respectively, via Harmonic Drive® systems which are represented by a model similar to that given in [35]. The position of actuator *i* is denoted as q_i^a. It is assumed that the actuator *i* is capable to deliver a couple τ_i to the motor shaft, which is elastically coupled to the link *j* of the robot (i = 1 or 2, j = 1 or 4). The flexibility of the drive system is modeled by a torsion spring with stiffness k_1. The gear ratio is denoted *n*. I^a is the axial moment of inertia of the motor *i* plus the Harmonic drive system.

In order to obtain relatively simple symbolic model for demonstrating the expected results, we propose to modelize the deformations of the robot by adding virtual joints on the links. We assume here that the link deformations of elements 2 and 3 are preponderant. Therefore, we add on them linear springs of stiffness k_2, that are directed along directions **BC** and **CD**, respectively. The displacements of the spring linked to element 2 (3, resp.) will be denoted as ε_1 (ε_2, resp.).

The singularity analysis of this manipulator [36] shows that the Type 2 singularities appear when links 2 and 3 are parallel (see also Fig. 2 in [1]). In both cases, the gained degree of freedom is an infinitesimal translation perpendicular to the links 2 and 3. However, if $L_2 = L_3$, the gained degree of freedom may become a finite rotary motion.

Taking into account that the gravity is directed along ***z*** axis (perpendicular to the plane of motions), the expression of the potential energy *V* may be written as:

$$V = 0.5\left(k_1\left((q_1^e - q_1^a)^2 + (q_2^e - q_2^a)^2\right) + k_2\left(\varepsilon_1^2 + \varepsilon_2^2\right)\right). \tag{9}$$

The expression of the kinetic energy is

$$T = 0.5\left(m\left(\dot{x}^2 + \dot{y}^2\right) + I^a\left(\left(\dot{q}_1^a\right)^2 + \left(\dot{q}_2^a\right)^2\right)\right), \tag{10}$$

where m is the mass of the payload and $\dot{x}$ and $\dot{y}$ are the velocities of point *C* along ***x*** and ***y*** axes, respectively.

Thus the dynamic model can be obtained from (3) and (4):

$$k_2\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\end{bmatrix} + \mathbf{C}^T\mathbf{A}^{-T}\mathbf{W_p} = \mathbf{0}\,,\ \mathbf{W_p} = m\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} \tag{11a}$$

$$k_1\begin{bmatrix}q_1^e - q_1^a/n\\ q_2^e - q_2^a/n\end{bmatrix} + \mathbf{B}^T\mathbf{A}^{-T}\mathbf{W_p} = \mathbf{0} \tag{11b}$$

$$\boldsymbol{\tau} = I^a\begin{bmatrix}\ddot{q}_1^a\\ \ddot{q}_2^a\end{bmatrix} - \frac{k_1}{n}\begin{bmatrix}q_1^e - q_1^a/n\\ q_2^e - q_2^a/n\end{bmatrix} \tag{11c}$$

Matrices **A**, **B** and **C** may be found from the loop closure equations:

$$f_1 = (x + a - L_1 \cos q_1^e)^2 + (y - L_1 \sin q_1^e)^2 - (L_2 + \varepsilon_1)^2 = 0 \quad (12a)$$

$$f_2 = (x - a - L_4 \cos q_2^e)^2 + (y - L_4 \sin q_2^e)^2 - (L_3 + \varepsilon_2)^2 = 0 \quad (12b)$$

from which it comes:

$$\mathbf{A} = \left[\frac{\partial f_i}{\partial \mathbf{x}}\right] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = 2\begin{bmatrix} x - L_1 \cos q_1^e + a & y - L_1 \sin q_1^e \\ x - L_4 \cos q_2^e - a & y - L_4 \sin q_2^e \end{bmatrix} \quad (13)$$

$$\mathbf{B} = -\left[\frac{\partial f_i}{\partial \mathbf{q}_e}\right] = -\begin{bmatrix} L_1(a_{11} \sin q_1^e - a_{12} \cos q_1^e) & 0 \\ 0 & L_4(a_{21} \sin q_2^e - a_{22} \cos q_2^e) \end{bmatrix} \quad (14)$$

$$\mathbf{C} = -\left[\frac{\partial f_i}{\partial \boldsymbol{\varepsilon}}\right] = 2\begin{bmatrix} L_2 + \varepsilon_1 & 0 \\ 0 & L_3 + \varepsilon_2 \end{bmatrix} \approx 2\begin{bmatrix} L_2 & 0 \\ 0 & L_3 \end{bmatrix} \quad (15)$$

with $\mathbf{x} = [x, y]^T$, $\mathbf{q}_e = [q_1^e, q_2^e]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2]^T$.

As the trajectory $\mathbf{x}(t)$ is known, from (11a), we can obtain the values of $\boldsymbol{\varepsilon}$. Introducing it into (12), the parameters $\mathbf{q}_e$ are computed. Finally, from (11b), the values of the actuated variables $\mathbf{q}_a$ are obtained. Differentiating it two times with respect to the time allows obtaining the values of the input torques.

Analyzing these expressions, it could be observed that the dynamic model depend on the position, velocity and acceleration of point *C*, but also of the jerk and its first derivative. Therefore, a ninth-degree polynomial has to be applied as a control law when the end-effector is not in singular configuration.

In order to avoid infinite values of the input torques when crossing a Type 2 singularity, Eq. (8) has to be satisfied. From matrix **A**, one can find that the twist of the infinitesimal displacement in the singularity can be written under the form:

$$\mathbf{t_s} = [-\sin\psi_1, \cos\psi_1]^T \quad (16)$$

Thus, the examined manipulator can pass through the given singular positions if the wrench $\mathbf{W_p}$ determined by (11) is orthogonal to the direction of the uncontrollable motion $\mathbf{t_s}$ described by (16).

Let us now consider the motion planning, which makes it possible to satisfy this condition. For this purpose the following parameters of manipulator's links are specified: a = 0.2 m, $L_1 = L_2 = L_3 = L_4$ = 0.25 m; m = 10 kg; k_1 = 25000 Nm/rad; k_2 = 350000 N/m. Moreover, we use the physical parameters of a real Harmonic Drive® system (HDUC-1U-11) coupled with an actuator Gamdrive (11-50-MB-SP-024-CR), that are $I^a = 0.064.10^{-4}$ kg.m² and n = 50.

With regard to the prescribed trajectory generation, the point *C* should reproduce a motion along a straight line between the initial position C_0 $(x_0, y_0) = C_0$ (0.1, 0.345) and the final point C_f $(x_f, y_f) = C_f$ (–0.05, 0.195) in t_f = 1.5 s (Fig. 2).

Thus, the given trajectory can be expressed as follows:

$$\mathbf{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 + s(t)(x_f - x_0) \\ y_0 + s(t)(y_f - y_0) \end{bmatrix} \quad (17)$$

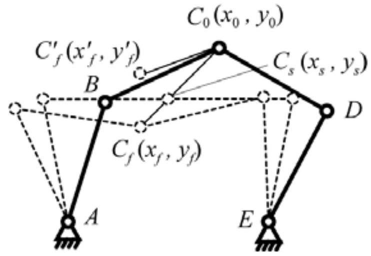


Figure 2. Initial, singular and final positions of the planar 5R parallel manipulator.

However, the manipulator will pass by a Type 2 singular position at point C_s $(x_s, y_s) = C_s$ (0, 0.245) (Fig. 2).

Developing the condition (8) for passing through the singular position for the planar 5*R* parallel manipulator at point C_s, we obtain:

$$\ddot{y} = 0 \quad (18)$$

Then, taking into account that the velocity and the acceleration of the end-effector in initial and final positions are equal to zero, the following thirteen boundary conditions are found:

$$s(t_0) = 0, \quad (19)$$
$$s(t_f) = 1, \quad (20)$$
$$s(t_s = 1 \text{ s}) = 2/3, \quad (21)$$
$$\dot{s}(t_0) = 0, \quad (22)$$
$$\dot{s}(t_f) = 0, \quad (23)$$
$$\dot{s}(t_s) > 0 \text{ (in our case } \dot{s}(t_s) = 1.5\text{)}, \quad (24)$$
$$\ddot{s}(t_0) = 0, \quad (25)$$
$$\ddot{s}(t_f) = 0, \quad (26)$$
$$\ddot{s}(t_s) = 0. \quad (27)$$
$$\dddot{s}(t_0) = 0, \quad (28)$$
$$\dddot{s}(t_f) = 0, \quad (29)$$
$$d(\dddot{s}(t_0))/dt = 0, \quad (30)$$
$$d(\dddot{s}(t_f))/dt = 0, \quad (31)$$

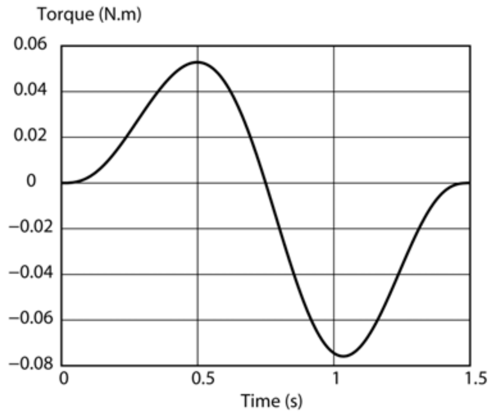
From (19)-(31), the following twelfth order polynomial trajectory planning is found:

$$s(t) = 14.96t^5 - 6.38t^6 - 128.38t^7 + 334.66t^8 - 381.71t^9 + 230.08t^{10} - 71.67t^{11} + 9.12t^{12} \quad (32)$$

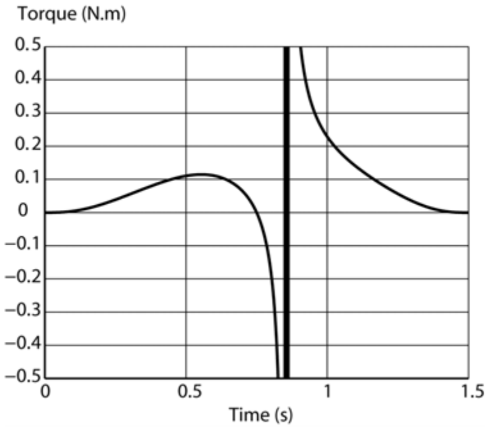
Thus the generation of the motion by the obtained twelfth order polynomial makes it possible to pass through the singularity without perturbation and the input torques remain in the limits of finite values.

In order to compare the different cases of trajectory planning, in Figs. 3 and 4 are given the values of the input torques obtained using the software ADAMS for the following numerical simulations:

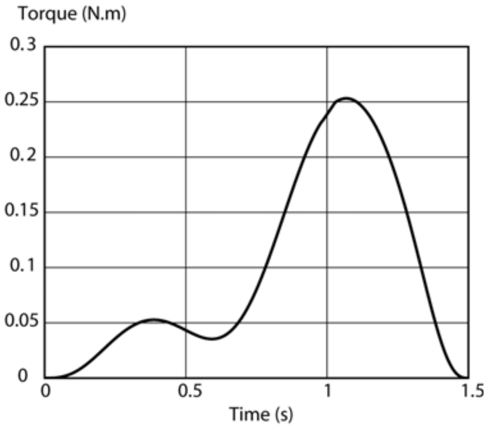
A: a trajectory between points C_0 and C'_f $(x'_f, y'_f) = C'_f$ (–0.05, 0.295) (Fig. 2) without meeting any singularity. For such a case, the following ninth order polynomial law is used $s(t)$= 16.59 t^5 – 36.87 t^6 + 31.6 t^7 – 12.29 t^8 + 1.82 t^9 for the trajectory planning out of the singular zone of the manipulator. In this case the values of the input torques are finite.



(a) Case A

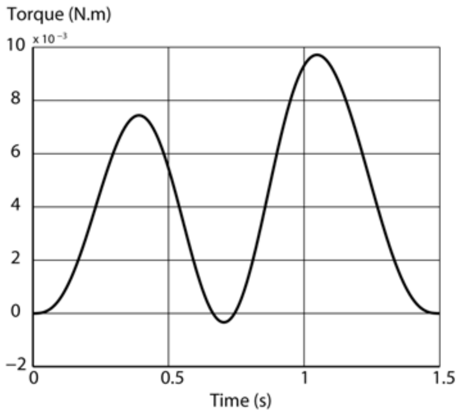


(b) Case B

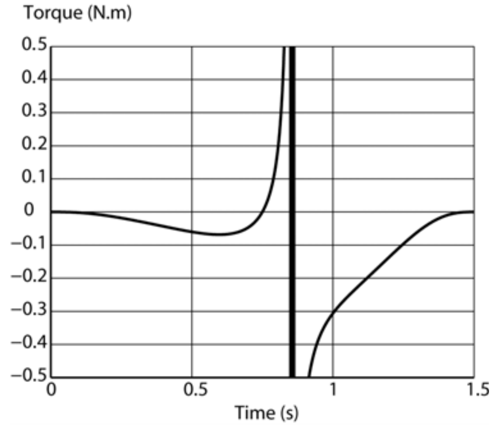


(c) Case C

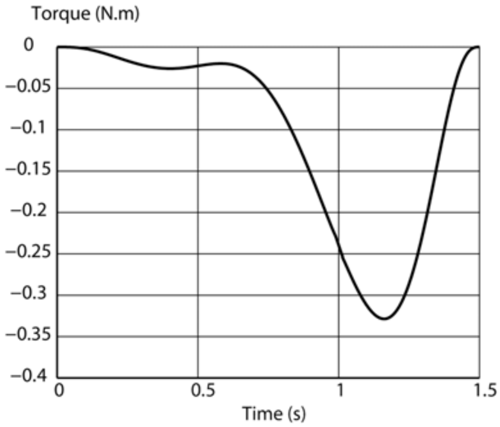
Figure 3. Torques values for the actuator 1.



(a) Case A



(b) Case B



(c) Case C

Figure 4. Torques values for the actuator 2.

B: the ninth order polynomial law $s(t) = 16.59\, t^5 - 36.87\, t^6 + 31.6\, t^7 - 12.29\, t^8 + 1.82\, t^9$ for the trajectory planning between C_0 and C_f inside the singular zone for the manipulator. In this case the values of the input torques close to the singular positions tend to infinity.

C: the twelfth order polynomial law of Eq. (32) for the trajectory planning of the manipulator inside the singular zone. The obtained results show that the values of the input torques are finite.

Thus, the numerical simulations show that the obtained optimal dynamic conditions assume the passing of the flexible manipulator through the singular position.

4 CONCLUSION

In our previous work, we have shown that any parallel manipulator can pass through the singular positions without perturbation of motion if the wrench applied on the end-effector by the legs and external efforts is orthogonal to the twist along the direction of the uncontrollable motion [1]. This condition was applied to the rigid-link manipulators. The obtained results showed that the planning of motion for assuming the optimal force generation can be carried out by a eight order polynomial law. In our other study [2] the rigid-link flexible-joint manipulators have been studied. It was shown that the degree of the polynomial law should be different, when the flexibility of actuated joints is introduced. The obtained results disclosed that the planning of motion for assuming the optimal force generation in the rigid-link flexible-joint manipulators must be carried out by a twelfth order polynomial law.

In this paper, we have expanded the information about the dynamic properties of parallel manipulators in the presence of Type 2 singularity by including in the studied problem the link flexibility and the payload. The obtained results have shown that the planning of motion for assuming the optimal force generation in the structurally flexible manipulators with payload must be also carried out by a twelfth order polynomial law.

The suggested technique was illustrated by a *5R* planar parallel manipulator. The obtained dynamic properties have been validated by numerical simulations carried out using the software ADAMS.

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