# Global Identification of Drive Gains and Dynamic Parameters of Parallel Robots - Part 1: Theory

Sébastien Briot[‡] and Maxime Gautier [†‡*]
[†] LUNAM, Université de Nantes, Nantes, FRANCE
[‡] Institut de Recherche en Communications et Cybernétique de Nantes
IRCCyN, UMR CNRS 6592, Nantes, FRANCE

**Abstract** Most of the papers dealing with the dynamic parameters identification of parallel robots are based on simple models, which take only the dynamics of the moving platform into account. Moreover the actuator drive gains are not calibrated which leads to identification errors. In this paper a systematic way to derive the full dynamic identification model of parallel robots is proposed in combination with a method that allows the identification of both robot inertial parameters and drive gains.

## 1 Introduction

Parallel robots are increasingly being used since a few decades. This is due to their main advantages compared to their serial counterparts that are: (i) a higher intrinsic rigidity, (ii) a larger payload-to-weight ratio and (iii) higher velocity and acceleration capacities (Merlet, 2006). In order to obtain these interesting properties, a good controller should be implemented. Several approaches could be envisaged (Amiral et al., 1996; Vivas and Poignet, 2005), but it appears that, for high-speed robots or when varying loads have to be compensated (e.g. in pick-and-place operations or machining), computed torque control is generally used (Khalil and Dombre, 2002). This approach needs a correct identification of the dynamic model of the robot with the load (Khalil et al., 2007), which can be obtained provided two main conditions are satisfied: (i) a well-tuned derivative band-pass filtering of actuated joints position is used to calculate the actuated joints velocities and accelerations, and (ii) the accurate values of actuator drive gains $g_\tau$ are accurately known to calculate the actuator force/torque as the product of

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the known control signal calculated by the numerical controller of the robot (the current references) by the drive gains. However, this is rarely the case and they need to be calibrated (Restrepo and Gautier, 1995; Corke, 1996).

Among all papers dealing with dynamic parameters identificiation for parallel robots, only few of them propose a systematic computation of the full Inverse Dynamic Identification Model (IDIM). In (Guegan et al., 2003), the authors propose an attempt to create a systematic IDIM based on a Newton-Euler approach. The closed loops are first virtually opened to compute the dynamic model of the tree structure and then, the closure constraints are imposed. However, the way to open the loop is not straightforward. In (Grotjahn et al., 2004; Diaz-Rodriguez et al., 2010), the authors propose methods for computing the IDIM based on the Jourdain's principle or Lagrange multipliers. But the way to identify the drive gains is not treated. Moreover, some jacobian matrices, whose computation is not straightforward, are not clearly derived.

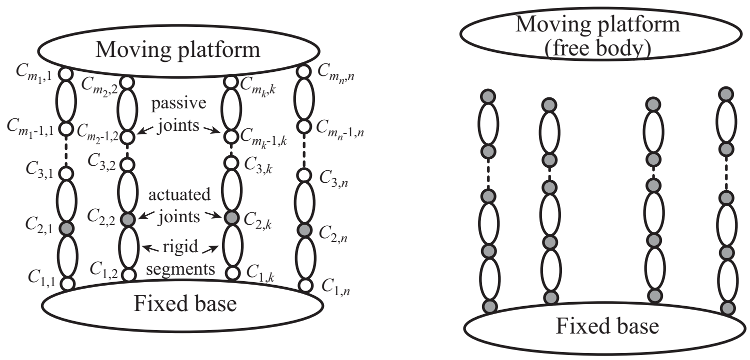
In this paper it is proposed a global approach for both the identification of parallel robots dynamic parameters and drive gains. This paper is the first part of our work on the identification of the parallel robots dynamic parameters and it presents the theoretical approach. A case study will be treated in (Briot and Gautier, 2012).

## 2 A Systematic Procedure for the IDIM Computation

A parallel robot is a complex multi-body system having several closed loops (Fig. 1a). It is composed of a moving platform connected to a fixed base by $n$ legs and $m$ elements. In this paper, a method similar to (Ibrahim and Khalil, 2010) is applied for the computation of IDIM of *non-redundant* parallel robots. The proposed method is decomposed into two steps: (i) all closed loops are virtually opened to make the platform virtually disassembled from the rest of the structure (Fig. 1b); each leg joint is virtually considered actuated (even for unactuated actual joints) so that the robot becomes a tree structure with a free body: the platform; the dynamic model of the tree structure and of the free platform is then computed using a systematic procedure based on the Newton-Euler principle and (ii) the loops are then closed using the loop-closure equations and the Lagrange multipliers.

### 2.1 IDIM of Tree Open Loop Robots

It is known that the complete rigid dynamic model of any open-loop tree structure can be linearly written in term of a $(n_t \times 1)$ vector with respect




(a) Kinematic chain (b) Virtual tree structure

Figure 1: A general parallel robot.

to the standard parameters $\chi_{st_t}$ (Khalil and Dombre, 2002),

$$\tau_{idm_t}\left(\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t\right) = \phi_{st_t}\left(\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t\right)\chi_{st_t} \tag{1}$$

where $\tau_{idm_t}$ is the $(n_t \times 1)$ vector of the virtual input efforts of the tree structure, $\phi_{st_t}$ is the $(n_t \times n_{st_t})$ jacobian matrix of $\tau_{idm_t}$, with respect to the $(n_{st_t} \times 1)$ vector $\chi_{st_t}$ of the standard parameters given by $\chi_{st_t} = [\chi_{st}^{1T}, \chi_{st}^{2T}, \ldots \chi_{st}^{nT}]$ that are described in (Khalil and Dombre, 2002) and $\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t$ are the vectors of the joint positions, velocities and accelerations, respectively.

Several methods can be used to systematically derive these equations. Here, an algorithm based on the use of the modified Denavit-Hartenberg robot geometric description and the Newton-Euler principle is applied. This modelling is known to give the dynamic model equations in the most compact form (Khalil and Dombre, 2002).

### 2.2 IDIM of Parallel Robots

The previous dynamic model does not take into account the closed loop characteristics of parallel robots: among all joint coordinates $\mathbf{q}_t$ and platform coordinates $\mathbf{x}$, only a subset denoted as $\mathbf{q}$ is independent (the actual actuated joints positions). As a result, vectors $\mathbf{q}_t$ and $\mathbf{x}$ can be computed as functions of $\mathbf{q}$ using the loop-closure equations (Merlet, 2006),

$$\mathbf{f}_t(\mathbf{q}, \mathbf{q}_t) = \mathbf{0}, \mathbf{f}_p(\mathbf{q}, \mathbf{x}) = \mathbf{0} \tag{2}$$

Using these equations, all joint and platform velocities and accelerations can be computed:

$$\dot{\mathbf{q}}_t = -\mathbf{A}_t^{-1}\mathbf{B}_t\dot{\mathbf{q}} = \mathbf{J}_t\dot{\mathbf{q}}, \mathbf{t} = -\mathbf{A}_p^{-1}\mathbf{B}_p\dot{\mathbf{q}} = \mathbf{J}_p\dot{\mathbf{q}}, \tag{3}$$

$$\ddot{\mathbf{q}}_t = -\mathbf{A}_t^{-1}(\dot{\mathbf{A}}_t\dot{\mathbf{q}}_t + \mathbf{B}_t\ddot{\mathbf{q}} + \dot{\mathbf{B}}_t\dot{\mathbf{q}}), \gamma = -\mathbf{A}_p^{-1}(\dot{\mathbf{A}}_p\mathbf{t} + \mathbf{B}_p\ddot{\mathbf{q}} + \dot{\mathbf{B}}_p\dot{\mathbf{q}}) \tag{4}$$

where $\mathbf{t}$ is the platform twist, $\gamma$ the platform acceleration screw and matrices $\mathbf{A}_t$, $\mathbf{A}_p$ ($\mathbf{B}_t$, $\mathbf{B}_p$, resp.) can be obtained through the differentation of the loop-closure equations (2) with respect to all joint coordinates $\mathbf{q}_t$ and the platform coordinates (actuated joints positions, resp.). It should be noticed that the computation of matrices $\mathbf{A}_t$ and $\mathbf{B}_t$ is generally not straightforward. Therefore, it is preferable to:

1. express the kinematic relation between the platform twist $\mathbf{t}$ and the velocities $\mathbf{t}_{tk}$ of all leg extremities $C_{m_k,k}$ (Fig. 1a), $\mathbf{t}_{tk} = \mathbf{J}_{tk}\mathbf{t}$,
2. express the kinematic relation between the velocities $\mathbf{t}_{tk}$ of all leg extremities $C_{m_k,k}$ and the velocities of all joints $\dot{\mathbf{q}}_p$, $\mathbf{t}_{tk} = \mathbf{J}_k\dot{\mathbf{q}}_p$,
3. combine these two relations with (3) in order to obtain $\dot{\mathbf{q}}_p = \mathbf{J}_t\dot{\mathbf{q}}$, with $\mathbf{J}_t = \mathbf{J}_k^{-1}\mathbf{J}_{tk}\mathbf{J}_p$. All the previous expressions are valuable as long as the robot does not meet any singularity.

To take into account the closure-loop constraints into the dynamic model of the parallel robot, the Lagrange multipliers $\lambda$ can be used (Khalil and Dombre, 2002) to compute the $(n \times 1)$ vector of the actuated joint force/ torque $\tau_{idm}$ of the closed-loop structure:

$$\tau_{idm} = -[\mathbf{B}_t^T, \mathbf{B}_p^T]\lambda, \text{ where } \begin{bmatrix} \mathbf{A}_t^T & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_p^T \end{bmatrix} \lambda = \begin{bmatrix} \tau_{idm_t} \\ \mathbf{f}_p \end{bmatrix} \tag{5}$$

$\mathbf{f}_p$ being the $(6 \times 1)$ vector of inertia forces of the platform plus the external loading (Ibrahim and Khalil, 2010). (5) can be rewritten as:

$$\begin{aligned} \tau_{idm} &= \mathbf{J}_t^T\tau_{idm_t} + \mathbf{J}_p^T\mathbf{f}_p = \mathbf{J}_t^T\phi_{st_t}(\mathbf{q}_t, \dot{\mathbf{q}}_t, \ddot{\mathbf{q}}_t)\chi_{st_t} + \mathbf{J}_p^T\phi_p(\mathbf{x}, \mathbf{v}, \gamma)\chi_p \\ &= \begin{bmatrix} \mathbf{J}_t^T\phi_{st_t} & \mathbf{J}_p^T\phi_p \end{bmatrix} \begin{bmatrix} \chi_{st_t}^T & \chi_p^T \end{bmatrix}^T = \phi_{st}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\chi_{st} \end{aligned} \tag{6}$$

where $\chi_p$ is the $(10 \times 1)$ vector of platform parameters, $\phi_p$ is the $(6 \times 10)$ jacobian matrix between $\mathbf{f}_p$ and $\chi_p$, $\chi_{st}$ is the $(n_{st} \times 1)$ vector of the global standard parameters of the parallel robot and $\phi_{st}$ the $(n \times n_{st})$ jacobian matrix between $\tau_{idm}$ and $\chi_{st}$.

The identifiable parameters are the base parameters which are the minimum number of dynamic parameters from which the dynamic model can be calculated (Khalil and Dombre, 2002). The minimal dynamic model can be written using the $n_b$ base dynamic parameters $\chi$ as follows:

$$\tau_{idm} = \phi(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\chi \tag{7}$$

where $\phi$ is a subset of independant columns in $\phi_{st}$ which defines the identifiable parameters.

Because of perturbations due to noise measurement and modelling errors, the actual force/torque $\tau$ differs from $\tau_{idm}$ by an error, $\mathbf{e}$, such that:

$$\tau = \mathbf{v}_\tau \mathbf{g}_\tau = diag(v_\tau^j) \begin{bmatrix} g_\tau^1 & \dots & g_\tau^n \end{bmatrix}^T = \tau_{idm} + \mathbf{e} = \phi(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\chi + \mathbf{e} \quad (8)$$

$\mathbf{v}_\tau$ is the $(n \times n)$ diagonal matrix of the actual current references of the current amplifiers ($v_\tau^j$ corresponds to actuated joints $j$) and $\mathbf{g}_\tau$ is the $(n \times 1)$ vector of the drive gains ($g_\tau^j$ corresponds to actuator $j$). Equation (8) represents the IDIM.

# 3 Identification Procedure

## 3.1 Recalls on Least Squares Identification of the Dynamic Parameters (IDIM-LS)

The off-line identification of the base dynamic parameters $\chi$ can be achieved given measured or estimated off-line data for $\tau$ and $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$, collected while the robot is tracking some planned trajectories. The model (8) is sampled, low pass filtered and decimated in order to get an overdetermined linear system of $(n \times r)$ equations and $n_b$ unknowns:

$$\mathbf{Y}(\tau) = \mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})\chi + \rho \quad (9)$$

where $(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$ are an estimation of $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$, obtained by band-pass filtering and sampling the measure of $\mathbf{q}$ (Gautier, 1997), $\rho$ is the $(r \times 1)$ vector of errors and $\mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$ is the $((n \times r) \times n_b)$ observation matrix.

Using the base parameters and tracking some 'exciting' reference trajectories (Gautier and Khalil, 1992), a well conditionned matrix $\mathbf{W}$ is obtained. The LS solution $\hat{\chi}$ of (9) is given by:

$$\hat{\chi} = \mathbf{W}^+ \mathbf{Y}, \text{ where } \mathbf{W}^+ = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \quad (10)$$

Standard deviations $\sigma_{\hat{\chi}_i}$ can be estimated assuming that $\rho$ is a zero mean independant noise (Gautier, 1997). The ordinary LS can be improved by taking into account different standard deviations on actuated joint $j$ equations errors (Gautier, 1997). This weighting operation normalises the errors in (9) and gives the weighted LS estimation of the parameters (IDIM-WLS).

## 3.2 Total Least Squares Identification

In the classical IDIM-LS, to compute vector $\mathbf{Y}$, the drive gains are supposed known. But usually the manufacturers give drive gain parameters with an uncertainty of about 10%, thus leading to identification errors. Therefore, it is preferable to introduce the drive gains into the base parameters and to use the Total Least Squares Identification (IDIM-TLS).

Details on the Total LS (TLS) identification method can be found in (Huffel and Vandewalle, 1991) and many papers of the same authors. This method has been applied in (Gautier et al., 1994) for the identification of the drive gains and the dynamic parameters on a two degrees of freedom (dof) serial robot but gives arguable results due to the lack of an accurate scale factor. In this paper a major improvement is proposed: the accurate scaling of parameters using the precise weighed value of an additional payload mass. However, by the use of the model (9) without any modification, the payload parameters are regrouped with the end-effector parameters and cannot be independently identified. In order to apply the proposed approach, the model (9) must be modified. This procedure is detailed below.

**IDIM Including a Payload and Drive Gains** The inertial parameters of the payload are easily added to the IDIM by considering the payload as a link $m+1$ fixed to the robot platform. The model (8) becomes:

$$\tau = \mathbf{v}_\tau \mathbf{g}_\tau = \begin{bmatrix} \phi & \phi_{uL} & \phi_{kL} \end{bmatrix} \begin{bmatrix} \chi^T & \chi_{uL}^T & \chi_{kL}^T \end{bmatrix}^T + \mathbf{e} \qquad (11)$$

where $\chi_{kL}$ is the $(n_{kL} \times 1)$ vector of the known inertial parameters of the payload (calculated with CAD or accurately measured), $\chi_{uL}$ is the $((10 - n_{kL}) \times 1)$ vector of the unknown inertial parameters of the payload, $\phi_{kL}$ is the $(n \times n_{kL})$ jacobian matrix of $\tau_{idm}$, with respect to the vector $\chi_{kL}$ and $\phi_{uL}$ is the $(n \times (10 - n_{kL}))$ jacobian matrix of $\tau_{idm}$, with respect to the vector $\chi_{uL}$.

**Solution of the IDIM-TLS** The identification of the dynamic parameters of the robot and the payload requires the achievement of two types of trajectories: (a) trajectories without payload and (b) trajectories with the payload fixed to the end-effector (Khalil et al., 2007). The sampling and filtering of the model IDIM (11) can be then written as:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{V}_{\tau a} \\ \mathbf{V}_{\tau b} \end{bmatrix} \mathbf{g}_\tau = \begin{bmatrix} \mathbf{W}_a & \mathbf{0} & \mathbf{0} \\ \mathbf{W}_b & \mathbf{W}_{uL} & \mathbf{W}_{kL} \end{bmatrix} \begin{bmatrix} \chi^T & \chi_{uL}^T & \chi_{kL}^T \end{bmatrix}^T + \rho \qquad (12)$$

where $\mathbf{W}_a$ is the observation matrix of the robot in the unloaded case, $\mathbf{W}_b$ is the observation matrix of the robot in the loaded case, $\mathbf{W}_{uL}$ is the observation matrix corresponding to the unknown payload inertial parameters, $\mathbf{W}_{kL}$ is the observation matrix corresponding to the known payload inertial parameters, $\mathbf{V}_{\tau a}$ is the matrix of $\mathbf{v}_\tau$ samples in the unloaded case, $\mathbf{V}_{\tau b}$ is the matrix of $\mathbf{v}_\tau$ samples in the loaded case.

Eq. (12) becomes:

$$\mathbf{W}_{tot}\chi_{tot} = \begin{bmatrix} -\mathbf{W}_a & \mathbf{V}_{\tau a} & \mathbf{0} & \mathbf{0} \\ -\mathbf{W}_b & \mathbf{V}_{\tau b} & -\mathbf{W}_{uL} & -\mathbf{W}_{kL}\chi_{kL} \end{bmatrix} \left[\chi^T, \mathbf{g}_\tau^T, \chi_{uL}^T, \delta\right]^T = \rho \tag{13}$$

where $\mathbf{W}_{tot}$ is a $(r \times (n_b + n + 11 - n_{kL}))$ matrix, $\chi_{tot}$ is a $(n_b+n+11-n_{kL})$ vector and $\delta$ is a scalar which should be equal to 1.

Without perturbation, $\rho = 0$ and $\mathbf{W}_{tot}$ should be rank deficient to get the solutions $\lambda\chi_{tot} \neq \mathbf{0}$ depending on a scale coefficient $\lambda$. However because of the measurement perturbations, $\mathbf{W}_{tot}$ is a full rank matrix. Therefore, the system (13) is replaced by the compatible system closest to (13) with respect to the Frobenius norm: $\hat{\mathbf{W}}_{tot}\hat{\chi}_{tot} = \mathbf{0}$, where $\hat{\mathbf{W}}_{tot}$ is the rank deficient matrix, with the same dimension as $\mathbf{W}_{tot}$, which minimizes the Frobenius norm $\|\mathbf{W}_{tot} - \hat{\mathbf{W}}_{tot}\|$ (Gautier et al., 1994) and $\hat{\chi}_{tot} = \begin{bmatrix}\hat{\chi}^T & \hat{\mathbf{g}}_\tau^T & \hat{\chi}_{uL}^T & \hat{\delta}\end{bmatrix}^T$ is the solution of the compatible system closest to (13).

There are infinity of vectors $\hat{\chi}_{tot} = \lambda\hat{\chi}_{tot}^n$ that can be obtained by a scale factor $\lambda$. A unique solution $\hat{\chi}_{tot}^* = \hat{\lambda}\hat{\chi}_{tot}^n$ can be found by taking into account that the last value of $\hat{\chi}_{tot}^*$ should be equal to 1, i.e. $\hat{\lambda} = 1/\hat{\delta}$. More information on IDIM-TLS can be found in (Gautier and Briot, 2012).

This section ends the theoretical part of this work. A case study is developped in (Briot and Gautier, 2012) and shows the effectivess of the method.

## 4 Conclusions

This paper has presented a global approach for both the identification of parallel robots dynamic parameters and drive gains. It is based on a IDIM-TLS technique using current reference and position sampled data while the robot is tracking one reference trajectory without load fixed on the robot and one trajectory with a known payload fixed on the robot, whose inertial parameters are measured. This paper has presented only theoretical derivations and a case study will be presented in (Briot and Gautier, 2012).

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