Identification of Consistent Standard Dynamic Parameters of Industrial Robots

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Abstract— **The dynamics of each link and joint of a robot is characterized by a set of 14 standard dynamic parameters (6 for the inertia matrix, 3 for the centre of mass coordinates, 1 for the mass and 4 for the drive chain inertia and friction). It is known that only a subset of the standard parameters, called the base parameters, are identifiable using the inverse dynamic model and the linear least squares techniques. Moreover, some of the base parameters are poorly identified because they poorly affect the joint torque. Thus they can be eliminated, leading to a new subset of dynamic parameters called the essential parameters. However, the identified values of the base or the essential parameters may be physically inconsistent regarding to the loss of the positive definiteness of the robot inertia matrix. Several methods have been developed in the past to verify the physical consistency of the identified parameters but they are complicated, time consuming and lead to non-optimal parameters. To overcome these drawbacks, a new method calculates a set of optimal standard dynamic parameters which are the closest to *a priori* consistent dynamic parameters obtained through CAD data given by the robot manufacturers. This is a straightforward method which is based on using the SVD and the Cholesky factorization and the linear least squares techniques.**

The new procedure is experimentally validated on an industrial 6 degrees of freedom Stäubli TX-40 robot.

I. INTRODUCTION

IENTIFICATION of robot dynamic parameters is a field that has been vastly studied in the past but for which several opened and fundamental questions still exist. One of them concerns the positive definiteness of the identified robot inertia matrix for some configurations.

Each robot link can be defined by a set of 10 inertial parameters plus 4 terms characterizing the drive chain of the joint. This set of parameters is called the *standard inertial parameters* [1]. The *base parameters* set is defined to be a minimum set of inertial parameters that are used for calculating the joint torque uniquely; they constitute also the dynamic identifiable parameters [2]–[4]. Some base parameters may almost be too small or are poorly excited to have a significant contribution to the joint torque/force. They are poorly identified and cancelled to keep a set of *essential parameters* of a simplified dynamic model without loss of joint torque/force model accuracy [5].

The authors of [6] noted that some sets of values of the base or essential parameters for a manipulator are physically impossible due to measurement noise: they determine the inertia matrix not to be positive definite for some configurations of the manipulator. Therefore it has been proposed in [7] a method for finding a set of virtual standard inertial parameters that can be related to the base or essential parameters and that guaranty the positive definiteness of the inertia matrix. However, this method is based on a trial and error algorithm and is quite complicated and time-consuming.

In our previous work [8] and in other works [9]–[11] the approaches are based on adding constraints to the system so that the inertia matrices are positive definite, the masses are positive and eventually that the center of mass are located into a convex hull that represents the segment. Because of the added constraints, these results are not optimal because the identified standard parameters do not minimize the norm of the model error.

All the previous works do not use *a priori* values of the standard parameters. However, it is now easy for robot manufacturers to get good *a priori* values of the robot dynamic standard parameters from their *CAD* data. This information should be taken as an advantage for finding a set of updated standard parameters as close as possible to the *a priori* parameter values, and corresponding to the actual robot parameters, taking into account the actual behaviour and the actual data of each robot which are not included in the nominal *CAD* data.

A method to calibrate the standard parameters with respect to *a priori* known values using the Singular Value Decomposition (*SVD*) of the regressor matrix is proposed. The obtained solution minimizes the residual norm error, thus it is one of the possible best solutions. It is also shown that if the *a priori* value of the parameters is physically consistent and well chosen, and if the measurement errors are small enough, then the calibrated parameters are physically consistent.

The result of the study on physical consistency of the set of base-parameter values gives an important piece of information to the knowledge on dynamics. It can be used for analyzing the integrity of a robot after a shock with its environment. Also it can be used to check if a set of base-parameter values, which are obtained through parameter identification or other methods, is physically consistent or not and to modify the set of base parameter values that was judged to be physically impossible. Hence, the result directly contributes to model-based control or motion simulation for

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manipulators, where the use of a dynamic model with non-positive inertia matrix leads to an unstable system.

II. The Inverse Dynamic Identification Model

The inverse dynamic model (*IDM*) of a rigid robot composed of n moving links calculates the (n×1) motor torque vector τ_{IDM}, as a function of the generalized coordinates and their derivatives. It can be obtained from the Newton-Euler or the Lagrange equations [1], [12]. It is given by the following relation:

$$\tau_{idm} = M(q)\,\ddot{q} + N(q,\dot{q}) \qquad (1)$$

where q, $\dot{q}$ and $\ddot{q}$ are respectively the (n×1) vectors of generalized joint positions, velocities and accelerations, $M(q)$ is the (n×n) robot inertia matrix, and $N(q,\dot{q})$ is the (n×1) vector of centrifugal, Coriolis, gravitational and friction forces/torques.

It is known that the dynamic model of any manipulator with n actuators can be linearly written in term of a (n×1) vector of standard parameters χ_{st} [1], [13], [14]:

$$\tau_{idm}(q,\dot{q},\ddot{q},\chi_{st}) = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} \qquad (2)$$

where:

IDM_{st} is the (n×n_{st}) jacobian matrix of τ_{idm}, with respect to the (n_{st}×1) vector χ_{st} of the standard parameters given by $\chi_{st} = \left[\chi_{st}^{1\,T}\ \chi_{st}^{2\,T}\ \ldots\ \chi_{st}^{n\,T}\right]^T$.

For rigid robots, there are 14 standard parameters by link and joint. For the joint and link j, these parameters can be regrouped into the (14×1) vector χ_{st}^{j} [1]:

$$\chi_{st}^{j} = \left[XX_j\ XY_j\ XZ_j\ YY_j\ YZ_j\ ZZ_j\ MX_j\ MY_j\ MZ_j\ M_j\ Ia_j\ Fv_j\ Fc_j\ \tau_{off_j}\right]^T \qquad (3)$$

where:

XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j are the 6 components of the inertia matrix of link j at the origin of frame j.

MX_j, MY_j, MZ_j are the 3 components of the first moment of link j,

M_j is the mass of link j,

Ia_j is a total inertia moment for rotor and gears of actuator j.

Fv_j, Fs_j are the viscous and Coulomb friction coefficients of the transmission chain, respectively,

$\tau_{off_j} = \tau_{offFS_j} + \tau_{off\tau_j}$ is an offset parameter which regroups the amplifier offset $\tau_{off\tau_j}$ and the asymmetrical Coulomb friction coefficient τ_{offFS_j}.

Because of perturbations due to measurement noise and modelling errors, the actual force/torque τ differs from τ_{idm} by an error, e, such that:

$$\tau = \tau_{idm} + e = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} + e \qquad (4)$$

where τ is calculated with the drive chain relations:

$$\tau = v_\tau\, g_\tau = \begin{bmatrix} v_\tau^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & v_\tau^n \end{bmatrix} \begin{bmatrix} g_\tau^1 \\ \vdots \\ g_\tau^n \end{bmatrix} \qquad (5)$$

v_τ is the (n×n) matrix of the actual motor current references of the current amplifiers (v_τ^j corresponds to actuator j) and g_τ is the (n×1) vector of the joint drive gains (g_τ^j corresponds to actuator j) that is given by *a priori* manufacturer's data or identified [15][16]. Equation (4) represents the Inverse Dynamic Identification Model (*IDIM*).

III. Weighted Least Squares Identification of Essential Base Parameters with QR factorization (*IDIM-WLS*)

The general identification problem after sampling and low-pass filtering (4) can be written as follows:

$$Y = W_{st}^{a}\chi_{st} + \rho_a \qquad (6)$$

Where, for r samples in total:

- Y is the (r×1) sampled vector of motor torques τ,
- W_{st}^{a} is the (r×n_{st}) sampled regressor IDM_{st},
- ρ_a is the (r×1) of errors due to measurement noise and modelling error.

The identification problem consists in finding χ_{st} that minimizes the square norm of the error ρ_a:

$$\min_{\chi_{st}} \|\rho_a\|^2 = \min_{\chi_{st}} \left\|Y - W_{st}^{a}\chi_{st}\right\|^2 \qquad (7)$$

Usually, the vector of standard parameters is not calculated directly when solving the linear problem (7) as there is a structural rank deficiency of W_{st}^{a} because the n_{st} columns of the regressor IDM_{st} are not independent: $rank(W_{st}^{a}) = n_b$ such that $n_b \le n_{st}$. Consequently, there exists infinity of solutions for χ_{st} from which only some are physically consistent: the mass positive, the inertia matrix positive definite, and the center of mass located inside the segment. It is thus common to identify the base parameters χ_b which are the minimal set of parameters that calculates the motor torque with the *IDIM* (2) and which can be identified using linear least squares .They are obtained by linear combinations of the standard parameters which depend on the choice of the independent columns in W_{st}^{a} and which can be determined for the serial robots using simple closed-form rules [2], or by numerical method based on the *QR* or *SVD* decomposition [3]–[4]. This leads to a non-unique minimal model, and a non-unique set of base parameters such that (6) becomes:

$$Y = W_b^{a}\chi_b + \rho \Rightarrow \hat{\chi}_b = W_b^{a+}Y \qquad (8)$$

with a rather small difference between $\|\rho\|$ and $\|\rho_a\|$ in (6) and where W_b^{a+} is the pseudo-inverse of W_b^{a} and $\hat{\chi}_b$ is the least squares (*LS*) solution of (8) which is computed using the *QR* factorization of W_b^{a}.

Standard deviations $\sigma_{\hat{\chi}_i}$ are estimated assuming that W_b^{a} is a deterministic matrix and ρ_a is a zero-mean additive independent Gaussian noise, with a covariance matrix $C_{\rho\rho}$, such that:

$$C_{\rho\rho} = E(\rho_a\rho_a^T) = \sigma_\rho^2 I_r \qquad (9)$$

E is the expectation operator and I_r, (r×r) identity matrix. An unbiased estimation of the standard deviation σ_ρ is:

$$\hat{\sigma}_\rho^2 = \left\| Y - W_b^a \hat{\chi}_b \right\|^2 \Big/ (r - n_b) \quad (10)$$

The covariance matrix of the estimation error is given by:

$$C_{\hat{\chi}\hat{\chi}} = E[(\chi_b - \hat{\chi}_b)(\chi_b - \hat{\chi}_b)^{\mathrm{T}}] = \hat{\sigma}_\rho^2 (W_b^{aT} W_b^a)^{-1} \quad (11)$$

$\sigma_{\hat{\chi}_i}^2 = C_{\hat{\chi}\hat{\chi}}(i,i)$ is the i^{th} diagonal coefficient of $C_{\hat{\chi}\hat{\chi}}$

The relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is given by:

$$\%\sigma_{\hat{\chi}_{ri}} = \sigma_{\hat{\chi}_i} / \left|\hat{\chi}_i\right| \ \text{ for } \left|\hat{\chi}_i\right| \neq 0 \ (i\text{-th coefficient of } \hat{\chi}_b) \quad (12)$$

The ordinary LS can be improved by taking into account different standard deviations on joint j equations errors [17]. Data in Y and W_b^a of (8) are sorted and weighted with the inverse of the standard deviation of the error calculated from ordinary LS solution of the equations of joint j [17].

Some small parameters remain poorly identifiable because they have no significant contribution in the joint torques. These parameters have no significant estimations and can be cancelled in order to simplify the dynamic model. Thus parameters such that the relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is too high are cancelled to keep a set of essential parameters χ_e of a simplified dynamic model with a good accuracy [5]. The essential parameters are calculated using an iterative procedure starting from the base parameters estimation. At each step the base parameter which has the largest relative standard deviation is cancelled.

A new *LS* parameter estimation of the simplified model is carried out with new relative error standard deviation $\%\sigma_{\hat{\chi}_{ri}}$.

The procedure ends when $max(\%\sigma_{\hat{\chi}_{ri}}) / min(\%\sigma_{\hat{\chi}_{ri}}) < r_\sigma$, where r_σ is a ratio ideally chosen between 10 and 30 depending on the level of perturbation in Y and W_{st}^a.

IV. Standard Essential Consistent Parameters Identification with SVD Factorization

A. Standard parameters identification with SVD

A solution of a linear over-determined system, such as the identification model (6), can be obtained using the *SVD*. As the standard identification model is considered, from the n_{st} columns of W_{st}^a a distinction is made between the n_b independent columns and the others. Thus the following decomposition is obtained:

$$W_{st}^a V_a = U_a \Sigma_a \,,\ \Sigma_a = \begin{bmatrix} \Sigma_1^a & 0_{n_b, n_{st}-n_b} \\ 0_{n_{st}-n_b, n_b} & \Sigma_2^a \end{bmatrix} \quad (13)$$

where:

$V_a \in R^{n_{st} \times n_{st}}, V_a = \left[V_1^a, V_2^a\right]$ is a matrix composed of two submatrices V_1^a and V_2^a of respective dimensions $n_{st} \times n_b$ and $n_{st} \times (n_{st} - n_b)$

$U_a \in R^{r \times n_{st}}, U_a = \left[U_1^a, U_2^a\right]$ is a matrix composed of two submatrices U_1^a and U_2^a of respective dimensions $r \times n_b$ and $r \times (n_{st} - n_b)$

$\Sigma_a \in R^{n_{st} \times n_{st}}$ is a diagonal matrix composed of the singular values of W_{st} sorted in decreasing order; Σ_a is decomposed into two submatrices Σ_1^a and Σ_2^a of respective dimensions $n_b \times n_b$ and $(n_{st} - n_b) \times (n_{st} - n_b)$.

In the ideal case, i.e. without noise and perturbations on the data, W_{st}^a must be rank-deficient and Σ_2^a a zero matrix. However, with measured data, this is not the case but the values of Σ_2^a are very small and can be set to zero. The system (13) thus becomes

$$W_{st} V = U \begin{bmatrix} \Sigma_1 & 0_{n_b, (n_{st}-n_b)} \\ 0_{(n_{st}-n_b), n_b} & 0_{(n_{st}-n_b), (n_{st}-n_b)} \end{bmatrix}, \quad (14)$$

where W_{st} is the rank deficient matrix closest to W_{st}^a with respect to the Frobenius norm and is given by [16]:

$$W_{st} = W_{st}^a - \sum_{k=n_b+1}^{n_{st}} s_k U_k^a V_k^{aT} \ , \quad (15)$$

with s_k is the k-th value on the diagonal of Σ_a and U_k^a (V_k^a, resp.) the k-th column of U_a (V_a, resp.) corresponding to s_k, and

$V \in R^{n_{st} \times n_{st}}, V = \left[V_1, V_2\right]$ is a matrix composed of two submatrices V_1 and V_2 of respective dimensions $n_{st} \times n_b$ and $n_{st} \times (n_{st} - n_b)$

$U \in R^{r \times n_{st}}, U = \left[U_1, U_2\right]$ is a matrix composed of two submatrices U_1 and U_2 of respective dimensions $r \times n_b$ and $r \times (n_{st} - n_b)$.

The rank-deficient system closest to the actual one (8) is thus described by:

$$Y = W_{st} \chi_{st} + \rho \ , \ \left\|\rho\right\| > \left\|\rho_a\right\| \quad (16)$$

with a rather small difference between $\left\|\rho\right\|$ and $\left\|\rho_a\right\|$.

By multiplying Y and W_{st} χ, respectively, on the left by U^T, the following relations are obtained:

$$U^T Y = G = \begin{bmatrix} U_1^T Y \\ U_2^T Y \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad (17)$$

$$U^T W_{st} \chi_{st} = \begin{bmatrix} \Sigma_1 & 0_{n_b, (n_{st}-n_b)} \\ 0_{(n_{st}-n_b), n_b} & 0_{(n_{st}-n_b), (n_{st}-n_b)} \end{bmatrix} \begin{bmatrix} V_1^T \chi_{st} \\ V_2^T \chi_{st} \end{bmatrix} = \begin{bmatrix} \Sigma_1 V_1^T \chi_{st} \\ 0_{(n_{st}-n_b), 1} \end{bmatrix} \quad (18)$$

Let us define vector Z as:

$$Z = V^T \chi_{st} = \begin{bmatrix} V_1^T \chi_{st} \\ V_2^T \chi_{st} \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \ \text{ or } \ \chi_{st} = V Z \,. \quad (19)$$

(18) can be rewritten as:

$$U^T W_{st} \chi_{st} = \begin{bmatrix} \Sigma_1 Z_1 \\ 0_{(n_{st}-n_b), 1} \end{bmatrix}. \quad (20)$$

As the product by U^T keeps the norm unchanged, the identification problem (16) can be expressed by the following equations:

$$\begin{aligned} \left\|\rho\right\|^2 &= \left\|Y - W_{st}\chi_{st}\right\|^2 = \left\|U^T Y - U^T W_{st}\chi_{st}\right\|^2 \\ &= \left\|G - \Sigma Z\right\|^2 = \left\|G_1 - \Sigma_1 Z_1\right\|^2 + \left\|G_2\right\|^2 \end{aligned} . \quad (21)$$

The unique solution $\hat{Z}_1$ to this problem is given by:

$$\hat{Z}_1 = \Sigma_1^{-1} G_1 = \Sigma_1^{-1} U_1^T Y \,, \quad (22)$$

and the family of all optimal solution $\hat{Z}$ is, for any Z_2

$$\hat{Z} = \begin{bmatrix} \hat{Z}_1 \\ Z_2 \end{bmatrix} \quad (23)$$

Thus, an optimal solution $\hat{\chi}_{st}$ to (21) is given by:

$$\hat{\chi}_{st} = V \hat{Z} = V_1 \Sigma_1^{-1} U_1^T Y + V_2 Z_2 \quad (24)$$

Introducing (24) into (21), it is shown that, for any optimal solution, the minimal norm of the error ρ is:

$$\|\rho\|_{min} = \|G_2\| = \|U_2^T Y\| \,. \quad (25)$$

Finally, the optimal solution $\hat{\chi}_{st}^{opt}$ that minimizes both norms of $\hat{\chi}_{st}$ and ρ at the same time is obtained for $Z_2 = 0$, i.e.:

$$\hat{\chi}_{st}^{opt} = V_1 \Sigma_1^{-1} U_1^T Y \quad (26)$$

where $V_1 \Sigma_1^{-1} U_1^T$ is the Moore-Penrose pseudo-inverse of W_{st}.

B. *Standard parameters closest to a priori values*

The minimal norm solution obtained by (26) is optimal in term of the error norm (25). However the consistency of the parameters, with respect to its physical meaning is not guaranteed. Here a new approach is proposed that takes benefits of the *a priori* values χ_{st}^{ref} of the inertial parameters calculated with *CAD* software from the manufacturers' data.

Let us denote as Y^{ref} the joint torques estimated with the *a priori* values χ_{st}^{ref}:

$$Y^{ref} = W_{st} \chi_{st}^{ref} \quad (27)$$

Substracting (27) to (6), it comes

$$Y - Y^{ref} = W_{st}(\chi_{st} - \chi_{st}^{ref}) + \rho \Leftrightarrow \Delta Y = W_{st} \Delta\chi_{st} + \rho \quad (28)$$

where the error ρ is the same as that of the system (6).

Similarly to (26), the optimal solution $\Delta\hat{\chi}_{st}^{opt}$ that minimizes the norm of $\Delta\hat{\chi}_{st}$ is given by:

$$\Delta\hat{\chi}_{st}^{opt} = V_1 \Sigma_1^{-1} U_1^T \Delta Y \quad (29)$$

which leads to

$$\hat{\chi}_{st}^{opt} = \chi_{st}^{ref} + V_1 \Sigma_1^{-1} U_1^T (Y - Y^{ref}) \,. \quad (30)$$

$\hat{\chi}_{st}^{opt}$ minimizes the norm of ρ given in (16) and the norm of $\chi_{st} - \chi_{st}^{ref}$ at the same time. $\hat{\chi}_{st}^{opt}$ is the optimal standard solution closest to a consistent solution χ_{st}^{ref}, then it is the best optimal standard solution that can keep the physical consistency of χ_{st}^{ref} if the minimal norm of $\chi_{st} - \chi_{st}^{ref}$ is small and if the measurement errors are small.

C. *Standard essential and consistent parameters*

The previous method does not take into account the fact that some parameters may almost be null and thus have no contribution to the system dynamics; or that some parameters can be identified with a very small confidence and have no significant values that lead to the loss of consistency of some standard parameters.

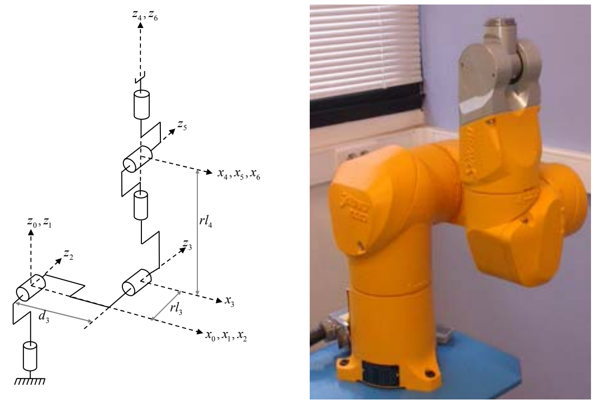

Fig. 1. Link frames of the TX-40 robot

TABLE I
GEOMETRIC PARAMETERS OF THE TX-40 ROBOT WITH THE PAYLOAD

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	q_1	0
2	0	$-\pi/2$	0	$q_2-\pi/2$	0
3	0	0	$d_3 = 0.225$ (m)	$q_3+\pi/2$	$rl_3 = 0.035$ (m)
4	0	$+\pi/2$	0	q_4	$rl_4 = 0.225$ (m)
5	0	$-\pi/2$	0	q_5	0
6	0	$+\pi/2$	0	$q_{6}+\pi$	0

To overcome this problem, let us take advantage of the correct knowledge that it is possible to have on the identified essential parameters denoted as $\hat{\chi}_e$ ($\hat{\chi}_e$ is composed of the n_e values of the essential parameters χ_e calculated with the *IDIM-WLS* method proposed in section III and of $(n_{st}-n_e)$ zeros). Weighting the matrix W_{st} in (6) by this vector leads to the new system:

$$Y = \left(W_{st}\, diag\left(\hat{\chi}_e\right)\right)\chi_{st}^e + \rho = W_{st}^e \chi_{st}^e + \rho \quad (31)$$

where χ_{st}^e is a vector of standard parameters weighted by χ_e, i.e. $\chi_{st} = diag(\hat{\chi}_e)\chi_{st}^e$.

The *SVD* of the weighted matrix W_{st}^e allows to calculate the kernel of the linear transformation defined by W_{st}^e, corresponding to the parameters with small influence on the joint torques in (31), and the image of the transformation which allows to identify the essential parameters which are significant wrt their confidence interval, adding a small increase of the norm error of ρ [5].

Thus, solving the system (31) with *SVD* and applying the previous method for the calibration of the standard parameters, the optimal solution becomes:

$$\hat{\chi}_{st}^{opt} = \chi_{st}^{ref} + diag(\hat{\chi}_e) V_{1e} \Sigma_{1e}^{-1} U_{1e}^T (Y - Y^{ref}) \,. \quad (32)$$

where U_{1e}, V_{1e} and Σ_{1e} are matrices obtained from the *SVD* of W_{st}^e, i.e.

$$W_{st}^e V_e = U_e \begin{bmatrix} \Sigma_{1e} & 0_{n_e,(n_{st}-n_e)} \\ 0_{(n_{st}-n_e),n_e} & 0_{(n_{st}-n_e),(n_{st}-n_e)} \end{bmatrix}, \quad (33)$$

where:

$V_e \in R^{n_{st} \times n_{st}}, V_e = [V_{1e}, V_{2e}]$ is a matrix composed of two submatrices V_{1e} and V_{2e} of respective dimensions $n_{st} \times n_e$ and $n_{st} \times (n_{st} - n_e)$

$U_e \in R^{r \times n_{st}}, U_e = [U_{1e}, U_{2e}]$ is a matrix composed of two submatrices U_{1e} and U_{2e} of respective dimensions $r \times n_e$ and $r \times (n_{st} - n_e)$

$\Sigma_{1e} \in R^{n_e \times n_e}$ is a diagonal matrix composed of the singular values of W_{st}^e ranked in decreasing order.

In the next section, the identification of the standard parameters of an industrial Stäubli TX-40 robot is presented. It will be shown that the best results are obtained when using the calibration that takes into account the essential parameters.

V. Case Study

A. Description of the TX 40

The Stäubli TX-40 robot (Fig. 1) has a serial structure with six rotational joints. Its kinematics is defined using the modified Denavit and Hartenberg notation (*MDH*) [19]. In this notation, the link j fixed frame is defined such that the z_j axis is taken along joint j axis and the x_j axis is along the common normal between z_j and z_{j+1} (Fig. 1). The geometric parameters defining the robot frames are given in Table I. The parameter $\sigma_j = 0$, means that joint j is rotational, α_j and d_j parameterize the angle and distance between z_{j-1} and z_j along x_{j-1}, respectively, whereas θ_j and r_j parameterize the angle and distance between x_{j-1} and x_j along z_j, respectively. Since all the joints are rotational then θ_j is the position variable q_j of joint j, except for joint two ($\theta_2 = q_2 - \pi/2$), joint three ($\theta_3 = q_3 + \pi/2$) and joint 6 ($\theta_2 = q_6 + \pi$), as shown in Table 1.

The TX-40 robot is characterized by a coupling between the joints 5 and 6 such that:

$$\begin{bmatrix} \dot{q}r_5 \\ \dot{q}r_6 \end{bmatrix} = \begin{bmatrix} N_5 = 45 & 0 \\ N_6 = 32 & N_6 = 32 \end{bmatrix} \begin{bmatrix} \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}, \begin{bmatrix} \tau_{c_5} \\ \tau_{c_6} \end{bmatrix} = \begin{bmatrix} N_5 & N_6 \\ 0 & N_6 \end{bmatrix} \begin{bmatrix} \tau_{r_5} \\ \tau_{r_6} \end{bmatrix} \quad (34)$$

where $\dot{q}r_j$ is the velocity of the rotor of motor j, $\dot{q}_j$ is the velocity of joint j, N_j is the transmission gain ratio of axis j, τ_{cj} is the motor torque of joint j, taking into account the coupling effect on the motor side, τ_{rj} is the electro-magnetic torque of motor j. The coupling between joints 5 and 6 also adds the effect of the inertia of rotor 6 and new viscous and Coulomb friction parameters Fvm_6 and Fcm_6, to both τ_{c5} and τ_{c6}.

It is possible to write:

$\tau_{c_5} = \tau_5 + Ia_6\,\ddot{q}_6 + Fvm_6\,\dot{q}_6 + Fcm_6\,\mathrm{sign}(\dot{q}_6)$ and

$\tau_{c_6} = \tau_6 + Ia_6\,\ddot{q}_5 + Fvm_6\,\dot{q}_5 + Fcm_6\left(\mathrm{sign}(\dot{q}_5 + \dot{q}_6) - \mathrm{sign}(\dot{q}_6)\right)$

where τ_j already contains the terms $(Ia_j\ddot{q}_j + Fv_j\dot{q}_j + Fc_j sign(\dot{q}_j))$, for j=5 and 6 respectively,

with $Ia_5 = N_5^2 Ja_5 + N_6^2 Ja_6$ and $Ia_6 = N_6^2 Ja_6$ (35)

Ja_j is the moment of inertia of rotor j.

(35) is introduced into (4) to obtain the *IDIM*

B. Identification results

In this section are presented the experimental results. As it is a calibration procedure, the choice of the *a priori* value χ_{st}^{ref} is crucial. However, in the manufacturer's datasheets, the friction parameters and the drive chain inertia Ia_j taking into account the gear box inertias are not given.

These values are extracted from a first identification of the dynamic parameters using the *IDIM-WLS* procedure described section III. They are given in bold font in Table II, with the *a priori* parameter values χ_{st}^{ref}. The TX40 has n_{st}=86 standard parameters, n_b=61 base parameters and n_e=31 essential parameters.

The standard parameters are calibrated using the approach presented above. The path of the trajectory used for identification consists of 11 intermediate points. The trajectory between the points is carried out using the trapezoidal acceleration interpolation function of the controller CS8C of the Stäubli robots. In Table II, the parameters $\hat{\chi}_{st}^{0b}$ are those computed using the matrix W_{st} (16) defined with the n_b=61 independent columns of the base parameters and the parameters $\hat{\chi}_{st}^{0e}$ are those calculated using the matrix W_{st}^e (31) defined with the n_e=31 independent columns of the essential parameters. The difference with respect to the *a priori* value is also shown. It can be clearly observed that the difference between the *a priori* parameters and those estimated using the essential parameters is smaller. In Fig. 2 are also plotted the joint torques calculated with (5) from the measure of the current reference and with the *IDIM* (2) computed with the parameters $\hat{\chi}_{st}^{0e}$. It should be mentioned that another trajectory is used for plotting these figures, i.e. the identification results are *cross-validated*. It can be observed that the joint torques are well estimated.

Let us now verify the physical consistency of the identified parameters. The identified parameters are computed at the joint centre position of each link. They are physically consistent if the identified mass is positive and the inertia matrix written at the center of mass (CoM) of each link is positive definite. We use the Huygens theorem matrix transformation formula to compute the inertia matrix J_j at the CoM, from the identified parameters according to:

$$J_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix} - \frac{1}{M_j} \begin{bmatrix} MY_j^2 + MZ_j^2 & -MX_jMY_j & -MX_jMZ_j \\ -MX_jMY_j & MX_j^2 + MZ_j^2 & -MY_jMZ_j \\ -MX_jMZ_j & -MY_jMZ_j & MX_j^2 + MY_j^2 \end{bmatrix} \quad (36)$$

The positive definitiveness of J_j can be tested either with eigenvalue decomposition, with the Sylvester theorem, or a Cholesky decomposition. Each method is equivalent; however, as noted in [7], the Sylvester theorem allows us to find conditions that the parameters must verify to obtain the positive definitiveness. In the case of a failed test, these conditions make it possible to adjust the parameters to obtain a positive definite matrix by modifying the inertial parameters

TABLE II

IDENTIFIED STANDARD DYNAMIC PARAMETERS OF THE TX-40.

Param.	χ_{st}^{ref}	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$	e_{bi}	e_{ei}	Param.	χ_{st}^{ref}	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$	e_{bi}	e_{ei}
ZZ_1	3,92e-02	1,24e+00	3,15e-01	*1,20*	*0,28*	MX_4	1,45e-02	-3,26e-02	-1,27e-02	0,05	0,03
Ia_1	**3,62e-01**	3,62e-01	3,62e-01	*0,00*	*0,00*	MY_4	-7,24e-03	-7,55e-03	-7,24e-03	0,00	0,00
Fv_1	**7,96e+00**	7,96e+00	7,93e+00	*0,00*	*0,03*	MZ_4	-5,86e-01	-5,86e-01	-5,86e-01	0,00	0,00
Fs_1	**6,79e+00**	6,81e+00	6,86e+00	*0,02*	*0,07*	M_4	3,62e+00	3,62e+00	3,62e+00	0,00	0,00
τ_{off1}	**0,00e+00**	3,14e-01	3,14e-01	*0,31*	*0,31*	Ia_4	**3,41e-02**	3,15e-02	3,03e-02	0,00	0,00
XX_2	1,63e-02	-4,71e-01	2,71e-02	*0,49*	*0,01*	Fv_4	**1,06e+00**	1,07e+00	1,07e+00	0,01	0,01
XY_2	7,85e-04	1,03e-02	6,19e-03	*0,01*	*0,01*	Fs_4	**2,72e+00**	2,60e+00	2,67e+00	0,12	0,05
XZ_2	-1,57e-02	-1,49e-01	-4,05e-02	*0,13*	*0,02*	τ_{off4}	**0,00e+00**	-6,06e-02	0,00e+00	0,06	0,00
YY_2	8,81e-02	8,81e-02	8,81e-02	*0,00*	*0,00*	XX_5	1,01e-03	1,92e-03	1,01e-03	0,00	0,00
YZ_2	3,24e-04	5,56e-03	3,24e-04	*0,01*	*0,00*	XY_5	0,00e+00	-9,30e-04	0,00e+00	0,00	0,00
ZZ_2	8,28e-02	1,08e+00	1,31e-01	*1,00*	*0,05*	XZ_5	0,00e+00	-2,37e-03	-1,58e-04	0,00	0,00
MX_2	3,92e-01	2,14e+00	9,59e-02	*1,75*	*0,30*	YY_5	1,00e-03	1,00e-03	1,00e-03	0,00	0,00
MY_2	-7,20e-03	1,05e-01	6,79e-02	*0,11*	*0,08*	YZ_5	-3,06e-06	6,73e-05	-3,06e-06	0,00	0,00
MZ_2	1,62e-01	1,62e-01	1,62e-01	*0,00*	*0,00*	ZZ_5	1,01e-03	3,93e-03	1,01e-03	0,00	0,00
Ia_2	**5,07e-01**	5,07e-01	5,07e-01	*0,00*	*0,00*	MX_5	0,00e+00	8,24e-03	0,00e+00	0,01	0,00
Fv_2	**5,92e+00**	5,93e+00	5,92e+00	*0,01*	*0,00*	MY_5	-3,06e-03	-1,05e-02	2,39e-03	0,01	0,01
Fs_2	**7,38e+00**	7,47e+00	7,42e+00	*0,09*	*0,04*	MZ_5	-1,02e-03	-1,02e-03	-1,02e-03	0,00	0,00
τ_{off2}	**0,00e+00**	8,60e-01	0,00e+00	*0,86*	*0,00*	M_5	1,02e+00	1,02e+00	1,02e+00	0,00	0,00
XX_3	2,23e-02	1,30e-01	6,91e-02	*0,11*	*0,05*	Ia_5	**3,61e-02**	3,22e-02	3,46e-02	0,00	0,00
XY_3	-1,95e-04	-6,97e-03	-1,95e-04	*0,01*	*0,00*	Fv_5	**1,24e+00**	1,24e+00	1,24e+00	0,00	0,00
XZ_3	-1,16e-02	2,20e-03	-1,16e-02	*0,01*	*0,00*	Fs_5	**2,62e+00**	2,60e+00	2,62e+00	0,02	0,00
YY_3	2,24e-02	2,24e-02	2,24e-02	*0,00*	*0,00*	τ_{off5}	**0,00e+00**	1,05e-01	0,00e+00	0,11	0,00
YZ_3	-2,22e-03	4,53e-03	-2,22e-03	*0,01*	*0,00*	XX_6	3,53e-04	4,71e-04	3,53e-04	0,00	0,00
ZZ_3	4,41e-03	1,08e-01	4,20e-02	*0,10*	*0,04*	XY_6	0,00e+00	8,46e-04	0,00e+00	0,00	0,00
MX_3	3,26e-02	8,41e-02	3,08e-02	*0,05*	*0,00*	XZ_6	0,00e+00	3,53e-04	0,00e+00	0,00	0,00
MY_3	2,44e-02	-6,31e-01	-1,15e-01	*0,66*	*0,14*	YY_6	3,53e-04	3,53e-04	3,53e-04	0,00	0,00
MZ_3	2,65e-01	2,65e-01	2,65e-01	*0,00*	*0,00*	YZ_6	0,00e+00	-4,41e-04	0,00e+00	0,00	0,00
M_3	4,07e+00	4,07e+00	4,07e+00	*0,00*	*0,00*	ZZ_6	0,00e+00	7,04e-04	0,00e+00	0,00	0,00
Ia_3	**8,29e-02**	1,02e-01	9,14e-02	*0,02*	*0,01*	MX_6	0,00e+00	1,07e-03	0,00e+00	0,00	0,00
Fv_3	**1,98e+00**	1,99e+00	2,01e+00	*0,01*	*0,03*	MY_6	0,00e+00	-3,71e-03	0,00e+00	0,00	0,00
Fs_3	**6,43e+00**	6,41e+00	6,37e+00	*0,02*	*0,06*	MZ_6	8,40e-03	8,40e-03	8,40e-03	0,00	0,00
τ_{off3}	**0,00e+00**	4,48e-01	0,00e+00	*0,45*	*0,00*	M_6	2,00e-01	2,00e-01	2,00e-01	0,00	0,00
XX_4	1,09e-01	5,60e-03	1,09e-01	*0,10*	*0,00*	Ia_6	**1,14e-02**	1,10e-02	1,12e-02	0,00	0,00
XY_4	2,90e-05	-3,66e-03	2,90e-05	*0,00*	*0,00*	Fv_6	**6,94e-01**	6,40e-01	6,37e-01	0,05	0,06
XZ_4	1,35e-03	-2,60e-03	1,35e-03	*0,00*	*0,00*	Fs_6	**0,00e+00**	4,20e-01	4,08e-01	0,42	0,41
YY_4	1,08e-01	1,08e-01	1,08e-01	*0,00*	*0,00*	τ_{off6}	**0,00e+00**	1,88e-01	1,74e-01	0,19	0,17
YZ_4	-1,17e-03	-6,64e-03	-1,17e-03	*0,01*	*0,00*	Fvm_6	**5,92e-01**	6,04e-01	5,98e-01	0,01	0,01
ZZ_4	4,07e-03	3,78e-03	4,07e-03	*0,00*	*0,00*	Fsm_6	**1,88e+00**	1,78e+00	1,81e+00	0,10	0,07

$e_{bi} = \left|\hat{\chi}_{st}^{0b} - \chi_{st}^{ref}\right|$, $e_{ei} = \left|\hat{\chi}_{st}^{0e} - \chi_{st}^{ref}\right|$. $norm(e_{bi})/norm(\chi_{st}^{ref})$=0.1591. $norm(e_{ei})/norm(\chi_{st}^{ref})$=0.0416.

that are in the null-space of the regressor, i.e. the non-base parameters.

The parameters are not independent, thus modifying one parameter results in the modification of all the non-base parameters and manipulations need precautions.

The Cholesky decomposition presents the advantage that a tolerance $\varepsilon \leq 0$ can be set in the algorithm and allows for taking into account noise and measurement error, which in the case of experimental data is of importance. It is similar to setting the tolerance that defines a numerical rank in the *SVD* or *QR* decomposition. The tolerance is chosen according to the error and the level of noise in the collected data. Results on the positiveness of inertia matrices using the Cholesky decomposition are shown in Table III. The parameters obtained with the base parameters $\hat{\chi}_{st}^{0b}$ need a tolerance $|\varepsilon| \geq 0.04$ to obtain definitive positive matrices for all the links, while the use of essential parameters needs only the zero tolerance.

TABLE III

TOLERANCE OF THE CHOLESKY FACTORIZATION AND NUMBER OF POSITIVE DEFINITE INERTIA MATRICES IN THE DIFFERENT CASES

Tolerance	$\hat{\chi}_{st}^{0b}$	$\hat{\chi}_{st}^{0e}$
0 strict	2	6
-0.01	3	6
-0.02	5	6
-0.04	6	6

VI. CONCLUSION

A new method for computing a set of standard essential and consistent dynamic parameters closest to *a priori CAD* values, using *SVD* factorization and LS techniques, was presented. This method was experimentally validated on an industrial Stäubli TX-40 robot and give extremely conclusive results. The positiveness of inertia matrices using the

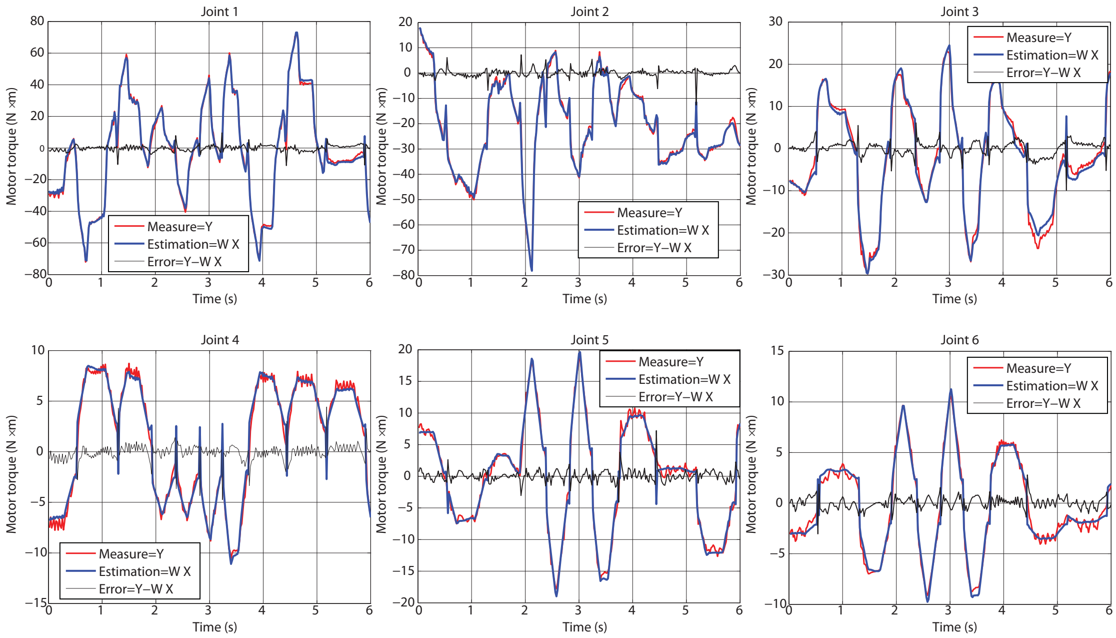

Fig. 2. Motor torques (joint side units) estimated from the measure of the current reference (red) and with *IDIM* (blue) of the TX-40.

Cholesky decomposition have shown that the standard parameters identified on the space spanned by the n_e columns of W_{st}^{e} corresponding to the essential parameters and closest to *a priori* consistent values, are consistent for all the links with a zero Cholesky tolerance. This is a strong result, which means that the essential parameters, which have significant identified values with respect to their small standard deviation (depending on measurement and modelling errors), are consistent because they lead to identify a set of standard essential consistent parameters. The base parameters which are not well identified are inconsistent because they lead to inconsistent standard parameters.

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