

# Specification and verification of embedded systems with Event B

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## Outline

### Plan

- 1 Introduction
- 2 Modelling with Event-B
- 3 Examples - Case studies
- 4 Case study: readers-writers



## Event-B: Some References

- *Modelling in Event-B: System and Software Engineering*, J-R. Abrial, Cambridge, 2010
- *Modelling and proof of a Tree-structured File System*. Damchoom, Kriangsak and Butler, Michael and Abrial, Jean-Raymond, 2008.
- *Applying Event and Machine Decomposition to a Flash-Based Filestore in Event-B*. Damchoom, Kriangsak and Butler, Michael; 2009.
- *Faultless Systems: Yes We Can!*, Jean-Raymond Abrial, 2009
- *Modelling an Aircraft Landing System in Event-B*, Dominique Méry, Neeraj Kumar Singh, 2014
- *Closed-Loop Modelling of Cardiac Pacemaker and Heart*, Dominique Méry, Neeraj Kumar Singh, 2012

## Event B Specification Approach

**Correct-by-construction**: build correctly the systems  
(abstraction, modelling, refinement, composition/decomposition, proof)

Some hints to formal methods:

- Formal methods are **rigorous engineering tools**.
- Formal methods are **means to build** executable code from software requirement documents (informal, natural language).
- **Requirement Documents** (provided by clients) **should be rewritten** after analysis and understanding into **Reference Document** (where every thing is made clear and properly labelled for traceability).

## B Method and Event B

- Event-B is an **extension of the B-method** (J-R. Abrial).
- It is devoted
  - for **system engineering** (both hardware and software), top-down approach
  - for **specifying and reasoning about complex systems** : concurrent and reactive systems.
- Event-B comes with a new modelling framework called **Rodin**. (like **Atelier B** tool for the classical B)
- The **Rodin platform** is an Eclipse-based open and extensible tool for B model specification and verification.  
It integrates various plugins: **B Model editors**, **proof-obligation generator**, **provers**, **model-checkers**, **UML transformers**, etc

## Event B Modelling and its dissemination

Yet **used in various case studies and industrial projects**:

- Train signalling system
- Mechanical press system
- Access control system
- Air traffic information system
- Filestore system
- Distributed programs
- Sequential programs
- Cardiac Pacemaker
- etc

## Event B Modelling: principles

**Observe** the behaviour of any system; what matters?

- A **set of changes** of its states.
- But, the **observation distance** does matter!  
(the details may be observed or not: parachutist paradigm)
- The **observation focus** does matter!  
(the observed changes are not the same)
- Different points of view = **several abstractions**.

## Remind B Specification Approach

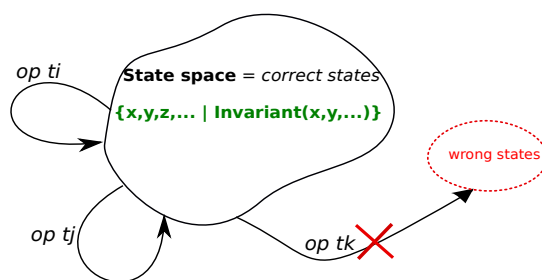


Figure: Do it right with B

<p><b>VARIABLES</b></p> <p><math>x, y, z, \dots</math></p> <p><b>INARIANT</b></p> <p><math>Inv(x, y, z, \dots)</math></p> <p><b>OPERATIONS</b></p> <p><math>t_i = \dots</math></p> <p><math>t_j = \dots</math></p> <p><math>t_k = \dots</math></p>
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## B Method: general development approach

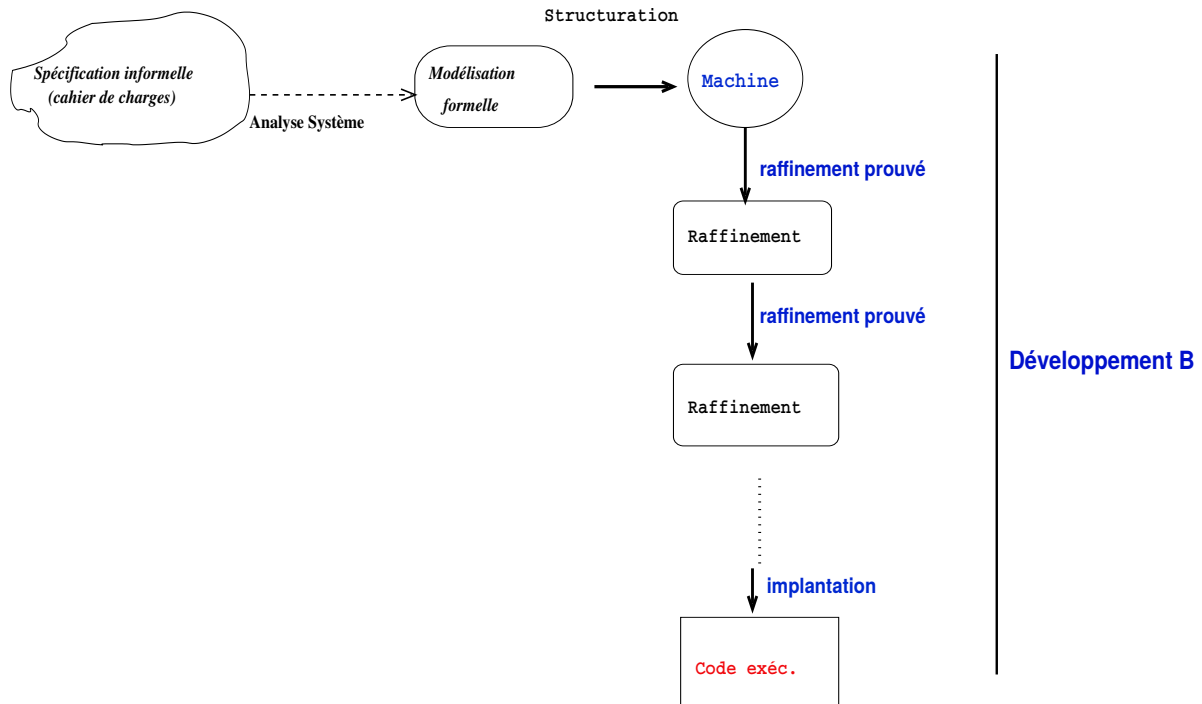


Figure: Development process with B

## Event B Specification Approach

Event B Specification: start with **Abstract system** or Abstract model

An **abstract system** is a mathematical model of an **asynchronous system behaviour**

System behaviour: described by **events** which are observed!

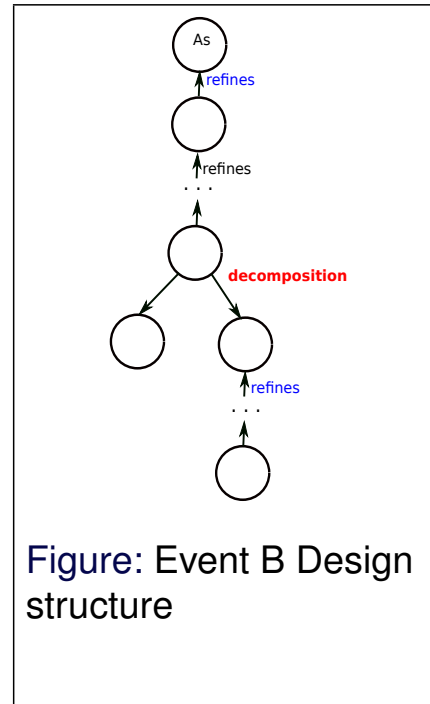
**Events are guarded actions/substitutions**

Event occurrences involve a State-transition model.

A **system model** is a state-based model equipped with events

## Event B Development Structuring

- Start with an Abstract system (or abstract model)
- **Refinement** of data and events  
The parachutist paradigm / microscope paradigm (JR Abrial)
- **Decomposition** (of a system into sub-systems, Hw, Sw)



## B Abstract System

Variables

Predicate

Events

```

SYSTEM
SETS ...
VARIABLES
...
INVARIANT
... predicate
INITIALISATION
...
EVENTS
...
END

```

but structured more efficiently using **Contexts** and **machines**.

## Remind! Capturing the correct state space and events

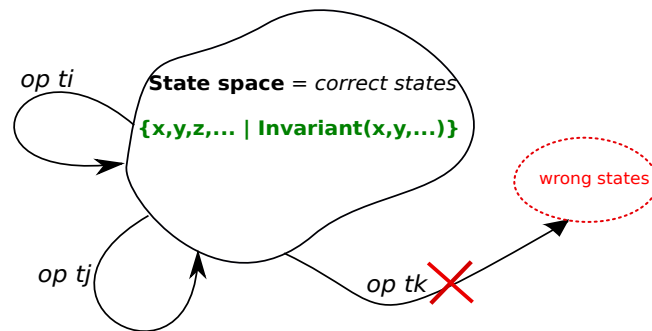


Figure: Events should preserve correct states

## Capturing a system behaviour - Events

- The **behaviour** of a discrete system is a **sequence of changes** (system transitions).
- The changes may be **internal** or enabled by **external** signals.
- Each **event describes the occurrence of a change** in the discrete system under modelisation.
- Event B uses **Guards** and **Actions** [Dijkstra]

event = when **Conditions** then **Effects**

- But, the behaviour of a system may/should be captured gradually.

## Formal Description of Events

An event has one of the following general forms (Fig. 5)

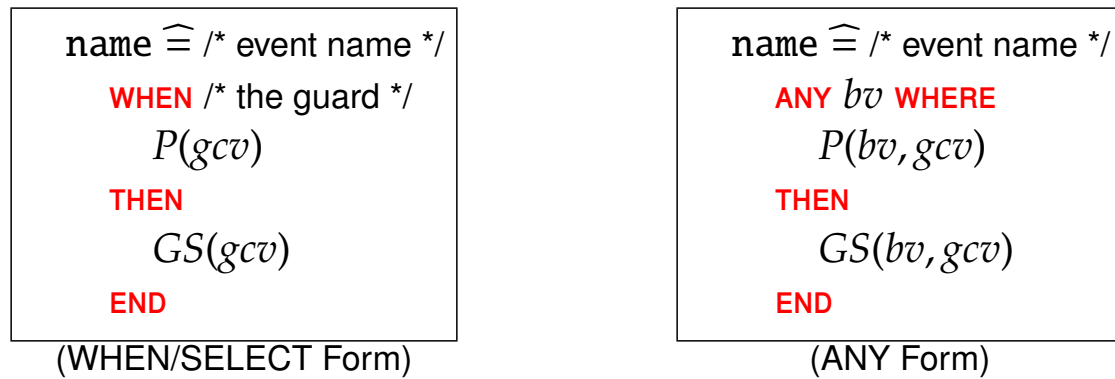
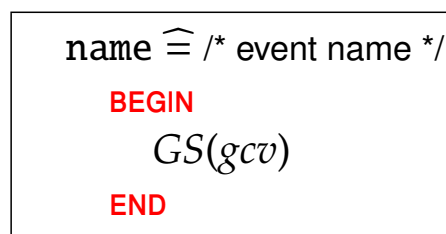


Figure: General forms of events

$gcv$  denotes the global constants and variables of the abstract;  
 $bv$  denotes the local bound variables of the event;  
 $P(bv, gcv)$  a predicate.

## Formal Description of Events

An event without guard has the following form:





## Abstract System (or a model, or a machine)

- The **guard** of an event with the WHEN form is:  $P(gcv)$ .
- The **guard** of an event with the ANY form is:  $\exists(bv).P(bv, gcv)$ .
- The WHEN form is a particular case of the other.
- The action associated to an event is modeled with a **generalized substitution** using the variables accessible to the event:  $GS(bv, gcv)$ .

## Abstract System : Semantics and Consistency

An abstract system describes a mathematical model that simulates the behaviour of a system.

Its **semantics** arises from the **invariant** and is ensured by **proof obligations** (PO).

The consistency of the model is established by such proof obligations.

### Consistency of an event B model

- PO: the initialisation establishes the invariant
- PO: each event of the abstract system preserves the invariant of the model

$I(gcv)$  the invariant and  $GS(bv, gcv)$  the generalized substitution modelling the event action.

## Abstract System : Semantics and Consistency

- the **initialisation** establishes the invariant;

$$[U]Inv$$

- each event preserves the invariant:**

In the case of an event with the ANY form, the proof obligation is:

$$I(gcv) \wedge P(bv, gcv) \wedge \text{prd}_v(S_e) \Rightarrow [GS(bv, gcv)]I(gcv)$$

Moreover the events (e) terminate:

$$I(Gcv) \wedge eGuard \Rightarrow \text{fis}(eBody)$$

(note  $eBody = S_e$ )

## Abstract System : Semantics and Consistency

The predicate  $\text{fis}(S)$  expresses that  $S$  does not establish *False*:

$$\text{fis}(S) \Leftrightarrow \neg [S]False$$

ie

$$I(Gcv) \wedge eGuard \Rightarrow \neg [S]False$$

The predicate  $\text{prd}_v(S)$  is the *before-after predicate* of the substitution  $S$ ; it relates the values of state variables just before ( $v$ ) and just after ( $v'$ ) the substitution  $S$ , also written  $BA_e(v, v')$ .

The  $\text{prd}_v(\text{ANY } x \text{ WHERE } P(x, v) \text{ THEN } v := S(x, v) \text{ END})$  is:

$$\exists x.(P(x, v) \wedge v' = S(x, v))$$

## Example: producer/consumer

Features: Concurrency and synchronization

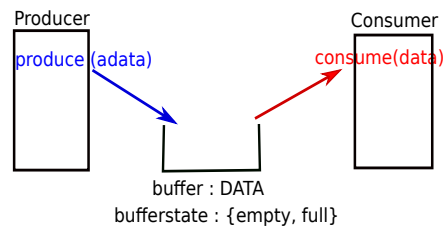


Figure: An overview of a producer-consumer

- Concurrent running of a process **consumer** which retrieves a data from a buffer filled by another process **producer**.
- The consumer cannot retrieve an empty buffer and the producer cannot fill in a buffer already full.

An event-driven model of the system is as follows:



## Example : producer/consumer

```

Machine ProdCons /* the abstract model */
sets
  DATA ;   STATE = {empty, full}
variables   buffer, bufferstate, bufferc
invariants
  bufferstate ∈ STATE ∧ buffer ∈ DATA ∧ bufferc ∈ DATA
initialization
  bufferstate := empty || buffer := DATA || bufferc := DATA
events
  produce ≙ /* if buffer empty */
    any dd where   dd ∈ DATA ∧ bufferstate = empty
    then   buffer := dd || bufferstate := full
    end ;
  consume ≙ /* if buffer is full */
    select   bufferstate = full
    then   bufferc := buffer || bufferstate := empty
    end
end

```

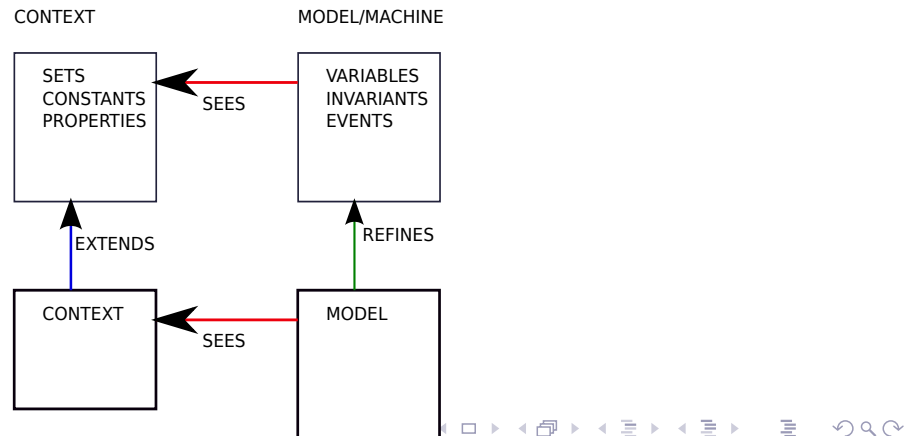


Figure: A Producer-Consumer Abstract System

## Structuring Event-B Models

An event-B model is structured with

- **Contexts** that contain carrier **sets**, **axioms** and **theorems** (seen by various machines)
- **Machines** which see the contexts and define a **state space** (**static part: variables + labelled invariants**) and a dynamic part made of some events.
- A **context may be extended**; a **machine may be refined**.



## Refinement: principles

- Data refinement  
(as usually: new variables + properties; binding invariant)
- Event Refinement (**extended**):
  - **Strengthening guards** (unlike with Classical B)  
More variables are introduced with their properties.
  - **Each event of the concrete system refines an event of the abstraction.**
  - Introduction of **new events** which refine **skip**, and use new variables.

## Refinement: principles

Let A with **Invariant:  $I(av)$**

```

evta ≡ /* Abs. ev. */
  when P(av)
  then GS(av)
  end

```

avec  $\text{prd}_v(\dots) = \text{Ba}(av, av')$

Refined with: **Invariant  $J(av, cv)$**

```

evtr ≡ /* Conc. ev. */
  when Q(cv)
  then GS(cv)
  end

```

avec  $\text{prd}_v(\dots) = \text{Bc}(cv, cv')$

Proof obligation:

$$I(av) \wedge J(av, cv) \wedge Q(cv) \wedge Bc(cv, cv') \Rightarrow \exists cv'. (Ba(av, av') \wedge J(av', cv'))$$

## Event B Tools

- First generation tools
  - Translation into classical B
  - [B4free](#), [Click'n'Prove](#)
- New generation tools: DataBase, Eclipse Plugins, ...
  - [Rodin](#) (From sveral EU Projects: Matisse, Deploy, etc)

## Refinement: structuring models

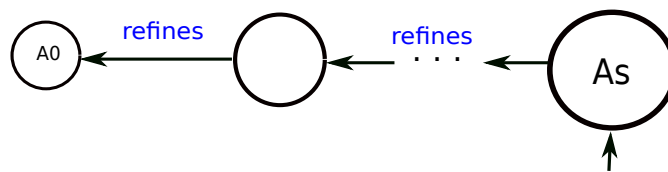
**Refinement= development technique:** various refinement strategies.

### Horizontal refinement (feature augmentation)

From a small and abstract to a larger abstract model.

Details are gradually introduced in an abstract model in order to make it more precise

(wrt to requirements).



## Refinement: structuring models

### Vertical refinement: From abstrat to more concrete models

Details are gradually introduced in an abstract model

The specifier introduces new variables and makes some choices

Events may be split : **event decomposition**

machines may be split too: **machine decomposition**

## Vertical Refinement: machine decomposition

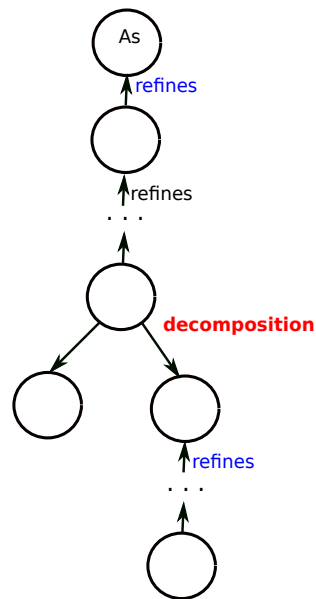


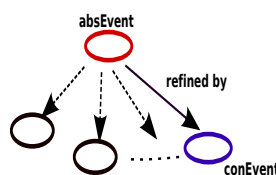
Figure: Vertical refinement with machine decomposition

## Vertical Refinement: event decomposition

A **coarse grain event** is analysed and described in a more detailed (**fine grain**) way.

Think about the transfer of a file via a network.

- A **given change** consists of:
  - start by **sub-change...**;
  - follow by **sub-change...**;
  - end by **sub-change...**;
- Hence, **at least one sub-change (an event), refines the abstract event.**



## Machine Decomposition: structuring models

A coarse grain model is analysed and described in a more detailed (fine grain) way.

Think about a system involving **software** and **physical devices**.

- A given model is made of **variables that model purely physical devices**, and events are associated only to these variables
- The splitting is based on **variables splitting** (but not always straightforward).
- Divide and conquer: a small model is more tractable than a huge one.

Decomposition enables one to break complexity, to structure and develop more easily.

## Machine Decomposition: structuring models

Machine variables and events are partitioned into sub-machines.

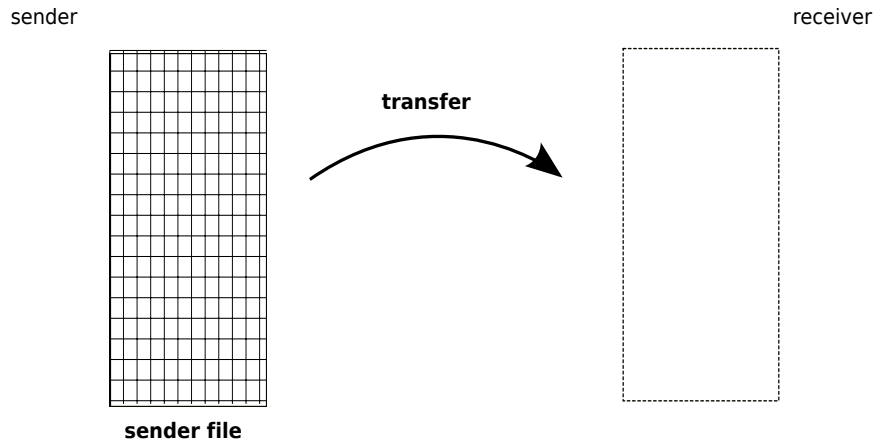
- **Decomposition with Abrial's style (shared variables)**: the sub-machines may interact with each other via **shared variables**. Shared variables are duplicated, new external-events are introduced in each machine that has a shared variable in order to ensure consistency of changes.
- **Decomposition with Butler's style**: the **variables are not shared**; an event which uses variables in separate machines, is shared (then separated-duplicated).  
The sub-machines may interact with each other via **synchronisation over shared parameterised events**.

*Event-B Model Decomposition*, C. Pascal(Systerel), R. Silva(Univ. of Southampton)

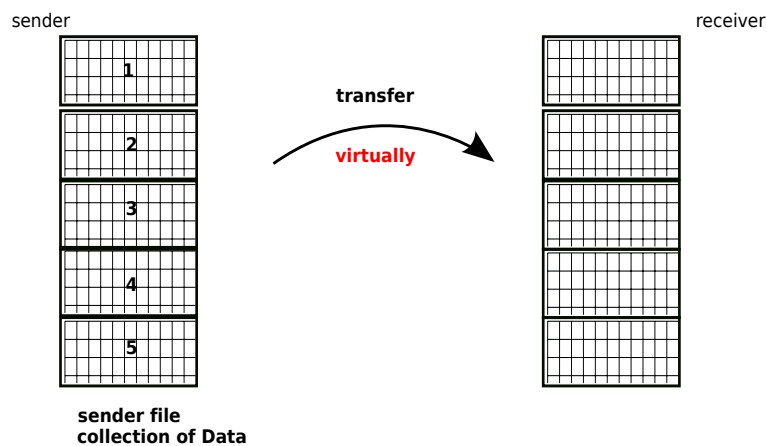


## Event-B Model - Example: File transfer protocol

Specification of a file transfer between two sites: a sender and a receiver.



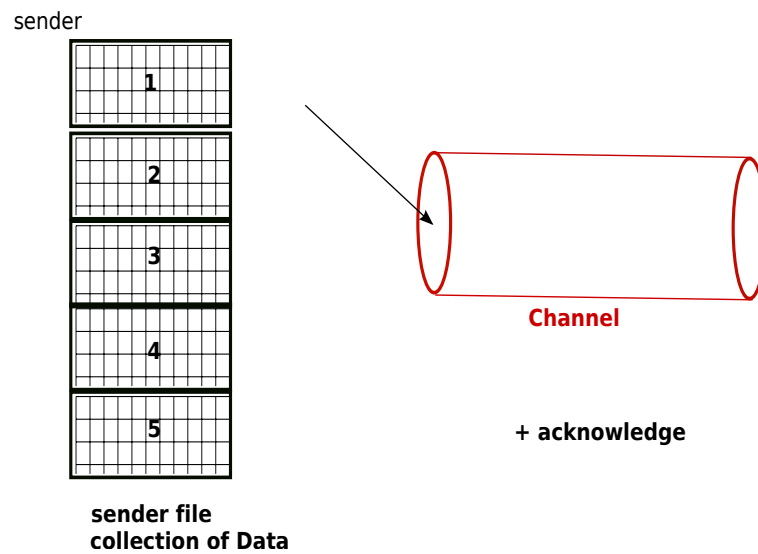
## Event-B Model - Example: File transfer protocol



A file is made of a set of data records.

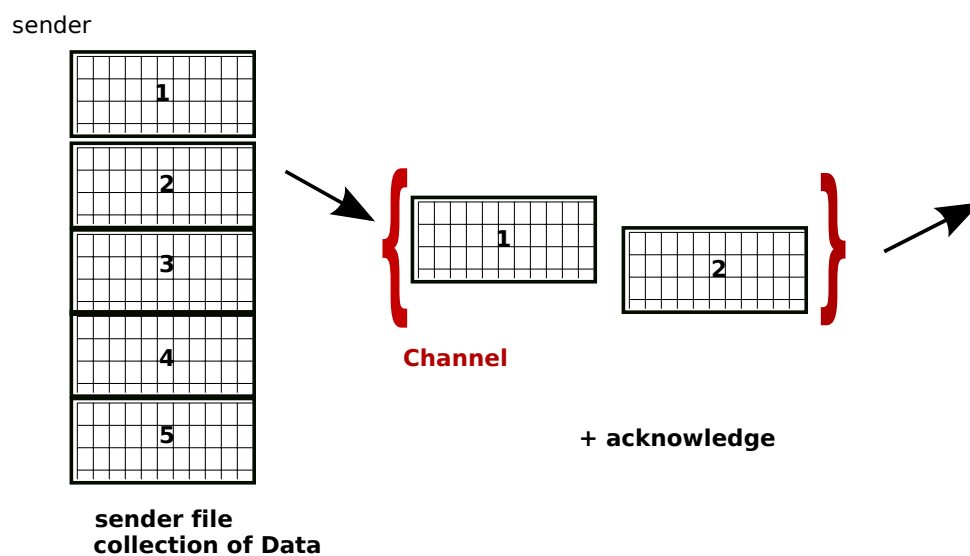
From a very abstract level, the transfer is done **instantaneously**.

# Event-B Model - Example: File transfer protocol



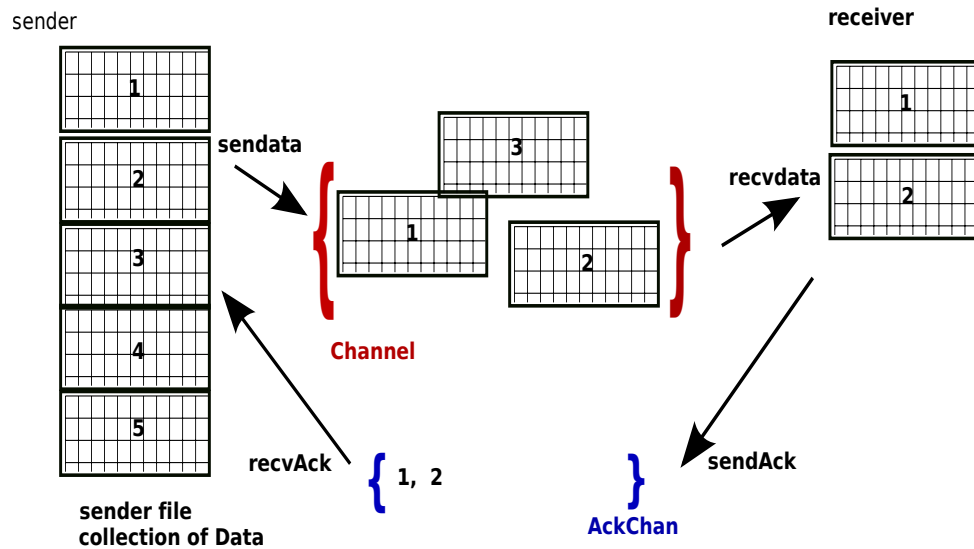
But, a file is made of a set of data records which are to be transferred through a channel.

# Event-B Model - Example: File transfer protocol



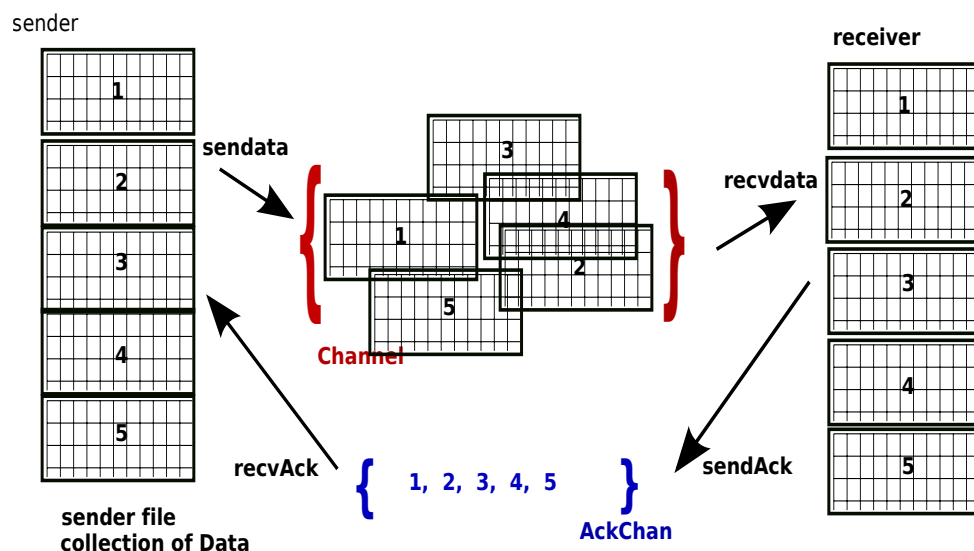
From a more concrete level, the transfer is achieved step by step, one record after the other.

## Event-B Model - Example: File transfer protocol



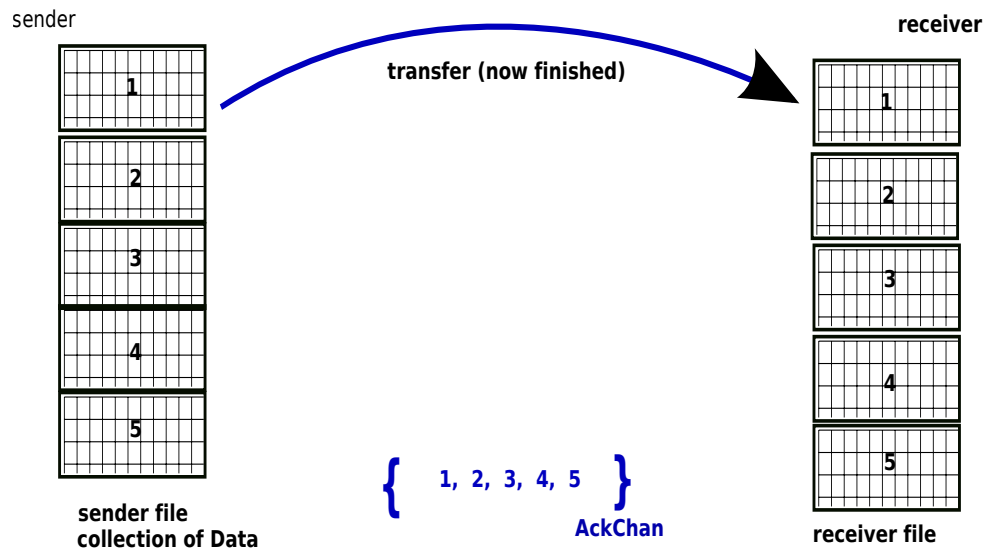
There are some intermediary operations, to **send data** on the channel from the sender side, to **receive data** from the channel from the receiver side. In the same way **acknowledgements** are sent/received.

## Event-B Model - Example: File transfer protocol



Only **after all the intermediary operations**, the transfer will be completed.

## Event-B Model - Example: File transfer protocol



## Event-B Model - Example: File transfer protocol

- Senderfile = some data records =  $1..nr \rightarrow DATA$   
 $\{1 \mapsto data1, 2 \mapsto data2, \dots\}$
- A channel is a set of such data records.
- At each time, the channel contains a part (set inclusion) of the sender's file
- The receiver acknowledges the received records numbers.
- The file transfer is completed when all the records are acknowledged.
- Failure: loss of data/ack in the channels.

**We have the model!**

# Event-B Model Example: File transfer protocol

```

MACHINE Transfer
SETS DATA
CONSTANTS nr /* file size : number of records
*/
PROPERTIES nr : NAT & nr > 1
VARIABLES
sf /* sender file */
, rf /* receiver file */

INVARIANT
& sf : 1..nr -> DATA /* all records of sf */
& rf : 1..nr +-> DATA /* probably part of
records of sf */
INITIALISATION
sf := {} || rf := {}

```

```

EVENTS
transf = /* instantaneous transfer, from far
way */
BEGIN
rf := sf
END

/* but, technically, we will need to anticipate
the intermediary events */
END

```



# Event-B Model Example: File transfer protocol

```

MACHINE Transfer
SETS DATA
CONSTANTS nr /* file size */
PROPERTIES nr : NAT & nr > 1
VARIABLES
sf /* sender file */
, rf /* receiver file */
INVARIANT
& sf : 1..nr -> DATA /* all records of sf */
& rf : 1..nr +-> DATA /* probably part of
records of sf */
INITIALISATION
sf := {} || rf := {}

```

```

EVENTS
transf = /* instantaneous transfer, from far
way */
BEGIN
rf := sf
END

/* the following events are introduced by
anticipation of the forthcoming gradual
refinement*/
; sendta = skip
; recdta = skip
; sendac = skip
; recvac = skip
/* the followings are events that simulate the
non-releiability of channels */
; rmvData = skip
; rmvAck = skip
END

```



# Event-B Model Example: File transfer protocol

```

REFINEMENT
Transfer_R1

REFINES Transfer

VARIABLES

cs /* current record to be sent */
, cr /* current record received */
, rf
, sf /* sender file */
, erf /* effectively received file */
, dataChan /* data channel */
, ackChan /* ack channel */
INVARIANT
cs : 1..nr+1 /* current to be sent */
& cr : 0..nr /* current received */
& cr <= cs /* current received is <= current
sent */
& cs <= cr+1 /* cr <= cs <= cr+1 */
& erf = (1..cr) <| sf
& dataChan <: (1..cs) <| sf
& ackChan <: 1..cr

```

```

INITIALISATION
cs := 1
|| cr := 0
|| rf := {}
|| sf := {}
|| erf := {}
|| dataChan := {}
|| ackChan := {}
EVENTS
transf =
WHEN
cs = (nr + 1) /* that is all cs are received
(last ack received) */
THEN
rf := erf /* not necessary, effective copy of
the received file in the receiver */
END

... (continued)
END

```



# Event-B Model Example: File transfer protocol

```

/* new events introduced (ie. we "forget" the
anticipation in the abstract model) */
; sendta =
WHEN
cs <= nr
THEN
dataChan(cs) := sf(cs)
/* now wait for the ack, before updating cs */
END

; recdta =
WHEN cr+1 : dom(dataChan)
THEN
erf(cr+1) := dataChan(cr+1)
|| cr := cr + 1 /* the next data to be received
*/
END

; sendac =
WHEN cr /= 0 /* send ack for the received cr
data */
/* may be observed repeatedly until the next
data */
THEN
ackChan := ackChan {cr}
END

```

```

recvac =
WHEN cs : ackChan /* ack for the already sent
cs */
THEN
cs := cs + 1 /* now the next to be sent */
END
/* Simulating non-reliability of channels,
data/ack may be loss */
; rmvData =
ANY ii, dd WHERE
ii |->dd : dataChan
THEN
dataChan := dataChan - { ii|->dd }
END
;
rmvAck =
ANY ii WHERE
ii : ackChan
THEN
ackChan := ackChan - {ii}
END

```



## Case Study : Multiprocess specification (Readers/writers)

- Description
  - Multiple processes: **readers, writers**
  - Shared resources between the processes
  - Several readers may read the resource
  - **Only one writer at a time**
- Property:
  - Mutual exclusion between readers and writers**
- Improvement:
  - no starvation** → as a new property  
(using refinements)

## Multiprocess specification

```

MACHINE
readWrite2
SETS
WRITER /* set of writer processes */
; READER /* set of reader processes */

VARIABLES
writers /* current writers */
, activeWriter
, waitingWriters
, readers /* current readers */
, waitingReaders
, activeReaders /* we may have svrl readers simultan. */

```

## Multiprocess specification

### INVARIANT

```
writers <: WRITER
& activeWriter <: WRITER & card(activeWriter) <= 1
& waitingWriters <: WRITER
& writers /\ waitingWriters = {}
& activeWriter /\ waitingWriters = {}
& activeWriter /\ writers = {}
/* merge */
& readers <: READER
& waitingReaders <: READER
& activeReaders <: READER & card(activeReaders) >= 0
& readers /\ waitingReaders = {}
& activeReaders /\ waitingReaders = {}
& activeReaders /\ readers = {}
/*-----safety properties -----*/
& not((card(activeWriter) = 1)&(card(activeReaders) >= 1))
```



## Multiprocess specification

### INITIALISATION

```
activeWriter := {}
|| waitingWriters := {}
|| activeReaders := {}

|| readers :: POW(READER)
|| writers :: POW(WRITER)
|| waitingReaders := {}
```





## Multiprocess specification

```

want2write = /* observed when a process wants to write */
ANY ww WHERE
ww : writers
& ww /: waitingWriters
& ww /: activeWriter
THEN
waitingWriters := waitingWriters \/ {ww}
|| writers := writers - {ww}
END
;
writing =
ANY ww WHERE
ww : waitingWriters
& activeReaders = {} & activeWriter = {}
THEN
activeWriter := {ww}
|| waitingWriters := waitingWriters - {ww}
END

```



## Multiprocess specification

```

endWriting =
ANY ww WHERE
ww : activeWriter
THEN
writers := writers \/ {ww}
|| activeWriter := {}
END
;
want2read =
ANY rr WHERE
rr : readers
& rr /: waitingReaders
& rr /: activeReaders
THEN
waitingReaders := waitingReaders \/ {rr}
|| readers := readers - {rr}
END

```



## Multiprocess specification

```

reading =
  ANY rr WHERE
  rr : waitingReaders
  & activeWriter = {}
  THEN
  activeReaders := activeReaders \/ {rr}
  || waitingReaders := waitingReaders - {rr}
  END
;
endReading =
/* one of the active readers finishes and leaves
the competition to the shared resources */
  ANY rr WHERE
  rr : activeReaders
  THEN
  activeReaders := activeReaders - {rr}
  || readers := readers \/ {rr}
  END

```



## Multiprocess specification

```

newWriter = /* a new Writer */
  ANY ww
  WHERE ww : WRITER
  & ww /: (writers \/ waitingWriters \/ activeWriter)
  THEN
  writers := writers \/ {ww}
  END
; leaveWriters = /* a writer leaves the group */
  ANY ww
  WHERE
  ww : writers
  THEN
  writers := writers - {ww}
  END

```



## Multiprocess specification

```

newReader = /* a new reader joins the readers */
ANY rr WHERE
rr : READER
& rr /: (readers\waitingReaders \/activeReaders)
THEN
readers := readers \/ {rr}
END
; leaveReader =
ANY rr WHERE
rr : readers & card(readers) > 1
THEN
readers := readers - {rr}
END

```

## Conclusion

- Initiation rapide à B et Event-B
- Reste à pratiquer, pratiquer, et pratiquer encore

## Event-B: Some References

- *Modelling in Event-B: System and Software Engineering*, J-R. Abrial, Cambridge, 2010
- *Modelling and proof of a Tree-structured File System*. Damchoom, Kriangsak and Butler, Michael and Abrial, Jean-Raymond, 2008.
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